博士論文

Strategic Behavior of Non-profit Maximizers
(非利潤最大化経済主体による戦略的行動)

益田　拓

Introduction

Maximizing profits is the ultimate goal for private firms. This is one of the most fundamental assumptions in the literature of industrial organization. Under this assumption, researchers have piled up studies about optimal production, pricing, research and development, advertising, entry and exit. Accordingly, researchers have assumed that private firms choose their actions so as to maximize their own profits. This sounds quite natural and valid. Why on earth do firms act differently despite their ultimate goal?

In 1987, however, one paper written by Fershtman and Judd shows the possibility that non-profit maximizing firms can acquire higher profits than profit maximizing firms in oligopoly. Consider a firm operated by a professional CEO hired by its owner. The contract between them specifies the incentive scheme in which the CEO is evaluated not only on profits but also revenues of the firm. Under such contract, the CEO operates more aggressively to sell more products than profit maximizing firms. The aggressiveness crowds out competitors and results in a higher market share and hence higher profits. This is the mechanism in which non-profit maximizers defeat profit maximizers. Given this insight, the next question is what is optimal to achieve the ultimate goal.

Many studies follow Fershtman and Judd to open up the strategic delegation literature. Some try to probe other criteria than revenues, such as market shares, sales or relative performance. Others apply the mechanism to other economic agents than private firms, including state-owned enterprises, nonprofit organizations, and governments. For example, state-owned enterprises are assumed to maximize social welfare, but it is found that the best objective function is not necessarily the welfare itself. Rather, under fairly general settings it is a mixture of welfare and corporate profits, which provides theoretical supports for privatization of the state-owned companies.

This thesis also belongs to the strategic delegation literature. Chapter 1 investigates the strategic relationships between corporate social responsibility of private firms and partial privatization of the state-owned firm. CSR can play the same role as the revenue component that private firms can commit. Thus the strategic delegation framework partially explains recent rise of CSR. In response, one research question is whether the government should privatize public firms. The other question is whether a government wishing to induce more CSR, should privatize the state-owned firms. It proves that the relationships can be either strategically complementary or substitutive, or even non-monotonic if there is a threshold where the strategic characteristics reverse.

Chapter 2 focuses on price competition with multimarkets, and privatization policy therein. There are two markets. One is served solely by a state-owned public firm, and the other is served by both public and private firms. The markets are linked by the production technology of the public firm. Using a relatively general model, we first show that privatization is never optimal in the absence of multimarket contacts (i.e., there is only one monopoly or duopoly market). However, in the presence of multimarket contacts, privatization can be optimal. We provide a parametric example for this possibility using a linear-quadratic specification. The characterization of optimal privatization demonstrates its non-monotonicity with respect to the relative sizes of the two markets and the degree of product differentiation.

Chapter 3 applies strategic delegation to the tax competition literature. There is no room for objective function adjustment if the countries are closed and there is no mutual relationship between them. However, circumstances change as globalization takes place; as the countries links to other countries through spillovers, trade and factor mobility. The model of tax competition is led by policy makers selected from citizens with different stance to other countries. The citizens will choose a political leader who makes them expect to bring the greatest benefit in relation to other countries. With this approach, our interest is on which of the altruistic or malicious, in other words hostile politician towards neighboring countries will be more likely to be elected as the policy-maker who represents the diverse citizens when countries have become connected in a single market. The results in the symmetric tax competition show that there exists an incentive for the median voter of the country to delegate the power to decide the tax policy to the altruistic individual. In our study allowing the asymmetries across the countries, it was also shown that the individual having malicious preferences could become representatives of the country.

Reference

Fershtman, C. and Judd, K.L. (1987), “Equilibrium Incentives in Oligopoly.”, *The American Economic Review*, Vol. 77, No. 5 (Dec., 1987), pp. 927-940.

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Chapter 1

Strategic Relationships between Corporate Social Responsibility and Privatization

Taku Masuda*

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1 Introduction

Corporate Social Responsibility, henceforth CSR, prevails nowadays. According to KPMG Survey of Corporate Responsibility Reporting 2017, 93% of the world's largest 250 companies and 75% of the whole sample of 4900 companies issue Corporate Responsibility reports. The numbers were 39% and 12% in 1999 respectively. As the numbers show, private firms, which are conventionally modeled as pure profit maximizers, pay attention to social factors which directly has no positive effects on profits. This holds true also in industries such as postal services, financial services, telecommunications, automobile manufactures, utilities and so forth. These industries are typical ones where the state-owned enterprises operate in addition to private ones. As CSR rising, the effects of privatization to CSR and CSR to privatization attracts researchers' attention. The research question is something like whether the state-owned firm should sell or buy its government share in response to increasing CSR, or in order to promote it.

Several papers use versions of strategic delegation game *à la* Fershtman and Judd (1987) to investigate the interaction of privatization and CSR. Ouattara (2017) and Kim et al. (2017) analyze strategic incentives of privatization to CSR. They show the result that privatization is a strategic substitute to CSR and if CSR is fixed at some high degrees then full-nationalization can be optimal in contrast to Matsumura (1998). Kim et al. (2017) additionally points out, that the optimal degree of privatization exhibits non-monotonic relationship with the degree of CSR if private firms have different

*Graduate School of Economics, The University of Tokyo, Email:taku121281@gmail.com

magnitudes of CSR. Itano (2017) studies the strategic relationship of CSR to privatization besides the one of CSR to privatization. He concludes that CSR is also a strategic substitute to privatization. The strategic relationships of CSR and privatization in mixed duopoly situations are confirmed as mutually monotonic and substitute. That implies, for example, that nationalization possibly induces more CSR.

