論文の内容の要旨
Thesis Summary

Title of Dissertation
High order hyperbolic approach for diffusion dominated flows
（拡散が支配的な流れに対する高次精度双曲型解法）

Name of Author
CHAMARTHI AMARESHWARA SAINADH
チャマーシー　アメーレシュワーラー　サイナド

In this thesis, high-order and high-resolution upwind methods are implemented successfully for the diffusion equation in the hyperbolic form on uniform meshes. Implementation of the ghost-cell boundary conditions for the new variables introduced for the hyperbolic approach are the key to attain high-order accuracy. The adequacy of linear upwind schemes for diffusion equation and the use of shock-capturing scheme like WENO does not have any adverse effect on the solution, unlike the total-variation diminishing (TVD) methods is clearly elucidated. The approach is further extended to advection-diffusion equation, and appropriate boundary conditions have obtained a consistent design accuracy of the third and fifth order. Implementation of WENO scheme for advection-diffusion equation by using the split hyperbolic method demonstrated the advantage of non-oscillatory schemes to capture sharp gradients in boundary layer type problems without spurious oscillations.

Second, the ideas and lessons learnt from isotropic diffusion are applied for development of hyperbolic approach for anisotropic diffusion equation. Preconditioned form of the hyperbolic system is found suitable for anisotropic diffusion with significant improvement in accuracy and steady state convergence. Appropriate length scale, L_r, and relaxation time, T_r, are found for the anisotropic transport equation and due to optimum T_r and L_r the simulations were made independent not only of degree of anisotropy and but also angle of misalignment. Consistent high-order accuracy was achieved for test cases with variable diffusion coefficients.

Finally, significant improvement is achieved by utilizing WENO scheme for the simulation of magnetized electron fluids as the results are much closer to the field-aligned mesh. For small anisotropies, linear upwind schemes as well as TVD-MUSCL

employed can be appropriate. Certain TVD schemes are found inappropriate for the magnetized electron fluids computations similar to the hyperbolic diffusion equation. Similar to the boundary layer problem WENO approach is found to be more suitable and robust to reduce the spurious oscillations associated with the sharp gradients with increasing anisotropic diffusion in electron fluid equations. Boundary conditions based on WENO extrapolation are found more suitable to prevent unphysical extrema for anisotropies higher than 500. For very high anisotropic diffusion problems, $\mu_{||}/\mu_{\perp}$ from 10^4 to 10^9 that can be seen in practical applications like tokamaks and space propulsion devices, all the schemes resulted in spurious oscillations on coarse meshes, say 96×96. The important contributions in this thesis are summarized as follows:

1. Hyperbolic approach for Diffusion and Advection-Diffusion equations:

    - High-order accuracy is obtained on cartesian meshes for wide range of problems by upwind schemes.
    - Detailed analysis is carried out to verify the suitability of shock-capturing schemes for diffusion equation and advantages of WENO scheme over TVD is clearly brought out.
    - Ghost cell boundary treatments are found to be appropriate for computations.
    - Sharp gradients in boundary layer flows are captured without oscillations on under-resolved grids for advection-diffusion.

2. Hyperbolic approach for Anisotropic Diffusion:

    - Numerical method is applicable for complex magnetic field shapes with low numerical pollution.
    - The computational approach is independent of degree of anisotropy as well as angle of misalignment.
    - Significant improvement is achieved by utilizing higher order linear upwind methods and WENO schemes for the simulation of magnetized electron fluids.
    - Spurious oscillations due to misalignment of the grid and magnetic field are drastically reduced through high-order schemes.
    - Through implementation of new boundary conditions, WENO extrapolation, positivity of space potential is obtained for moderate degree of anisotropies.