

Some Perspectives on Turbulence Modeling

乱流モデリングの展望

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An attempt to improve existing Reynolds-averaged models is given. In this method called a zonal approach, the flow region dealt with is divided into several subregions. Different sets of model constants are adopted in each subregion, leading to good estimate of turbulence quantities.

I. Overview

The facts that nearly all flows of interest to engineers are turbulent, at least in part, and that the phenomena involved in turbulent flows are highly complex and nonlinear, give rise to the need for turbulence models. A wide range of models have been proposed over the years; a classification scheme for them was given by Bardina et al (1981).

A prime difficulty in turbulent flows arises from the fact that they are neither entirely deterministic nor entirely chaotic. We believe that if either were the case, the turbulence problem would have yielded to analysis by this time. In fact, despite over a hundred years of effort devoted to the problem, we are still a long way from having reliable methods of predicting turbulent flows with satisfactory accuracy.

Relatively simple flows, such as those in pipes and over flat surfaces, can be described entirely in terms of one or, perhaps, a small number of parameters such as the Reynolds number based on a single geometric length scale. For these, correlations can be developed and, when available, correlations are the best way to predict a turbulent flow. They are not useful, however, for problems involving many parameters; unfortunately, nearly all problems in high technology require several parameters to define the geometry. Integral methods were developed

specifically to handle flows in complex geometries and are an excellent choice for problems to which they are applicable. In this case, the principal difficulty is that the correlations used in integral methods need to be reworked each time a new complication is added. This requires new experimental data (or, perhaps, new simulations) and considerable time. The need for a more universal method should be obvious.

In an attempt to fill this need, a number of one point closure models have been proposed and developed over the past fifteen years. It has been hoped that these models could be universal i.e. that one model would suffice to predict all flows, but there is no a priori reason why this should be the case. It has been demonstrated that a single one point closure model can predict a wider range of flows than any correlation or integral method but complete universality has not been demonstrated and, in the author's opinion, it is unlikely that a sufficiently accurate universal model can be constructed. Indeed, we shall show below that such a model would need to be very complex and difficult to use.

The variation in the physics of a flow from one region to another suggests that it might be wise to tie the modeling to the local physical nature of the flow. That is, it may be useful to use one model in a region that resembles a free shear flow and another in a boundary layer-like region. This procedure simplifies the process of finding models (and the models themselves) but requires a method

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of tying the regions together. Finding methods for these readjustments may be as difficult as finding the models for the individual regions. In this paper we shall show, for a few simple cases, how this might be accomplished.

II. The Case Against Universal Modeling

In the preceding section, we stated an opinion that the construction of universal models for turbulence is improbable. Here, we would like to present some evidence for this statement. In particular, we shall demonstrate some difficulties with the $k-\epsilon$ and Reynolds stress models. The argument will be made for homogeneous turbulent flows. It may be argued that these flows are not representative of engineering flows, but recent work by Rogers and Moin (1986) has shown that the process of turbulence production in homogeneous shear flow is essentially the same as that found in many inhomogeneous shear flows. Although this is only one case, we believe that it validates the argument that a turbulence model ought to apply to homogeneous as well as inhomogeneous flows. We will deal with homogeneous flows because they are simpler.

Bardina et al (1981) performed full simulations of isotropic turbulence subjected to solid body rotation. In this flow the Reynolds stress tensor remains isotropic and the $k-\epsilon$ and Reynolds stress models are essentially identical. The standard $k-\epsilon$ model predicts no effect of rotation on the rate of decay of turbulence while numerical simulations clearly show that the decay rate is reduced under the influence of rotation. This effect may be accounted for by adding a term to the model dissipation equation.

The situation is even more serious for isotropic turbulence subjected to uniform compression; this case was considered by Wu et al (1985). Again the Reynolds stress tensor remains isotropic but the length scales of the turbulence are reduced by the action of the compression; this may be a consequence of the low Reynolds numbers used but the effect appears to be independent of Reynolds number over the range studied. At the same time the dissipation is reduced because the viscosity is decreased during the compression. In this case, the $k-\epsilon$ model fails because the dissipation cannot play the dual role of determiner of the length scale and

destruction rate of the turbulent kinetic energy. It is apparently able to handle this dual role in most shear flows.

Further evidence against present turbulence models was presented by Lee and Reynolds (1985). They made full simulations of homogeneous flows subjected to various types of irrotational strain. Among other things, they found that the nature of the return to isotropy following the removal of the strain depended on the type of strain to which the flow had been subjected. In other words, the relaxation to isotropy of the flow cannot be described entirely in terms of the anisotropy of the Reynolds stress tensor. This means that more variables must be introduced into the model, making it very complex. They suggested that a model using twenty two variables might suffice but did not construct a complete model of that type. Furthermore, there is no guarantee this model would work for more complex flows without further modification.

