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A Note on Finite Element Synthesis of Structures (Part 1)
——Shift Synthesis of Vibration Eigenpairs——
有限要素法による構造シンセシスに関するノート（第1報）
——振動固有値・固有ベクトルのシフト・シンセシス——

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1. Introduction

A new approach is proposed for the strctural synthesis by means of the finite element method for the purpose to determine the structural parameters required to realize certain objectives of the structure under interest. It has been made easy to analyse the structural response with respect to the given set of the structural parameters by use of numerical methods such as the finite element method and boundary element method, owing to the fact that the uniqueness of the solution holds for linear systems. On the other hand, the structural synthesis in the sense of the determination of the structural parameters for the given set of objective response is still a difficult matter to cope with in inverse problems of structures, because that a same response can be attained by different combinations of the structural configuration and parameters.

A number of literatures have been published so far on the techniques of structural optimization[1),2),3),4),5)]. The basic notion employed to develop the techniques is to determine the design variables so that a functional, usually comprised of the error defined somehow in regard to the difference between the objective design and baseline design, is minimized. This notion seems to result in many numbers of the iteration required until the design variables converge to the objective design. The reason is that the design variables are likely to be diversified in the course of the error minimization. This note presents two formulations of the shift synthesis based on the notion that the design variables are determined so that the objective design is sought as near the baseline design as possible while the constraint conditions set for the attainment of the objective design is imposed in terms of the Lagrange multiplier method. The numerical example is concerned with the determination of the bending stiffness to realize the determinate shift of the eigenvalue and eigenvector of undamped flexural vibration of beams.

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2. Statement of problem

Suppose that a baseline design or prototype design is well identified so that the stiffness matrix $[\overline{K}]$ and mass matrix $[\overline{M}]$ are known together with the eigenvalue $\overline{\lambda}$ and eigenvector $\{\overline{\phi}\}$ as the solution of the following eigenvalue problem Eq. (1).

$$([\overline{K}]-\overline{\lambda}[\overline{M}])\{\overline{\phi}\}=\{0\} \qquad (1)$$

The upper bar indicates the values for the baseline design hereafter. The change of the eigenpairs in the vicinity of the baseline design can be approximated by means of the Taylor series expansion with respect to the design variables α_n and truncated at the first order of them. The problem is to determine the design variables of N in total number to attain the objective eigenpairs of a certain mode.

3. Formulation based on vectorical constraint

Suppose that the objective eigenvalue λ and eigenvector components of l in number $\{\phi\}_l$ are prescribed. A functional Π_1 is introduced in the form of Eq.(2) so that an objective function comprised of the sum of squared design variables is to be minimized under the conditions of a scalar constraint for the eigenvalue and a vectorical constraint for the eigenvector by use of two Lagrange multipliers μ_1 and

μ_2. This formulation means that the objective structure is sought as near the baseline design as possible and the objectives of λ and $\{\phi\}_l$ are attained by the first order approximation of the eigenpair change.

$$\Pi_1=\sum_{n=1}^{N}\alpha_n^2+\mu_1(\lambda-\bar{\lambda}-\sum_{n=1}^{N}\lambda'_n\,\alpha_n)^2+\mu_2\lfloor\varepsilon\rfloor\{\varepsilon\} \tag{2}$$

In the above, $\{\varepsilon\}$ indicates a vector of l degrees of freedom corresponding to the difference between the objective vector components and the baseline ones as given below.

$$\{\varepsilon\}=\{\phi\}_l-\{\bar{\phi}\}_l-\sum_{n=1}^{N}\{\phi_n^I\}_l\alpha_n \tag{3}$$

The first order rates of change, in other word, sensitivities of the eigenvalue λ_n^I and eigenvector $\{\phi_n^I\}_l$ with respect to the design variables α_n can be computed on the basis of the first order perturbation technique[6].

4. Formulation based on scalar constraints

In this formulation, the objective eigenvector components of l in number are dealt with individually so that all the objectives including the eigenvalue are indicated as x_j standing for scalar vaiables of $L=1+l$ in total. The functional Π_2 in this formulation is constituted by the same notion of minimizing the change of the design variables as given in Eq. (4) by use of the Lagrange multipliers of L in number. In Eq. (4), x_1 indicates the objective eigenvalue and x_j ($j=2\sim L$) the objective eigenvector components.

$$\Pi_2=\sum_{n=1}^{N}\alpha_n^2+\sum_{j=1}^{L}\mu_j(x_j-\bar{x}_j-\sum_{n=1}^{N}x_{jn}^I\,\alpha_n)^2 \tag{4}$$

