特 集 1
研究解説
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Large Eddy Simulation at Very High Reynolds Numbers
高レイノルズ数におけるラージ・エディ・シミュレーション

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Reynolds numbers of turbulent flows in large-scale regions are very high. In their large eddy simulation, sufficiently narrow filters cannot be adopted owing to the limitation of computer resources. Coase filters give rise to various difficulties concerning the choice of model constants and the imposition of noslip boundary conditions. Some methods for resolving such difficulties are provided.

I. Introduction

Wide varieties of approaches to simulating turbulent flows have been developed over the past twenty years. In the most fundamental approach, full or direct turbulent simulation (FTS), the Navier−Stokes equations are solved numerically; the results contain the complete details of the flow. Large eddy simulation (LES) solves filtered Navier−Stokes equations and yields the large scale components of the fluid motion. In time−or Reynolds−averaged Navier−Stokes (RANS) calculations (also called one point closures), the Navier−Stokes equations are averaged over an ensemble of realiztions of the flow.

Among these methods, FTS is essentially exact (if careful attention is paid to numerical methods), LES requires modeling for the small scales (which may contain anywhere from 10 to 90 percent of the kinetic energy of the turbulence) and RANS computations demand modeling of all the turbulent motions. Because turbulence models are uncertain, as the fraction of the turbulence represented by a model increases, the accuracy expected of a calculation decreases. On the other hand, the more exact approaches require considerably more computation time and expense; these are often beyond what most users can afford.

For practical reasons, one should use the least expensive method capable of producing the desired accuracy. However, as the preceding paragraph shows, there is a trade−off between cost and accuracy so one may be forced to choose between a method that is not accurate enough and one that is too expensive. Because RANS models are not always sufficiently accurate, LES has been suggested as an alternative to them. In this paper, we shall investigate issues connected with LES of very high Reynolds number flows, for which filtering removes appreciable part of the energy of the turbulence. This is an important topic because, in many applications, it is essential to predict the (steady and unsteady) forces that arise from the largest scale turbulent motions. Fields with interest in this subject include architectural aerodynamics, unsteady aircraft aerodynamics and meteorology, among others. LES has been applied to all of these areas. There are important differences between this type of calculation (eg. Murakami et al, 1985) and the kind of LES used to investigate the fundamental physics of turbulent flows (eg. Moin and Kim, 1982). It is these differences that we intend to focus on.

II. Filtering

In the standard approach to LES, the part of the velocity field to be resolved is defined by spatial filtering; i.e. the filtered velocity field $\bar{u}$ (written

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in one dimention) is :

$$\bar{u}(x)=\int G(x,x';\Delta)u(x')dx' \qquad (1)$$

which may be thought of as a running average of the velocity, u, over a volume whose linear dimention is the filter width, Δ. For present purposes, the choice of filter, G, is not important. However, it is important to note that, although they may be quite anisotropic, the small structures or eddies are similar in all three demensions and time. Consequently, filtering in any one dimention (or time) tends to remove the components of the velocity field which vary rapidly in the other directions and in time.

The primary issue we want to investigate is what happens when the width, Δ, of the filter becomes large. It would be useful if, in this limit, the LES equations reduce to RANS equations. Unfortunately, this does not happen in the usual formulation of LES, because filtering on such a large scale also smoothes the mean velocity field. The limit we desire will obtain if the flow is homogeneous in at least one spatial direction and is filtered only in that direction. However, we prefer to avoid this restriction.

An alternative, especially for statistically steady flows, is to filter in time. Although this approach has been mentioned a number of times, it has not been out into pratice because it is computationally inconvenient. Furthermore, as noted above, spatial filtering removes the small time scale fluctuations of the velocity field so time filtering is unnecessary when spatial filtering is used. For our purposes, it is convenient to adopt temporal filtering for nominally steady flows because this allows a Reynolds averaged formulation to emerge as a limit of LES.

The ideal method of deriving the RANS equations is via ensemble averaging ; the need for spatial or temporal averaging is thereby eliminated and the difficulties described above are avoided. Unfortunately, as LES is designed to reproduce a single flow realization, ensemble averaging cannot serve our purpose.

III. Smagorinsky Model

Now let us consider what happens when the width of the filter is increased. As suggested above, we shall think in terms of time filtering but, because they are better developed, we will use ideas derived from the spatial filtering formulation.

In the limit of zero filter width, LES reduces to FTS and no modeling is required. As the width of the filter is increased, some of the small scale motions become part of the unresolved or subgrid scale field and a model must be introduced to account for them. Eddy viscosity models have been widely used for this purpose ; they represent the subgrid scale Reynolds stresses, in terms of the large scale field by :

$$\tau_{ij}=-\nu_T\left(\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\partial\bar{u}_j}{\partial x_i}\right)=-2\nu_T S_{ij} \qquad (2)$$

where ν_T is the subgrid scale eddy viscosity. In turn, the eddy viscosity can be written :

$$\nu_T=C_S{}^2 q\Delta \qquad (3)$$

where q is a velocity scale of the subgrid motions and, Δ is the filter width, the appropriate length scale for the subgrid motions and C_S is the single model parameter.

