

Doctorate Dissertation  
博士論文

$O(N)$  method for  
spatio-temporal boundary integral equation method  
and  
a refined evolution law of the frictional strength  
- Toward realistic fault modeling

(現実的な断層モデリングに向けた、  
摩擦強度発展則の精密化と  
時空間境界積分方程式法の $O(N)$ 法の開発)

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Department of Earth and Planetary Science, Graduate School of Science,  
The University of Tokyo  
東京大学大学院 理学系研究科 地球惑星科学専攻

Daisuke Sato  
佐藤 大祐

## Abstract

As the computational sciences and geosciences develop, physics-based fault modeling has become improved. In this research, I treat two major issues related to the significant components of the physics-based fault modeling: elasticity and friction.

In the elastic part, I solve the numerical cost problems in the algorithm of the spatiotemporal boundary integral equation method (ST-BIEM). Because of its accurate description of the complex fault geometries and the stress singularities around fault edges, BIEM has been widely used in the earthquake sciences. However, the numerical cost of the spatiotemporal formulations is scaled by  $O(N^2M)$  for the given number of elements  $N$  and that of time steps  $M$ , and the numerical cost thus become a major issue of the ST-BIEM when large scale computations is required for the realistic modeling. To overcome this cost problem, I propose a novel algorithm that achieve the numerical cost of  $O(N \log N)$  for the elastodynamic ST-BIEM, for the first time. Our propositions are called the FDP=H-matrices, the basis of which comprises the Fast Domain Partitioning Method (Ando, 2016) and the H-matrix method (Hackbusch, 1999), and physically corresponds to the plane-wave expansion. FDP=H-matrices greatly reduce the current numerical cost problem in modeling the fault.

In the friction part, I propose a refined evolution law of the rate and state friction law (the RSF law). The RSF law is known as an established friction law at the low slip rate around or smaller than mm/sec, and applied wide range of physics-based fault modeling. However, currently available evolution laws of the state variable in the RSF law cannot describe the representative tests to give the basis of the RSF law. This problem results in the large uncertainty in the fault modeling related to the choice of the evolution laws. Assembling the experimental requirements, I propose an evolution law, called the modified-composite law, that can explain both experiments consistently. The key trick of the modified-composite law to describe both the experiments is to make the cutoff velocity to suppress the time dependent healing term introduced by (Kato & Tullis, 2001) inversely propositional to the state variable. More generally, the trick can be viewed as a switching between the aging and slip-induced evolution of the friction coefficient depending on the distance from the steady state measured by the frictional state variable. The modified-composite law will clear the current ambiguity in the modeling caused by the evolution law choice.

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# Chapter 1

## General Introduction

### 1.1 Short summary of this chapter

In recent years, the fault motions have become clarified in detail as the seismic and geodetic observation networks have been improved (e.g., Barbot et al., 2012; Ando et al., 2017). Requirements of advanced fault modeling provoke various problems in materials and computational sciences. This study treats two major problems among them for realistic fault modeling. The first is the numerical cost problem of the boundary integral equation method (Bonnet, 1999) in elastodynamic modeling. The second is the ambiguity of the time evolution law of the rate- and state-dependent friction law (Dieterich, 1979). Before explaining my results, in this chapter, I briefly summarize the background of this study.

### 1.2 General background and scope of this study

Micromechanical understanding of the earthquake is one of the ultimate objectives of the earthquake science. Such physical models of seismic activities can be traced back, for example, to the spring-slider model (Scholz, 2002) simply connecting a spring (responding elastically) and a slider (receiving resistance from the frictional surface). This model imitates the elastofrictional fault motions, and quantifies the classical theory (elastic rebound theory (Reid, 1910)) that explains seismic activities as unstable slips of the plates induced by the tectonic loading. By quantifying the qualitative theory by a model, we can compare observations with the theory, and

know the physical process of the earthquake, like (Anderson, 1905, 1951) who connected the fault system with the tectonic stress field by using the Coulomb criterion.