5. Determination of α_n and μ_j

The design variables α_n and Lagrange multipliers μ_j can be determined by the following conditions so that the aforemmentioned functional Π is minimized.

$$\frac{\partial\Pi}{\partial\alpha_n}=0 \qquad n=1\sim N \tag{5}$$

$$\frac{\partial\Pi}{\partial\mu_j}=0 \qquad j=1\sim L \tag{6}$$

The result is summarized in the matrix form of Eq. (8) for the case of Eq. (4) by use of the first order approximation of the squared term in the form of Eq. (7) wiith respect to α_n in order to linearize Eq. (5) in regard to α_n and μ_j

$$\sum_{j=1}^{L}\mu_j\ \{(x_j-\bar{x}_j)^2-2(x_j-\bar{x}_j)\sum_{n=1}^{N}x_{jn}^I\,\alpha_n \tag{7}$$

$$\left(\begin{array}{cccc|c} 2 & & & & \\ & 2 & & & -2(x_j-\bar{x}_j)x_{jn}^I \\ & & \ddots & & \\ & & & 2 & \\ \hline \text{SYM.} & & & & 0 \end{array}\right)\left\{\begin{array}{c}\alpha_1\\ \vdots\\ \alpha_N\\ \hline \mu_1\\ \vdots\\ \mu_L\end{array}\right\}=\left\{\begin{array}{c}0\\ \vdots\\ 0\\ \hline -(x_1-\bar{x}_1)^2\\ \vdots\\ -(x_L-\bar{x}_L)^2\end{array}\right\} \tag{8}$$

The values of α_n and μ_j obtained as the solution of Eq. (8) are not exact because that the first order approximation is employed to estimate the eigenpair change and to calculate the functional. It is necessary to repeat the procedure by means of changing the baseline design obtained by the current values of α_n until the solution of $\alpha_n=0$ for $x_j=\bar{x}_j$ is obtained.

6. Numerical examples

Two numerical examples are given to show the comparison and validity of the formulations in regard to the vibration of a cantilever and that of a truck chassis modeled by beam elements. The moment of inertia of the section of the beam is taken as the design variables while the other parameters such as the beam element length, cross-sectional area of the beam, Young's modulus and mass are kept constant.

A stiffness matrix for the beam bending is formulated newly by taking the moment of inertia I at two nodes as variable and assuming the linear distribution of the moment of inertia in an element for the purpose to chase the change of the beam shape by the elements of small number. It turns out that the change of the moment of inertia at a node causes the change of the stiffness matrices of the elements on both side of the node.

Cantilever beam

The first numerical example is concerned with a cantilever of 100mm in length and 415 Hz in the natural frequency for the first mode of the initial design with the uniform moment of inertia[7]. The problem aimed at is to reduce the eigenvalue to $\lambda=0.5\bar{\lambda}$ by use of five finite elements with six design variables of the nodal moment of inertia. Figure 1 shows the convergence of the eigenvalue due to the structure modification based on the formulation stated in section 3. It is seen that the objective eigenvalue of $0.5\bar{\lambda}$ can be attained by small iteration number of eight. Figure 2 shows the convergence to the objective eigenvector components of $w_a=0.9\bar{w}_a$ and $w_b=0.9\bar{w}_b$ for the two points of A and B indicated in Fig. 1. All the eigenvector is normarized so that the largest deflection component of the eigenvector remains unity in this study. This figure is obtained by the formulation based on the vectorical constraint of $\lfloor \phi \rfloor_t = \lfloor w_a, w_b \rfloor$. This formulation gives rise to meandering convergence and requires many iteration numer. Figure 3 shows the result of the same shift obtained by the formulation based on the scalar constraints stated in section 4. It is obvious that the scalar type formulation is superior to the vector type one due to the fast and monotonic convergence.

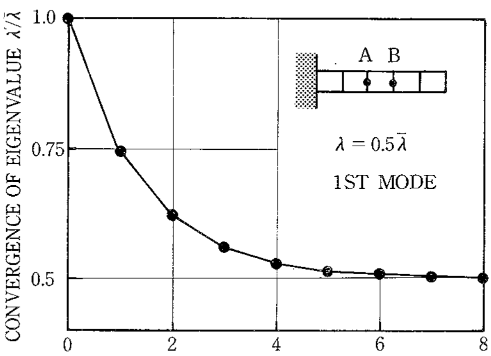

Figure 1　Convergence of eigenvalue due to structure modification

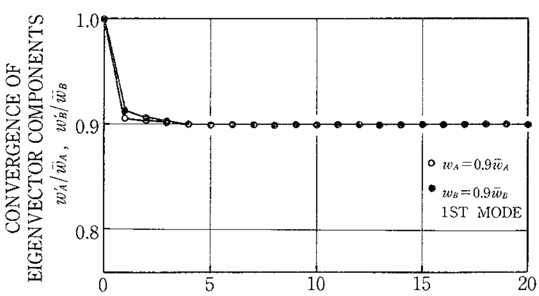

Figure 3　Convergence of eigenvector components due to structure modification (scalar constraints)

