研 究 速 報
UDC 624.04 : 519.65 : 534.5

A Note on Finite Element Synthesis of Structures (Part 2)
——Indeterminate Shift Synthesis of Vibration Eigenvalues and Eigenvectors——
有限要素法による構造シンセシスに関するノート (第2報)
——振動固有値・固有ベクトルの不確定シフト・シンセシス——

Shigeru NAKAGIRI* and Keiko SUZUKI*
中 桐　滋・鈴 木 敬 子

1. Introduction

Attention has been paid to the modification of both the eigenvalues and eigenvectors in problems of the optimization of dynamic responses of structures[1),2),3)].The vibration mode shapes are taken as the objective or constraint conditions in order to improve driving comfort or to decrease the noise of automobiles. An attempt was made to formulate the algorithm to determine the structural modification required to attain the objective mode shape on the basis of a new notion that the objective design is searched as near the baseline structure as possible[4)]. On the other hand, the objective mode shape hardly is set determinately on sure grounds. What is desired, for instance, is to enlarge or diminish some parts of the vibration modes to some extent, and there is left some fuzziness in the decision of designers. The fuzziness cannot be ignored in cases that the structural system has uncertain factors. Measurement errors and structural uncertainties are taken into account in problems of the system identification regarding to the modal analysis[5)]. It is rather easy to change some eigenvector components in far field, but difficult to change or set adequately the components in near field. When the objective components are set at important points, the realizable mode shape is determined of its own accord. It is therefore plausible to allow a certain latitude in setting the eigenvector components of limited number instead of setting many components determinately from the viewpoint of the eigenpair synthesis.

This paper presents a method to determine the structural modification required for the indeterminate shift of the eigenpair from a determinate baseline structure and deals with the implementation by the finite element method with respect to an undamped vibration eigenvalue problem. The method is based on the aforementioned notion. The approach to the objective design is approximated by the first-order sensitivities of the finite element solution, and is employed as the constraint conditions incorporated by the Lagrange multiplier method. The objective design is indeterminate corresponding to indeterminate setting of the objective eigenpair. The indeterminacy of the objective design is expressed in the form of the standard deviation of the structure shape.

*Dept. of Applied Physics and Applied Mechanics, Institute of Industrial Science, University of Tokyo

2. Statement of problem

Suppose that the stiffness matrix $[\overline{K}]$ and mass matrix $[\overline{M}]$ of the baseline structure or at a certain step of the iteration are given, and that the eigenvalue $\bar{\lambda}$ and eigenvector $\{\bar{\phi}\}$ of L degrees of freedom are known as the solution of the following real eigenvalue problem. The upper bar indicates the baseline or known terms.

$$([\overline{K}]-\bar{\lambda}\ [\overline{M}])\{\bar{\phi}\}=\{0\} \quad (1)$$

Any of the objective eigenvalue or eigenvector components are indicated by x_j ($j=1$ through J) generally. The expectations are denoted by x_j^*, and the indeterminacy of the objectives is represented by small probabilistic variables ε_j whose expectations are zero as given below.

$$x_j=x_j^*(1+\varepsilon_j) \quad (2)$$

The problem in this paper is to determine the variance of the objective design corresponding to the given x_j^* and statistical property of ε_j. The superscript $*$ indicates the expectation terms hereafter.

3. Iterative search using first-order sensitiveities

The structural modification is to be determined by the use of the well chosen design variables α_n ($n=1$ through N) in order to attain the indeterminate objectives x_j from $\bar{x}_j$ of the determinate baseline structure. Such strategy is taken that the design variables are determined to obtain the objective expectations x_j^* at first, and then the variation of the design variables is evaluated in the vicinity of the expectations for the indeterminate objectives. The shift of the eigenpair in a certain vibration order is dealt with in this study.