In modern-day studies, on the other hand, the observation network has been well developed (Obara, 2002) and various striking results have been clarified one after another. In the research based on the seismic observations, examples include not only the classical scaling law of the earthquakes (Kanamori & Anderson, 1975) and earthquake statistics (Gutenberg & Richter, 1944; Utsu et al., 1995), but also more detailed estimates of the earthquake source parameters (e.g., the stress drop (Ide & Beroza, 2001) and rupture velocity (Geller, 1976)) and the spatiotemporal pattern of the fault slip (Ide & Takeo, 1997). Moreover, novel phenomena have been also discovered and elucidated in recent years, as exemplified by the discovery of the slow slip (Obara, 2002) and the detection of the preseismic slow fault slip (Ellsworth et al., 1995). In geodetic observations, long-term after slip transient (e.g., Barbot et al., 2009) and post-seismic deformation (e.g., Moore et al., 2017) have been precisely captured by the global navigation satellite systems (GNSS). Furthermore, a precise spatial pattern of coseismic surface deformation has become captured by the interferometric synthetic-aperture radars (InSAR) (e.g., Simons et al., 2002).

As above, obtainable data has been markedly improved, and the significance of the realistic modeling has increased in the recent research of the earthquake (Barbot et al., 2012). Along this line, for example, fault modeling has become naturally linking the complex shape of the fault with the surface displacement in studies of complex fault systems such as inland faults (e.g. Ando et al., 2017). In addition, contact points are born between the observation and material and computational science, such as in the modeling that links the post seismic transient with the physical properties of asthenospheric viscous motions (Moore et al., 2017). Furthermore, some ambitious physics-based fault modeling is now targeting on reproducing the actual fault movement including a series of seismic activities (earthquake cycle) including coseismic ground motions and long-term post-seismic and inter-seismic transients (Barbot et al., 2012).

As above, the fault modeling has achieved remarkable growth. However, as we will see below, the fault modeling still requires various challenges. Consequently,

potentially unphysical assumptions and simplifications have been introduced associated with previous limitations in theoretical and numerical methods. In this study, I aim to settle two big issues of such fault modeling; those issues are concerning the major components of the modeling, the elasticity and friction.

Chapter 2 is concerning the elasticity. In order to treat the elastodynamic problems, the boundary integral equation method (BIEM) has been widely used for fault modeling (e.g., Das, 1980; Geubelle & Rice, 1995; Kame & Yamashita, 1999; Lapusta et al., 2000; Aochi & Fukuyama, 2002; Noda et al., 2009; Ando et al., 2017). Since the fault modeling formulated by the fracture mechanics deals with the stress singularity around the fault tips, in order to resolve them accurately, researchers have developed various numerical methods, such as the finite element method (FEM) with the mesh-refinement technique (Tago et al., 2012), and the high-order finite difference methods (FDM) (Kozdon et al., 2013). Among them, BIEM is known as its high accuracy (Tada & Madariaga, 2001; Aochi & Fukuyama, 2002) and to reduce the problem into solving only for the solution along the fault (Kozdon et al., 2013). Though BIEM has been used mostly for solving the uniform elastic media, recent BIEM formulations can include various extensions, such as the half-space geometry (Zhang & Chen, 2006; Hok & Fukuyama, 2011), elastic inhomogeneity (Kame & Kusakabe, 2012; Kusakabe & Kame, 2017), fault branches (Kame & Yamashita, 1999; Ando & Yamashita, 2007), and off-fault nonlinear inelasticity (Yamashita & Fukuyama, 1996; Barbot & Fialko, 2010). However, such potential of BIEM has been not yet brought out sufficiently, because the time domain BIEM to resolve the seismic waves, called the spatiotemporal BIEM (ST-BIEM) in this study, requires the huge numerical cost. For example, the abovementioned elaborate earthquake-cycle simulation (Barbot et al., 2012) has been applied only to the planar fault geometry for which the fast algorithm is already developed (Perrin et al., 1995). To maximize the utility of (ST-)BIEM, in this paper, I develop the algorithm for ST-BIEM whose computational complexity and memory consumption is almost scaled by the element number  $N$  (precisely  $N \log N$ ), which is the almost fastest cost order of the BIEM simulation, and had never been achieved for the elastodynamic time-domain formulations of the earthquakes.