Finally, it is important to note that, just as the choice between correlations, integral methods, and one point closure methods may depend on the kind of problem being solved, the adequacy of the $k-\epsilon$ model may depend on the problem to which it is being applied. In many high technology applications, accuracy of less than a few percent is necessary while, in other applications, including many environmental flow predictions, twenty or thirty percent may be considered adequate accuracy. The latter applications demand far less from a model and may even permit construction of a 'universal' model; this is an open question at the present time. In any case, it is important to state the desired accuracy prior to choosing a model.

In the remainder of this paper, we shall put attention on models designed for high accuracy. By considering the most challenging applications of turbulence models we shall automatically obtain models adequate for the easier cases. The models may be more complex than necessary for the less demanding applications it may be possible to get by with simpler models. We shall also assume implicitly that numerical methods of sufficient accuracy are applied to the problem. There is no point in discussing the accuracy of a turbulence model if the numerical errors dominate the effect of the

model. It is important to note that many published calculations used inadequate numbers of grid points so that their conclusions with respect to turbulence models must be regarded with suspicion.

III. The Zonal Strategy

The strategy we shall adopt with respect to turbulence modeling was outlined above. We shall regard a flow as a collection of 'zone' each of which has some distinct physical characteristics. We hope to be able to define a number of 'pure' zones with distinct characteristics. The best way to define such zones is by means of flows containing only a single zone; we shall call these 'simple' turbulent flows as distinguished from the more interesting 'complex' flows that may contain many zones.

To clarify this notion, consider a moderately complex flow such as the two dimensional flow behind a backward facing step. In the entry channel, there are two attached boundary layers and a potential flow (or, perhaps, a fully developed plane channel flow). The separation at the step face produces a free shear layer and a recirculation region below it. Then there is a reattachment region which leads to a new boundary layer. On the no-step wall the boundary layer faces an adverse pressure gradient and may or may not separate. This flow contains several zones including ones which closely resemble mixing layers, attached boundary layers, and potential flow; in addition, there are separation, reattachment, and recirculation zones. Thus, this relatively simple flow contains many of the elements found in far more complex flows.

The zonal method begins by establishing models for each pure zone and then proceeds to the construction of methods of connecting the zones. Although the zonal concept can be applied to any type of turbulence model, we shall adopt the $k-\epsilon$ model as the base from which to begin. This choice allows maximum advantage to be taken of work done within the past fifteen years and allows use of many codes which have been written to solve the equations of fluid mechanics with the $k-\epsilon$ model. It has been chosen in preference to the Reynolds stress or algebraic models because the latter have yet to display a sufficient advantage with respect to the $k-\epsilon$ model to warrant the additional complication they introduce.

The definitions of the zones are not known a priori i.e. it is not known how much change in the flow is required to demand labeling as a separate zone. Indeed, some of the zone definitions turn out to be surprising while regions that intuition suggests should be separate zones turn out to be well represented by a single model. In constructing the models, we must let the data speak for itself.

The zonal strategy provides some immediate benefits. In the construction of turbulence models intended to be universal, it is necessary to use data from a variety of different flows in setting the constants. Indeed, this gives these models the ability to handle a range of flows. On the other hand, if the argument that different zones require models is correct, this procedure also introduces 'contamination' i.e. unwanted information from other zones, into the model for each particular zone. The zonal strategy, by insisting on separate models for each zone, should produce more accurate models for each zone.

The zonal strategy is not without disadvantages and is open to criticism on at least two important points. First, by allowing different models or model constants in each zone, it increases the number of constants in the model. Clearly, given enough freedom, almost any model can fit a variety of flows. Second, the number of readjustments is greater than the number of 'pure' zones. Indeed, the number of possible readjustment is approximately one-half the square of the number of zones.

These criticisms should be answered. Final answers cannot be provided at present. For now, we merely want to argue that the strategy is worth pursuing. The best answer we can give to the first argument is that the complexity of turbulence phenomena demands complexity in their modeling. We hope to approach the construction of these models in a logical and systematic manner, adding complexity only when absolutely necessary. Furthermore, we support attempts to construct new types of models and hope that useful models of some generality will be developed. On the second point of the preceding paragraph, we note that the zonal strategy makes sense only if the readjustment regions can be treated with simple models. The indication to date is that

this is possible and we must hope that this remains the case in the future.

IV. Achievements to Date

Work on the construction of zonal models was begun approximately three years ago and the results are encouraging enough to warrant further work on the approach. In view of the large number of possible zones and readjustments, the task ahead is large and construction of a set of models capable of handling the wide variety of engineering flows (let alone those in other fields) will require considerable time and effort.