According to the change of the structural parameters described by the design variables, the eigenpair is changed. The change from the baseline structure to the objective design is approximated by the first-order Taylor series expansion in the form of Eq. (3).

$$x_j^* = \bar{x}_j + \sum_{n=1}^{N} x_{jn}{}^{\mathrm{I}} \alpha_n \tag{3}$$

The first-order sensitivities $x_{jn}{}^{\mathrm{I}}$ can be evaluated for $\bar{x}_j$ separately with respect to the design variables. The minimization of the squared sum of the design variables is employed for the determination of them so that the objective design is searched near the baseline structure. In doing so, the equations (3) are incorporated as the equality constraint conditions by means of introducing Lagrange multipliers μ_j in number J. The governing equation for α_n and μ_j is given on the basis of a functional of Eq. (4) and the differentiation of it with respect to them. The result is summarized in the form of Eq. (5),

$$\pi = \sum_{n=1}^{N} \alpha_n^2 + \sum_{j=1}^{J} \mu_j \left(x_j^* - \bar{x}_j - \sum_{n=1}^{N} x_{jn}{}^{\mathrm{I}} \alpha_n\right) \tag{4}$$

$$[H]\{y\} = \{W\} \tag{5}$$

where

$$[H] = \left[\begin{array}{ccc|ccc} 2 & & 0 & -x_{11}{}^{\mathrm{I}} & \cdots & -x_{J1}{}^{\mathrm{I}} \\ & \ddots & & \vdots & & \vdots \\ 0 & & 2 & -x_{1N}{}^{\mathrm{I}} & \cdots & -x_{JN}{}^{\mathrm{I}} \\ \hline & \text{SYM.} & & & 0 & \end{array}\right] \tag{5-1}$$

$$\{y\} = \left\{\begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_N \\ \hline \mu_1 \\ \vdots \\ \mu_J \end{array}\right\} \qquad \{W\} = \left\{\begin{array}{c} 0 \\ \hline \bar{x}_1 - x_1^* \\ \vdots \\ \bar{x}_J - x_J^* \end{array}\right\} \tag{5-2}$$

Such determination of α_n and μ_j should be iterated by renewing the baseline structure until $x_j^* = \bar{x}_j$ is obtained, because Eq. (3) is the first-order approximation of the change of the eigenpair.

4. Evaluation of structural indeterminacy

The objective design is indeterminate corresponding to the indeterminate objectives expressed by Eq. (2). An assumption is made that the indeterminate objective design can be expressed in the form of Eq. (6) based on the fluctuation of α_n and μ_j, which are obtained after the iteration stated in Section 3 is converged, with respect to ε_j. The vector $\{W\}$ is modified in the same form of Eq. (7).

$$\{y\} = \{y^*\} + \sum_{j=1}^{J} \{y_j{}^{\mathrm{I}}\} \varepsilon_j \tag{6}$$

$$\{W\} = \{W^*\} + \sum_{j=1}^{J} \{W_j{}^{\mathrm{I}}\} \varepsilon_j \tag{7}$$

The sensitivities $x_{jn}{}^{\mathrm{I}}$ of the objective design is indeterminate, but the indeterminacy of the matrix [H] is neglected in this study, because of the first-order approximation of x_j, so that the expectation matrix [H] of the objective design is used.

Then the indeterminacy of the objective design obtained after the iteration converged can be evaluated by means of determining the unknowns $y_j{}^{\mathrm{I}}$ in regard to ε_j. The governing equations for $y_j{}^{\mathrm{I}}$ are derived from the perturbation technique applied to Eq. (5) as follows.

$$[H]\{y_j{}^{\mathrm{I}}\} = \{W_j{}^{\mathrm{I}}\} \tag{8}$$

Now that the sensitivities of the design variables $\alpha_{nj}{}^{\mathrm{I}}$ are calculated as the upper part of $\{y_j{}^{\mathrm{I}}\}$, the variance of the design variables at the objective design is evaluated by the first-order approximation of Eq. (9),

$$\mathrm{Var}[\alpha_n] = \sum_{j=1}^{J} \sum_{k=1}^{J} \alpha_{nj}{}^{\mathrm{I}} \alpha_{nk}{}^{\mathrm{I}} E[\varepsilon_j \varepsilon_k] \tag{9}$$

where $E[\varepsilon_j \varepsilon_k]$ means the second-order moments of the probabilistic variables ε_j repersenting the indeterminacy of the objective eigenpair.