Chapter 3 is concerning the friction. The rate- and state-dependent friction law (the RSF law) (Dieterich, 1979) has been now widely used in the fault modeling (Lapusta et al., 2000). The RSF law has greatly succeeded in describing various slow fault motions (the slip rate of which is on the order of millimeter per second or smaller), such as the thickness of the seismogenic layer (Marone & Scholz, 1988; Scholz, 1998), afterslip transient (Marone et al., 1991; Perfettini & Avouac, 2004), and aftershock (Dieterich, 1994; Perfettini & Avouac, 2004). However, the problem is known to exist in the currently proposed representative time evolution laws (the evolution laws) of a variable in the RSF law (the state variable); representative evolution laws called the aging law (Dieterich, 1979) and the slip law (Ruina, 1983) cannot consistently describe both the representative experiments, called the velocity step test and slide-hold-slide test (Kato & Tullis, 2001). Moreover, recent studies have reported the simulation results become qualitatively uncertain by depending on the choice of the evolution laws (e.g., Ampuero & Rubin, 2008). This uncertainty arising from the evolution laws must be resolved in order for the realistic fault modeling. In this study, I propose an evolution law to describe two experiments consistently and show my proposition settle the frictional model uncertainty in the earthquake simulation.

In Chapter 4, I discuss the potential applications of the above two topics and related future works. I consider the potential impact of Chapter 2, Chapter 3 and refer to what becomes possible based on these knowledges.

In the following subsections, I overview the preceding studies to set the purpose of this study more clearly. The elastic modeling is first overviewed. The study on the constitutive law on the frictional fault is overviewed next.

### **1.3 Research topic 1. Elasticity**

Here we focus on the elastic fault modeling and explain the boundary integral equation method (also the called boundary element method), which has been widely used in the seismology. In the explanation of them, I introduce the cost problem of this method and motivate the study concerning Chapter 2.

### 1.3.1 Boundary integral equation method modeling earthquake

The boundary integral equation method (BIEM) is a quite popular method to simulate the rupture propagation problem (Kozdon et al., 2013). BIEM reduces the targeted problem to a problem with regard to the boundary only. Since this reduction of the problem is useful for the mesh generation to discretize complex objects (Ando et al., 2017), the reduction of the number of discretized elements (Nishimura, 2002). Those are advantages of BIEM over various volume based methods such as the discontinuous Galerkin finite element method (Tago et al., 2012).

The further remarkable is the advantage of BIEM in modeling the earthquakes. For example, BIEM models the fine structures of fault geometries well, frequently concerned with the rupture phenomena (Scholz, 2002; Ando et al., 2009), such as fault elongation (Kame & Yamashita, 1999), fault branching (Kame et al., 2003; Ando & Yamashita, 2007), and splay faulting (Scholz et al., 2010). In addition, the earthquake deformation is  $10^{-4}$  times smaller than the fault segment size (Scholz, 2002) so that the earthquake is appropriately modeled by a linear problem except for the boundary condition on the fault; the large deformation caused by the earthquake cycles is also eventually treated by BIEM with some remeshing technique, since the deformation is merely an accumulation of such small deformations caused by respective events. Although the kernel of the integral equation becomes sometimes complicated, the kernel is now already analytically known in various settings by extensive researches (e.g., Cochard & Madariaga, 1994; Tada & Madariaga, 2001; Aochi & Fukuyama, 2002; Tada, 2006).