A. Homogeneous Flows

The first flows considered were the homogeneous turbulent flows. This choice was dictated by the desire for a quick test of the concept of zonal modeling. As the statistical state of a homogeneous flow is independent of the spatial coordinates and depends only on time, its state can be described by ordinary differential equations and testing is greatly simplified. Unfortunately, homogeneous flows do not obey the Boussinesq relationship between the Reynolds stress and the applied strain and therefore require a model of the Reynolds stress type. We are thus committing a minor violation of one of the principles set out above.

In order to apply the zonal concept to homogeneous flows, it was assumed that each flow begins as isotropic turbulence; this is the case in nearly all experiments and full turbulence simulations performed to date. We further assumed that the flow reaches a fully developed state in which the structure of the Reynolds stress tensor does not change with time, in this state, the Reynolds stress tensor is simply the product of the turbulent kinetic energy and a constant tensor which depends on the nature of the applied strain. It is not known whether this assumption is valid for all homogeneous flows (they have not been allowed to develop long enough) but it appears to be a reasonable basis for a simple model.

One then regards the isotropic state as the initial zone and the fully developed state as a second zone. Interest centers on the readjustment from the first zone to the second. Simple first order lag models were used for this purpose and were found to perform adequately. Indeed, all of the data for

homogeneous flows could be fit by this type of model.

B. Free Shear Flows

The next simplest flows after the homogeneous flows are free shear flows which are two dimensional in the mean. Many such flows are possible but only a few well known ones have been measured in sufficient detail to provide a data base for turbulence modeling. These include jets (both into still fluid and coflowing streams), wakes, and mixing layers. Both planar and axisymmetric versions of jets and wakes have been studied.

It has been known for some time that the standard $k-\epsilon$ model is capable of predicting the planar jet issuing into still fluid quite accurately but that it does not perform as well for the far field of the wake or for jets with co-flowing free-streams. This suggests that it might be possible to improve the predictions by introducing a zonal type model.

The first problem is to choose the zones and construct the models for them. The momentum integral equation can be used as a guide. It suggests that the near and far fields should be treated differently. For the near field, the standard model is adequate and, in accordance with the principle that only necessary changes be made, it is adopted for this zone. Fitting the far field requires changes to two constants in the $k-\epsilon$ model. Again, a first order lag readjustment between the near and far field zones was adopted.

The modified model was applied to a variety of planar flows including the free jet, several co-flowing jets and the mixing layer and predicted all of them to within the experimental uncertainty.

An additional strain is introduced in axisymmetric flows. This is due to stretching of the ring vortices in the near field of the flow and results in a significant change in the nature of the turbulence. As a result, the standard $k-\epsilon$ model is not accurate for any of the axisymmetric free shear flows. However, we found that, in the far field, the far field model for planar flows works quite well. By following the derivation of the model carefully, we were able to locate the deficiency in the model to correct it by changing a single constant. A first order lag to connect the near and far fields sufficed to produce a model capable of predicting the axisymmetric free shear flows.

It would be useful to test the model against a more complex free shear flow but, unfortunately, none of the candidate flows has been measured with sufficient accuracy to permit comparison with the model.

C. Other Flows

The $k-\epsilon$ model works very well for the turbulent boundary layer on a flat plate. However, when additional strains such as pressure gradients or curvature are added, the accuracy is seriously degraded. This suggests that a zonal type of model might be useful. The authors' group is looking into such models and the early indication is that zonal modeling will fare as well in the boundary layer as in the free shear flows.

Our group is also looking at the application of zonal modeling to the backward facing step problem. A zonal approach to this problem was described earlier. A study of this flow has been initiated and it appears that careful assessment of the data together with judiciously chosen modifications to the model ought to yield accurate prediction of this flow.

It is worth noting in passing that most of the predictions of the turbulent flow over a backward facing step that were submitted to the 1980-81 Stanford Conference on Complex Turbulent Flows predicted a reattachment length considerably below the experimental value. We recently found, as part of the zonal modeling study, that these computations used inaccurate numerical methods. Redoing the calculation with a more accurate numerical method and a larger number of grid points removed approximately half of the difference between the predictions and the experimental results. This affirms the earlier statement that there is little point in improving turbulence model if the numerical methods are inadequate.

V. Conclusions

We have demonstrated that zonal modeling concept can be applied to homogeneous turbulence and free shear flows. This effort is only an indication of what might be achieved by zonal modeling and is more a proof of concept than a complete work. A great deal remains to be done before we have models capable of doing the tasks required in design engineering.

We believe that the zonal concept provides a logical framework within which accurate and useful turbulence models can be developed. It is an engineering approach (as to the more scientific approach taken by researchers looking for universal models) but it offers the possibility of yielding models with the desired accuracy within a reasonable time. We support the search for better universal-type models but recommend the zonal approach in the absence of such a model. Indeed, improved general purpose models would also ease the task of zonal modeling.

Finally, we want to reiterate the point that accurate numerical methods are a necessary preliminary to the accurate simulation of turbulent flows with any model.

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