5. Numerical example

Figure 1 illustrates the finite element modeling of a truck chassis. Thirty beam elements of uniform length are used by taking the Young's modulus equal to 206 GPa and the uniform moment of inertia $\bar{I}=0.141\times10^{-4}\mathrm{m}^4$ for the baseline structure. The mass matrix is generated by the distributed load shown in Fig. 1, the self-weight of the chassis being neglected. The stiffness matrix is formulated under the assumption that the moment of inertia is distributed linearly within an element in order to express the beam shape with the elements of small number. Only the nodal monents of inertia are taken as the design variables as given below.

$$I_n=\bar{I}_n(1+\alpha_n) \tag{10}$$

The natural frequencies of the baseline structure are 4.42, 9.83 and 23.66 Hz up to the third order.

5.1 Case study of determinate shift

Figure 2 shows the fast convergence of the iteration (blank circles) in the case that the eigenvalue is increased to 120% and the deflection components of the eigenvector at the points A and B in Fig. 1 decreased to 80% of the baseline structure determinately. The solid circles in the figure mean the case that the constraint conditions are introduced in a quadratic form[4]. Such normalization condition of the eigenvector is taken that the deflection component at the point P is kept equal to unity. Figure 3 shows that two statge setting is needed to cover such a large decrease of the deflection component at the point A to 16% of the baseline value. The resultant mode shape and distribution of the moment of inertia are shown in Fig. 4. The design variables determined by Eq. (3) sometimes gives rise to unnatural prediction such as negative moment of inertia because of the first-order approximation in the process of the iteration when the set shift is too large. The difficulty can be overcome by means of setting the objective by

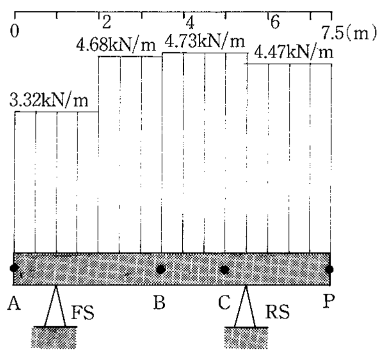

Figure 1 Baseline Structure of Truck Chassis Beam Model

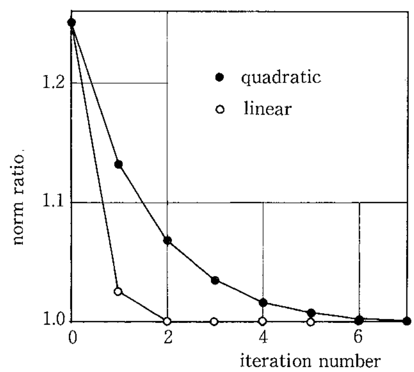

Figure 2 Comparison of Convergence of First-Order Sensitivity Based Schemes

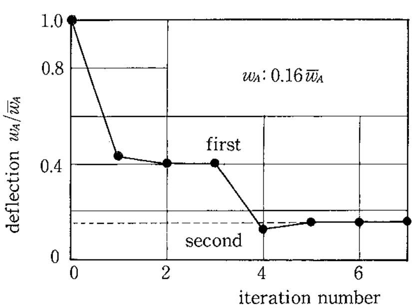

Figure 3 Convergence of Two-Stage Setting of Large Shift

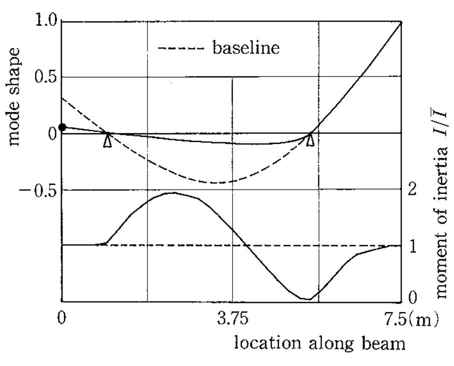

Figure 4 Mode Shape and Distribution of Moment of Inertia after Structural Modification

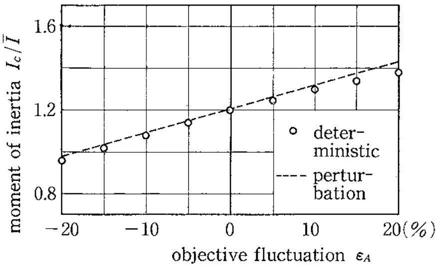

Figure 5 Deterministic Solutions and Perturbation Solution Regarding Objective Fluctuation

40% twice, while the moment of inertia is kept positive, no matter how small it is, as shown in Fig. 4. It can be said that the algorithm can cover the change of the objectives by 60%.