Due to these advantages, BIEM has been used in many situations, such as the inversion of the fault motions (sometimes called the dynamic rupture simulation of the earthquake (Ando et al., 2017), dynamic modeling of the earthquake (Peyrat et al., 2001), modeling of spontaneous seismic ruptures (Madariaga et al., 1998), dynamic inversion (Di Carli et al., 2010), or other various terms). Examples include the 1992 Landers earthquake (Aochi & Fukuyama, 2002), 1996 and 2004 Parkfield earthquake (Barbot et al., 2012), and 2016 Kumamoto earthquake (Uchide et al., 2016). The current improvement allows us to treat the complex fault geometry frequently

observed in the inland earthquake cases, e.g. the 2014 Tottori earthquake (Ando et al., 2017). The spatiotemporal BIEM (ST-BIEM) is also useful for investigating the seismicity and the fault source physics, such as the recurrence time scale of the repeating earthquake (Chen & Lapusta, 2009) and the wave radiation pattern caused by the dynamic rupture (Kaneko & Lapusta, 2010).

ST-BIEM is also applicable to modeling the long-period fault motions including the preseismic, coseismic, postseismic, and interseismic periods (the earthquake cycle simulation) (Lapusta et al., 2000). This is a way to model the whole history of the fault system, showing various time-scale phenomena; the slip rate becomes cm per sec in the coseismic periods while it is cm per year in the interseismic period. The approach of (Lapusta et al., 2000) using the planar fault ST-BIEM based on the spectral formulation (Perrin et al., 1995) is the current standard model of the earthquake cycle simulations (e.g., Barbot et al., 2012).

Moreover, since the problem setting is reduced to an equation regarding faults only, fault boundary conditions such as the friction law are frequently studied with BIEM. Examples include the simulated earthquake nucleation governed by the low speed friction law (Ampuero & Rubin, 2008) and the dynamic rupture governed by the high-speed friction law (Noda & Lapusta, 2013). In addition to being highly accurate, BIEM is also helping researchers to grasp the essence of the earthquake by simplifying the problem into the integral equation governing the fault motion.

### 1.3.2 Numerical Cost Problem of Fault Modeling

Accompanying the model improvement, however, the numerical cost has become problematic. This is because many extensions require the additional boundaries and then the cost becomes larger, as in the modeling of the half space geometry (Ando & Okuyama, 2010; Hok & Fukuyama, 2011) and the modeling of the spatially heterogeneous elasticity (Kame & Kusakabe, 2012). The numerical cost easily and rapidly increases than the homogeneous cases by imposed additional boundaries, because the cost is on the quadratic order of the number of the original and additional elements; note that the use of the currently available analytic form of the half space

kernel (Zhang & Chen, 2006) is much more time-consuming since the expression is obtained in the frequency domain, which requires the Fourier transform in each time step for solving the transient dynamic rupture problems (Ando & Okuyama, 2010). Therefore, many modeling simulations have been limited to the planar fault or quasi-dynamic approaches (quasi-static kernel plus the radiation damping term (e.g., Ohtani et al., 2011)) where the cost problems are resolved (Perrin et al., 1995; Hackbusch 1999); in those cases, the cost is almost on the order of the number of elements. The planar fault simulation cannot include half plane effects in general and the distribution of the strike and dip directions. The quasi-dynamic approaches in general fail to capture the high speed faulting in the rupture processes. The fully dynamic approach is required to resolve the dynamic behavior including the seismic wave radiation (Lapusta et al., 2000), which affects the stress drop (Ide et al., 2011) and the rupture velocity (Lapusta & Liu, 2009).

Although fully dynamic nonplanar simulations have been performed by various studies (e.g., Aochi & Fukuyama, 2002; Ando et al., 2017), the model flexibility is still restricted. Actually, the nonplanar simulation is at least almost element-number heavier than the planar case simulations; in the current largest scale simulation, the number of elements  $N$  is of  $N \sim 10^5$  (Ando, 2016), and thus the nonplanar simulation is roughly  $10^5$  times heavier than the planar cases. Obviously, the numerical cost has been a substantial restriction of the modeling (Nishimura, 2002).