There remains the problem of determining the scale velocity, q. For this purpose, the familiar argument relating the energy drain to the velocity and length scales can be used. At the high Reynolds numbers under consideration here, viscosity is not important in the larger subgrid scales and we have :

$$\varepsilon=\frac{Q^3}{L}=\frac{q^3}{\Delta} \qquad (4)$$

where Q is the velocity scale of total turbulent kinetic energy and L the integral or large scale of the turbulence. Combining these equations with

$$Q=L(S_{ij}S_{ij})^{\frac{1}{2}} \qquad (5)$$

which is derived by equating production and dissipation, gives :

$$\nu_T=C_S{}^2\Delta^{4/3}L^{2/3}(S_{ij}S_{ij})^{1/2} \qquad (6)$$

which is a version of the Smagorinsky model. In equation (5), ensemble averaged strain is approximated by Sij in equation (2) since the filter width is large. We have given the spatial filter form of the model but it is not difficult to convert it to a form appropriate to temporal filtering i.e. Δ is replaced by $q\Delta t$.

In the form of the model given by Eq. (6), the length scale is $(L\Delta^2)^{1/3}$ rather than the more commonly used Δ. In most simulations, the difference is not of great importance because the ratio L/Δ is approximately constant. However, (6) should require a different model constant.

IV. Broad Filters and Reynolds Averaged Models

In the limit of a very broad temporal filter, all of the turbulent kinetic energy lies in the unresolved motions. LES then reduces RANS. Eddy viscosity (Boussinesq) models have long been used in RANS. These models have the form (2), where u is the time mean velocity (the infinite width time−filtered velocity). Furthermore, the eddy viscosity takes form similar to (3):

$$\nu_e = C_\mu QL \tag{7}$$

where Q is the velocity scale of the complete tubulence field and L is a length scale usually taken to be the integral scale.

In writing equation (7), we are implicitly assuming that a method of computing the length scale is available. The method of doing this is of the major difficulties in turbulence modeling; a number of such procedures have been proposed. Of these, the zero- and two-equation models are far and away the most popular. The latter has been preferred in recent years because it is easier to apply to a wide range of flows. However, for the purposes of this paper, it is sufficient to assume that the length scale is somehow known i. e. we shall assume that the model for the length scale is accurate.

Thus, for temporal filtering applied to statistically steady fows, the limits of the LES equations when the filter width becomes large are just the familiar RANS equations. The only important difference is that the length scale in the eddy viscosity formula is not the filter width but a turbulence length scale. Under these circumstances, the model length (or time) scale can become smaller than the filter width.

We note in passing that, when eddy viscosity models are used in RANS calculations, the effect of the model is to replace the actual high Reynolds number flow by an effective low Reynolds number flow; however, the eddy viscosity is not spatially constant. Indeed, the objective of turbulence modeling is the determination of an eddy viscosity distribution which yields the mean velocity field of the turbulent flow i.e. the goal is to produce an effective laminar flow that has the tubulent mean velocity profile.

Experience shows that this is not easy. If too little viscosity is introduced, the computed flow may not be stable and an unsteady or divergent solution may be obtained when a steady one is expected. While the problem may be due to the numerical methods, insufficient effective viscosity is a common cause of the problem. In other words, one may be performing a large eddy simulation when a time−average calculation is intended.

Most importantly, let us note that, assuming it leads to a steady solution, the eddy viscosity of Eq. (7) must be an upper bound to the eddy viscosity that can be used in a large eddy simulaiton. This follows from the fact that, by definition, this eddy viscosity is large enough to remove all of the turbulence from the resolved scales and leave a steady resolved flow. Since, in this limit, the filter width is larger than any scale of the turbulence, the eddy viscosity of Eq. (7) can become also smaller than that given by Eq. (6).

An interesting situation occurs in simulations in which the cutoff wavenumber is below the inertial subrange (or the filter width is larger than the integral scale). Some of the turbulence remains unresolved so the simulated flow should be unsteady. However, the eddy viscosity must be smaller than that provided by the Smagorinsky model. This is usually accounted for by reducing the model constant but the above argument suggensts that using the integral scale rather than the filter width or grid size as the length scale in the model would be more appropriate.

In most RANS calculations, the grid size used in the numerical solution is smaller than the model length scale; this is required for numerical accuracy. In this case, the Smagorinsky eddy viscosity based on the grid size (rather than the filter size) is smaller than the eddy viscosity provided by a RANS model. This is consistent with the point made above i.e. that the principal feature distinguishing LES from RANS is the magnitude of the eddy viscosity. Indeed, one may be able to use the same numerical code for both types of calculations by varying the eddy viscosity.