5.2 Structural synthesis for indeterminate shift

This section deals with the indeterminate setting of the eigenvector. Figure 5 shows the comparison of the moments of inertia at the point C for the increase of the deflection at the point A to 120% ($\varepsilon_A=0$) with the latitude of 20%. It is seen that the perturbation solution (broken line) by Eq. (8) can follow the solutions obtained by repeating deterministic synthesis for the deflection increase (blank circles). This evidences that the indeterminate objective design can be simulated by the first-order perturbation for the latitude of about 10% of the objectives. Figure 6 depicts the indeterminate distribution of the moment of inertia. The expectations of the objective deflection components are the increase to 120% at the point A and decrease to 80% at the point B. This causes the decrease of the natural frequency expectation to 3.62Hz. The latitude of the objectives is input as the coefficients of variation of ε_A and ε_B equal to 5%. The solid line in Fig. 6 means the expectation of the moment of inertia. The chain line and broken line are the one-sigma bounds (the standard deviation added to or subtracted from the expectation) for the fully correlated case and uncorrelated case of the probabilistic variables. It should be noted that the one-sigma bounds thus obtained have no information of the spatial correlation of the moment of inertia.

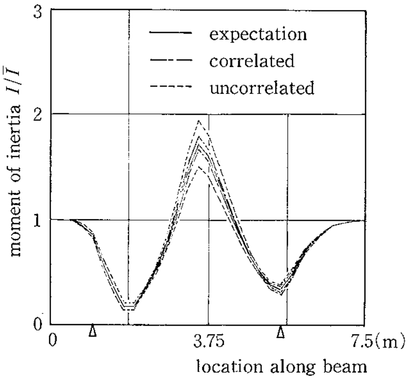

Figure 6 Indeterminate Distribution of Objective Moment of Inertia

6. Concluding remarks

A method of synthesis is presented to determine the structural modification with the latitude for the shift of the eigenpair prescribed in small number. This method is devised in order to overcome the difficulties of the structural synthesis for eigenpair arising from determinate setting of inadequate objectives in large number. An iterative algorithm is proposed for the synthesis on the basis of the first-order sensitivities of the finite element solution in conjunction with a new notion that the objective design is searched near the baseline structure. The effectiveness and fast convergence of the algorithm are evidenced by the numerical example of the shift of the vibration eigenvalue and eigenvector of the simple finite element model of a truck chassis.

(Manuscript received, February 3, 1988)

References

1) Vanhonacker, P., Differential and Difference Sensitivities of Natural Frequencies and Mode Shapes of Mechanical Structures, AIAA J., Vol. 18, No. 12 (1980), pp. 1511-1514.
2) Yamakawa, H. et al., Optimum Designs of Structures in Dynamic Response (2nd report, Optimum Design Method II), in Japanese, Trans. JSME, Vol. 48, No. 435 (1982), Ser. C, pp. 1750-1758.
3) Kerstens, J.G.M., Vibration of Modified Discrete Systems: The Modal Constraint Method, J. of Sound and Vibration, Vol. 83, No. 1 (1982), pp. 81-92.
4) Nakagiri, S. and Suzuki, K., A Note on Finite Element Synthesis of Structures (Part 1), "Seisan-Kenkyu", Monthly Journal of Inst. of Industrial Sci., University of Tokyo, Vol. 39, No. 11 (1987), pp. 460-463.
5) Ohkuma, M. and Nagamatsu, A., Research on Modal Analysis (6th report, Decision of Weighting Function in Curve Fit), in Japanese, Trans. JSME, Vol. 52, No. 484 (1986), Ser. C, pp. 3198-3206.