In order to overcome the cost problems of the spatiotemporal boundary integral equation method, this study develops almost  $O(N)$  methods for ST-BIEM, where the memory and computation costs become of almost  $O(N)$  (precisely  $O(N \log N)$ , detailed in Chapter 2), called the fast domain partitioning hierarchical matrices (FDP=H-matrices). FDP=H-matrices are based on the fast domain partitioning method (FDPM) (Ando et al., 2007; Ando, 2016), which is a fast algorithm of ST-BIEM based on the physical property of the kernel, and the hierarchical matrices (H-matrices) (Hackbusch, 1999), which is an almost  $O(N)$  method for spatial BIEM. The theoretical background of FDP=H-matrices is the plane wave expansion (Ergin et al., 1999) and is a natural extension of FDPM. FDP=H-matrices achieve almost  $O(N)$  for arbitrary fault geometries and allow us to study complex geometries with more efficient nu-

merical costs.

Although I limited the explanation to the elastic motions predominantly governing the fault motions, the proposition of this study is applicable to the fault model including the inelastic effects, based on the proposition of Barbot & Fialko (2010). Since the integral equation method of Barbot & Fialko (2010) expresses the inelastic or nonlinear effects (e.g., nonlinear viscosity) as the equivalent single force imposed to the linear elastic medium, their formulation can be combined with BIEM I here explained. FDP=H-matrices can be applied to various fault modeling.

## **1.4 Research topic 2. Friction**

The friction plays an essential role in the fault motions by determining the strength of the fault. In particular, the rate- and state-dependent friction law (the RSF law) is an important friction law that has been used widely for modeling the fault (Lapusta et al., 2000; Barbot et al., 2012). Here, I overview the previous studies of the RSF law, by focusing on the model uncertainty caused by ambiguity in the evolution law of the RSF law (Ampuero & Rubin, 2008), and motivate the study in Chapter 3.

### **1.4.1 Simple static dynamic friction law to Rate and state friction law**

The friction law is the fundamental constitutive law on the fault (Scholz, 2002). Among various friction laws, the RSF law have been successfully applied to various problems as mentioned earlier. I here briefly review the experimental development leading to the RSF law and the application of the RSF law to show the background of our study.

I begin with the most classical friction law, the Amonton Coulomb law (e.g., Scholz, 2002). It captures the yielding and the abrupt decrease of the yielding stress upon slip onset. In this law, the yielding stress is proportional to the normal stress and the constant of proportionality, called the friction coefficient, that have two values. The yielding condition gives a static value called the static friction coefficient, and the stress at the yielded area (different from the yielding stress) gives a dynamic value called the dynamic friction coefficient; hence the Amonton Coulomb law is



sometimes called the simple static dynamic friction law (Scholz, 2002). When the Amonton Coulomb law is adopted, the slip rate on the fault is zero before the yielding and the ratio of the shear stress and the normal stress is fixed at the dynamic friction coefficient as long as the slip rate is nonzero.

Rabinowicz (1951, 1956) found there is a non-zero characteristic distance required to decrease the friction coefficient from the static one to the dynamic one, called the critical slip weakening distance ( $D_c$ ), and found that the coefficient increases from the dynamic value to the static value temporally in proportion to the log time. In particular, the characteristic length scale  $D_c$  in the slip weakening provides decisively significant improvement from the Amonton Coulomb law, by introducing the spatial characteristic scale of fracturing called the cohesive zone (Scholz, 2002). Because of such finite spatial scale, the frictional fault can be solved independently from a high-frequency cutoff (corresponding to a grid-size in simulations) without the ill-posedness; this is striking compared with the Amonton Coulomb law where the friction coefficient abruptly changes at the crack tip so that the frictional formulation yields the grid-size dependence at the tip when using the classic Amonton Coulomb law. Theoretically, the introduction of  $D_c$  also means to introduce the non-zero fracture energy scale (Griffith & Eng, 1921) determined as the product of the stress drop (multiplied by some prefactor), while such an energy scale is depending on the artificial grid-size in the Amonton Coulomb law. The introduction of the critical slip weakening distance is thus essential to keep the consistency between the friction and the fracture mechanics (Ohnaka & Matsu'ura, 2002).