V. High Reynolds Number Large Eddy Simulation

Now suppose that the intent is to simulate very high Reynolds number flows. Such flows may contain very large eddies which produce forces that oscillate on relatively long time scales; it may be desirable to explicitly simulate these eddies and the forces they generate. For example, we might want to simulate the large unsteady vortices and forces generated when a boundary layer flows over an obstacle such as a building. (There are several different types of vortices in such flows.) Meteorology provides other examples; in one obvious case, it is essential to track the progress of a storm without explicitly simulating the smaller scale motions. Still another example is the flow within the cylinder of an internal combustion engine; the vortices differ from one cycle to the next and it may be important to capture this variation by doing several LES realization.

In these examples, LES is employed because it can simulate the important large eddies but cost considerations and abilties of the computer do not permit the use of full simulation. In simulations of these flows, a large fraction of the turbulent kinetic energy resides in the subgrid scales. The filter width is comparable to the grid size rather than being much larger than it as is the case for RANS eddy viscosity models, but it is also comparable to the integral scale of the turbulence. The arguments about the scaling of the subgrid scale made earlier no longer apply.

Because the filter width is now comparable to the integral scale of the turbulence and the wavenumber cutoff is now near the peak of the energy spectrum, the velocity scale should no longer vary with the filter size. The Smagorinsky eddy viscosity can take the form:

$$\nu_\tau = C_S^2 L^2 (S_{ij} S_{ij})^{1/2} \qquad (8)$$

for these flows. Again, it is possible that, if only a small range of flow parameters is used, the difference will not be noticed.

As noted earlier, it then follows, that for the simulation of very high Reynolds number flows, the subgrid scale eddy viscosity should be smaller than that predicted by the usual Smagorinsky model. The fact that a number of authors have found it necessary to reduce the value of the Smagorinsky coefficient may be related to this observation. Indeed, many simulations including meteorology (Deardorff, 1974), channel flow (Horiuti, 1986) and architectural flows (Murakami et al, 1985) have used $C_S = 0.1$ while homogeneous flows and theory suggest that 0.2 is more appropriate. In fact, as the Reynolds number is increased further, a still smaller value of C_S may be appropriate.

VI. Wall Conditions

In treating wall bounded flows, one has to choose between resolving the wall region by including no−slip boundary conditions at the surface and removing the wall region from consideration by applying artificial boundary conditions at some distance from the wall. In the latter case, the artificial condition is usually applied in the logarithmic or buffer region of the boundary layer. The first type of simulation is preferred when the purpose is to study the dynamics of turbulence production near walls while the second type is preferred in engineering applications aimed at computing a few important integral effects.

In simulations with no−slip boundary conditions, the Smagorinsky model is usually modified near the wall. This modification usually takes the form of reducing the length scale by a van Driest damping factor:

$$L = y(1 - \exp(-y/A))^2 \qquad (9)$$

where y is the distance to the wall and A is a "sublayer thickness": other forms of the damping factor have been proposed. The van Driest factor was originally introduced into RANS models in order to account for the reduction in the turbulence length scales near the wall. It is difficult to justify this factor in LES ; indeed, it may not be correct in RANS simulations (Norris and Reynolds, 1975). The philosophy of LES suggests that the distance to the wall should not be introduced as directly as Eq. (9) suggests ; it is difficult to justify the presence of an integral parameter such as the sublayer thickness in the model.

Actually, it is the turbulence intensity (unresolved or total) that is reduced near the wall. Relationships (2) and (6) are no longer correct. They overpredict the subgrid scale Reynolds stress and need to be modified near the wall. No simple method of doing so has yet been suggested. In the absence of such a model, we suggest use of a turbulent kinetic energy subgrid scale model in calculations that include no−slip conditions.

For simulations that use artificial boundary conditions, the best such condition is due to Schumann (1973). It assumes proportionality between the velocity at the point at which the condition is imposed and the shear stress at the wall, τ_w :

$$u(x,y_1)=\langle u(y_1)\rangle \tau_w(x)/\langle \tau_w\rangle \qquad (10)$$

where ⟨ ⟩ indicates an average of a quantity over a plane parallel to the wall. The velocity in the direction normal to the wall is assumed to be zero. However, the last assumption cannot be correct because sweep and burst events, which involve considerable motion normal to the wall, are known to carry most of the shear stress in the region in which the condition is to be imposed. Thus, improved boundary conditions should be sought ; Piomelli et al (1986) have proposed such models but have not yet tested them.

VII. Conclusions

We have shown that a number of approaches to the simulation of turbulent flows, including full simulation, large eddy simulation and one point closures, can be regarded as members of a continuum of methods parameterized by the fraction of the turbulence resolved. This viewpoint suggests that large eddy simulations of very high Reynolds number flows may require a different subgrid scale models than lower Reynolds numbers flows. It also shows that the differences between RANS and LES may not be as great as some authors suppose. Finally, further development of the artificial boundary conditions usually applied in very high Reynolds number LES is needed.

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