Truck chassis model

The second example deals with the flexural vibration of a truck chassis modeled by use of 30 elements simply supported at two points. The initial momement of inertia $\bar{I}$ is taken equal to 0.141×10^{-4} m^4, and the weight loaded on the chassis is 64.1 kN resulting in the natural frequencies of 4.42 and 9.83 Hz for the first and second modes [7]. The mode shape and resulted distribution of the moment of inertia corresponding to the eigenveector shift of $w_a=0.8\bar{w}_a$ and $w_b=0.8\bar{w}_b$ are shown in Fig. 4. Those with the additional shift of the eigenvalue to $1.2\bar{\lambda}$ are shown in Fig. 5.

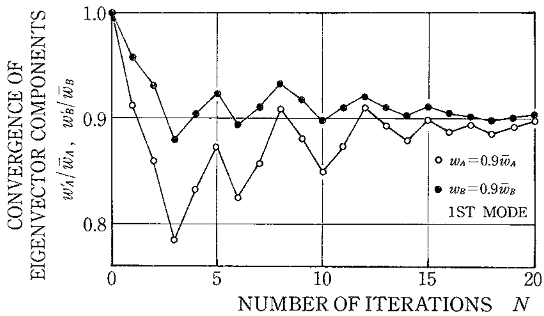

Figure 2　Convergence of eigenvector components due to structure modification (vectorical constraint)

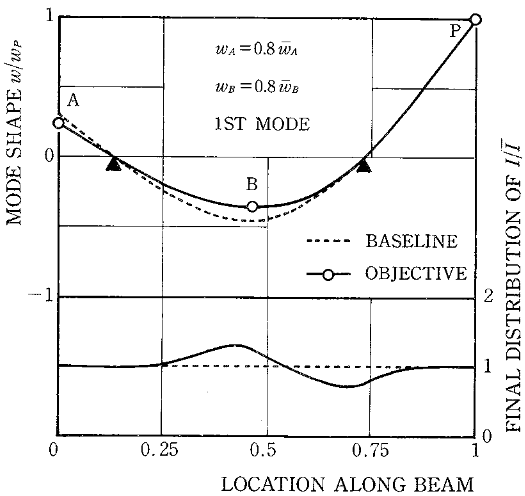

Figure 4　Mode shape and distribution of moment of inertia after structure modification for eigenvector change only

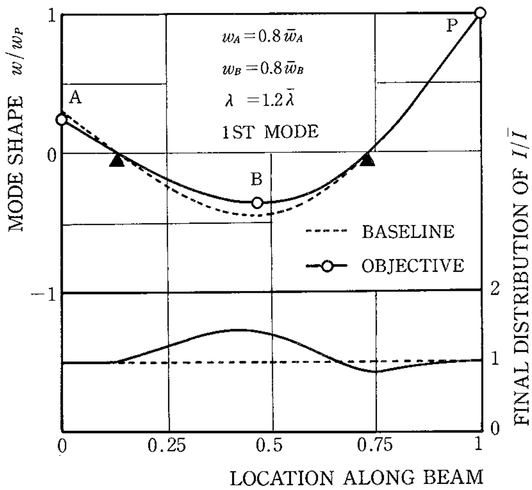

Figure 5 Mode shape and distribution of moment of inertia after structure modification for eigenpair change

7. Concluding remarks

This note presents two formulations for the shift synthesis, by which the structural modification required for the given shift of the eigenpairs of undamped linear vibration systems is obtained, on the basis of the finite element modeling and sensitivity study. The basis of the formulation is to seek the objective design near the baseline one while the response change to the objective design is chased by the first order approximation regarding the design variables, which is employed as the constraint conditions and incorporated in terms of the Lagrange multiplier method. It is shown that the formulation based on the scalar constraints results in the fast and monotonic convergence.

The fast convergence implies that the formulation can be applied to large scale problems. It is worthy to note, however, that the combination of the objective eigenpair should be chosen judisciously as well as the design parameters. Even in the simple cases described herein, the structural modification is unable when the deflection and rotation components at two adjacent nodes are prescribed randomly so that the smooth shape of the eigen mode is violated. It is therefore recommendable to choose the eigenvector components of limited number and to assign indeterminacy to the objective eigenpairs in case of large scale problems.

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