As the extension of the above slip weakening and time-healing friction, the RSF law (Dieterich, 1979) can be positioned (Nakatani, 2001). Since its origination by Dieterich (1979), the RSF law has been improved by some researchers (e.g., Ruina, 1983; Linker & Dieterich, 1992; Nagata et al., 2012). Dieterich (1979) found the instantaneous response of the friction coefficient to the velocity (called the direct effect), which was noticed to be independent of the slip weakening and the log time healing found in Dieterich (1978). To represent the direct effect, the slip weakening, and the log time healing, (Dieterich, 1979) formulated the RSF law, made of two equations. The first, called the constitutive law, separates the change in the friction

coefficient into the direct effect and the contribution from the temporally evolving variable, called the state variable, describing the other effects than the direct effect. The second, called the evolution law, describes the temporal evolution of the state variable. The evolution law has been greatly improved over decades. The time derivative form of the evolution law is obtained by Ruina (1983). His proposed evolution law is frequently called the Ruina law (Marone, 1998b) or slip law (Ampuero & Rubin, 2008). The slip law is slightly different from the evolution law proposed by Dieterich (1979) (expressed in the time derivative form by Okubo & Dieterich (1986)) (frequently called the Dieterich law (Marone, 1998b) or aging law (Ampuero & Rubin, 2008)). See the review (e.g., Marone, 1998b) and its listing references for the details. In addition, Linker & Dieterich (1992) and Nagata et al. (2012) respectively found the normal and shear stress dependence of the state variable. After the above findings, the dependence of the friction coefficient on the velocity, time, slip, and stress has been described in the unified formulations of the RSF law. The microscopic physics has also gradually clarified experimentally (e.g., Heslot et al., 1994; Dieterich & Kilgore, 1994; Nakatani, 2001).

Based on such laboratory-verification, the RSF law has been applied to a wide range of seismic phenomena relating to the slow fault motion as mentioned earlier. Although the applicable velocity range of the RSF law is relatively low velocity range (0.1 nm/sec  $\sim$  mm/sec) (Kilgore et al., 1993; Marone, 1998b, e.g.) compared with the seismic slip rate (cm/sec), such low velocity range is exactly the significant velocity range to determine the stability of the fault (Scholz, 2002); such stability actually determines the existence of the earthquake in the given tectonic setting, since the earthquake can be understood as the unstable rapid (stick-)slip motion (Ruina, 1983) caused by the velocity small perturbation. More classical friction laws than the RSF law (such as the slip weakening friction law) is marginal against the small perturbations, and thus cannot explain such stability of the fault. Therefore, the stability determination of the fault is the big contribution of the RSF law in the fault modeling. In addition, the relatively slow motions (but faster than the plate motions), such as the slow slip and the after slip are exactly in the velocity range of the RSF law, and so the RSF law gives the good prediction concerned with such slow motions (e.g.,

Marone et al., 1991; Dieterich, 1994; Perfettini & Avouac, 2004). Because the RSF law describes such slow motions, long time scale fault motions have been also developed along the line of the RSF law (e.g., Lapusta et al., 2000), sometimes with some extensions of the high-speed friction law (e.g., Noda & Lapusta, 2010).

### 1.4.2 Ambiguity of evolution laws

This study addresses the problem of the evolution law of the RSF law that seems to be a big problem for the current situation. In the current RSF law, two formulations, called the aging law and slip law, are widely used. However, it is known that either velocity step test or slide-hold-slide test cannot be described by those laws, though those two tests are the standard laboratory experiments to establish the RSF law (Marone, 1998b). Due to such ambiguity of the evolution law, simulated fault motions are even qualitatively uncertain. Recent reports of such problems are seen in the simulated rupture nucleation (Ampuero & Rubin, 2008) and the response of the characteristic earthquakes to the megathrust earthquake (Ariyoshi et al., 2014). To overcome those difficulties, Kato & Tullis (2001) proposed an evolution law, called the composite law, to compromise them; the composite law is now standardly used in the extensive work of Japan Agency for Marine-Earth Science and Technology (JAMSTEC) group (Hori & Kaneda, 2004; Kodaira et al., 2006; Hori & Miyazaki, 2011) for modeling the Nan-kai Trough, where a large earthquake is estimated to occur in the near future. However, as shown in Chapter 3, even the composite law involves another problem different from those of the aging and slip laws. The ambiguity of the plausible evolution law is thus a big problem for fault modeling currently progressing. I aim to improve the evolution law for the model accuracy. In Chapter 3, extending the previous study (Kato & Tullis, 2001), I propose a refined evolution law, called the modified-composite law.

Note that the high speed friction is out of the scope in this study. At the high speed around cm/sec, which is the characteristic slip rate on rupturing faults (Scholz, 2002), the frictional heating results in the phase transition such as the melting (Tsutsumi & Shimamoto, 1997; Rice, 2006; Goldsby & Tullis, 2011), pore pressure re-

sponse (Rice, 2006), and gel formation (Di Toro et al., 2011). The friction law is also affected by the mesoscopic structures such as the topographical fractality (Ohnaka, 2003; Aochi & Matsu'ura, 2002), wearing (Scholz, 2002), dissipations caused by fault gauges (Kuwano & Hatano, 2011), and even plasticity (effective thickness of faults) at crustal scales (Yamashita and Fukuyama, 1996; Andrewes, 2005; Marone et al., 2009). Although those are beyond the scope of this study, solving the problem of the RSF law will be useful to integrate the high speed friction laws with the RSF law governing the slow speed friction to capture the whole picture of the fault motions.

## 第 2 章

本章については、

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## 第 3 章

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5 年以内に雑誌等で刊行予定のため、非公開。

## Chapter 4

### General Discussion

In this study, for the realistic modeling, I have solved the cost problem of the spatiotemporal boundary integral equation method in terms of the algorithm, and the long-standing problem concerning the evolution law of the rate- and state-dependent friction law. The elastic part of this study (Chapter 2) will become the basis to construct the model based on the recent fine observation, mentioned in (Chapter 1). The friction part of this study (Chapter 3) will contribute to the fundamentals of the fault modeling. The potential implications of these knowledges are referred to in this section.

For the dynamic modeling of the seismic source, the numerical cost has been the large issue (Ando et al., 2016). As mentioned earlier, such a cost problem is actually deeply related to the inevitable difficulty in the computational mechanics solving elastodynamic problems (Nishimura, 2002). To overcome such a profound issue, I developed an algorithm FDP=H-matrices (Chapter 2). This achieves the almost smallest time complexity and memory consumption of  $N \log N$ , where  $N$  is the number of elements. This drastically reduces the costs and will maximize the flexibility of the realistic fault modeling.

Let us estimate how the cost reduction from that of the original BIEM of  $O(N^2M)$  to that of FDP=H-matrices of  $O(N \log N)$ . Using the same setting in Fig. 2.13 of Chapter 2, we plotted the  $N$  dependence of the memory consumptions in the original ST-BIEM and BIEM with FDP=H-matrices in Fig. 4.1, where the memory to store the kernel is plotted as a function of the number of elements  $N$ . The plot is terminated

around  $N \sim 10^6$ , which order is 10 times larger than the previously adopted largest  $N$  in the fault modeling Ando (2016). As  $N^2M$  costs becomes of  $N \log N$ , the costs to solve the same  $N$  systems will be reduced to be  $O(MN/\log N)$  times smaller. This cost reduction helps directly the parameter studies as required in the dynamic fault modeling (Urata et al., 2017). This will also have an impact on the seismic hazard assessments (e.g., that for anticipated Nankai-Tonankai earthquakes (Hok et al., 2011)), where numerous simulations are required to wholly cover the possible hazard scenarios. Moreover, when  $N$  and  $M$  are fixed, the memory and computation costs will be reduced to be  $O(NM \log N)$  times larger by using FDP=H-matrices.

The constitutive law on the modeled fault will become more realistic. Previously, the reasonable choice of previous evolution law had been problem-dependent Ampuero & Rubin (2008); Ariyoshi et al. (2014), though many researchers thought those evolution law has been pronounced to be adequate in some level (Ampuero & Rubin, 2008). Probably, the slip law is reliable in the problems where the energetics at the fault tip as nucleation phenomena, as pointed by Ampuero & Rubin (2008); on the other hand, the reliability of the aging law is in the long time dynamics where the time-dependent healing of the frictional strength plays an important role (Ariyoshi et al., 2014). Those ambiguities on the evolution law will be eliminated by the modified-composite law (Chapter 3). In particular, the earthquake cycle simulations require the precise description of both the time healing and the slip transient (Kato & Tullis, 2001), and thus the reconsideration will be profoundly needed urgently. In near future, I will apply the proposed evolution law to the simulation and will examine its effectiveness in the fault modeling. Also, this result has implication to the experiments. This is also a potential implication of this study.

My research contributed to the deeper understanding of the elastofrictional physics that fundamentally govern the fault motion. Combining the knowledges obtained from these research, I will next tackle more realistic problems. These will make numerically demanding fault modeling accessible, such as fully dynamic fault modeling of the complex fault system as that of inland earthquakes, and fully dynamic earthquake cycle simulations of the megathrust earthquakes. These simulations will provide new tests to the theory, and to our understanding of the earthquakes.



## 図 4.1

本章については、

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# Chapter 5

## General Conclusion

Integrating the computer sciences with seismic wave surveys, geodetic surveys, geological surveys, and laboratory experiments, it has become possible to model the shallow earth by the physical simulation based on the observational information. On the other hand, the detailed observations now require for the earthquake modeling to solve the problems exceeding the current modeling knowledges. In this study, to overcome such difficulties, I aimed to solve the problems concerning the numerical costs of the dynamic elastic simulations and the formulation of the frictional constitutive laws.

In Chpter 2, concerning the elasticity in the modeling, I treated the boundary integral equation (BIEM), which is a highly accurate scheme of the fracture problems, and especially focused on the spatiotemporal BIEM (ST-BIEM) where seismic waves can be described. ST-BIEM requires  $O(N^2M)$  costs to convolve the integral equation and to store the kernel of the integral equation, when simulating the  $N$ -element systems of the  $M$  time steps. These costs have remarkably restricted the modeling flexibility (Ando, 2016). For example, most of the current modeling relies on planar fault simulation accelerated by established methods (e.g., Perrin et al., 1995). As a remedy, I proposed an algorithm of almost the best numerical cost order almost  $O(N)$  (precisely  $O(N \log N)$ ), called FDP=H-matrices. This method can mostly erase out the cost problem of ST-BIEM. In fact, the dependence of the cost order on the number of elements in FDP=H-matrices is mostly the same to the volume-based methods; considering the decrease of the number of elements caused

by the dimensional reduction from the volume to the boundary, FDP=H-matrices can be potentially faster than volume based methods. Utilizing this algorithm, the modeling will become to include more details of the observations.

In Chapter 3, concerning the friction in the modeling, I took up the rate- and state-dependent friction law (the RSF law) and proposed an evolution law to solve the model uncertainty associated with the choice of the evolution laws. The RSF law is a law based on the low speed friction experiments, and it explains a wide range of seismic phenomena such as determination of the locked zone of fault, the cycle of repeating earthquakes, afterslips, and aftershocks. Also, it has been standardly adopted in the model simulations. Nevertheless, the current evolution law has partly mismatched with two representative experiments, namely, velocity step test and slide-hold-slide test. It has resulted in the uncertainty of simulations. For the situation, I aim to overcome it and proposed an evolution law called the modified-composite law to describe both experiments. The proposition successfully explained both the velocity step and slide-hold-slide tests. The problem of model uncertainty will be advanced toward the resolution by the proposal of this research.

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# 付録Ⅰ

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## 付録Ⅱ

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