In this paper, we show that the relationships can be non-monotonic, or precisely, there exists a threshold where the strategic substitute becomes complement. The new findings are due to generalization of functional specifications which are used in the literature. We generalize the specification in two ways; cost functions and an abstraction of CSR. In the literature, the cost functions are quadratic with coefficient 1/2. We introduce $\gamma > 0$ so the coefficient is $\gamma/2$, and show that $\gamma = 1$ is a threshold that the strategic relationships are always monotonic for $\gamma \geq 1$. The other generalization is about an abstraction of CSR. In the literature some papers use consumer surplus while other[1] use social welfare as representation of CSR, but never both. We test both to see if they support the same results. It turns out not, though they share the same property that firms become aggressive by taking CS or SW into consideration. That means there exists other crucial difference between two as representation of CSR. We figure out that it is whether being sensitive to competitors' actions or not. Considering CS makes firm less sensitive to competitors, whilst SW makes more sensitive than pure profit maximizers. That the sensitivity can be a determinant factor of the strategic relationships is also new finding of our study, not only to mixed oligopoly literature but to strategic delegation literature as well. Summarizing the above, in order to judge the strategic relationships of CSR and privatization, the results in the paper recommend policy makers to carefully check competitors' cost efficiency and sensitivity to others actions.

The specific game model is introduced in Section 2. The equilibrium under CS type CSR comes in Section 3. Then, Section 4 investigates welfare type of CSR, instead of consumer surplus type, whether the type matters to the results. Section 5 concludes.

2 Model

2.1 Game flow

The game consists of two stages. In the first stage, a public firm choose a degree of privatization to maximize welfare while a private firm a degree of CSR to maximize its profit. The choice is done simultaneously. In the second stage, both firms simultaneously choose production quantities to maximize their own objective functions, which are possibly different from pure welfare or profit ones depending on the result of the first stage. An equilibrium concept is a subgame perfect equilibrium, so the analysis takes place backwards.

[1]For instances, Yasui and Haraguchi (2018), Ghosh and Mitra (2014) and Matsumura and Ogawa (2014)

2.2 Specification

Following the literature, this study employs simple linear demand and quadratic cost formulations. Precisely, both firms have following profit functions,

$$\pi_i = (1 - q_i - q_j)q_i - \frac{\gamma q_i^2}{2}, \quad i = 0, 1. \tag{1}$$

Here, $i = 0$ represents a state-owned public enterprise and $i = 1$ a competing private firm and q_i each amount of production. On top of a canonical formulation, we introduce γ in the cost functions. This signifies an efficiency of firms' production technology. In the literature, $\gamma = 1$ is often assumed because of its parsimony. However as shown later, generalizing γ provides new findings which are in a stark contrast to results in the literature. Thus γ is allowed to take any strictly positive value, i.e., $\gamma > 0$.

As an abstraction of corporate social responsibility act, the private firm takes care of consumer surplus in its objective function, let alone own profit. Thus, the objective function to be maximized in the production stage is

$$U_1 = \alpha_1 \pi_1 + (1 - \alpha_1)CS, \tag{2}$$

$$where \;\; CS = \frac{(q_0 + q_1)^2}{2}. \tag{3}$$

α_1 indicates an inverse degree of CSR of the private firm. The lower is α_1, the more CSR is the private concerned with. To guarantee positive productions for both firms with any γ, the study restricts an attention to $\alpha_1 \in [1/2, 1]$.

For the public sector, following Matsumura (1998), its objective function has a possibility of partial privatization in the following form,

$$U_0 = \alpha_0 \pi_0 + (1 - \alpha_0)SW, \tag{4}$$

$$where \;\; SW = \pi_0 + \pi_1 + CS. \tag{5}$$

In turn, $\alpha_0 \in [0, 1]$ is a degree of privatization, so higher α_0 means the public firm sells larger amount of its stock to private investors and thus takes care its own profit more. The public firm maximizes U_0 in the production stage.

Through the second stage, an equilibrium pair of quantities $(q_0^*(\alpha_0, \alpha_1), q_1^*(\alpha_0, \alpha_1))$ and resultant values materialize. Anticipating them, both firms choose each α_i to maximize each objective function. Namely, the public firm's problem is,

$$\max_{\alpha_0} \; SW\left(q_0^*(\alpha_0, \alpha_1), q_1^*(\alpha_0, \alpha_1)\right) \tag{6}$$

and the private one is,

$$\max_{\alpha_1} \; \pi_1\left(q_0^*(\alpha_0, \alpha_1), q_1^*(\alpha_0, \alpha_1)\right). \tag{7}$$

3 Equilibrium

3.1 Production stage

Given α_i, each firm maximizes each objective function with its own quantities. The FOCs are,

$$\begin{aligned}\frac{\partial U_0}{\partial q_0} &= \alpha_0 \frac{\partial \pi_0}{\partial q_0} + (1-\alpha_0)\frac{\partial SW}{\partial q_0} \\ &= \alpha_0(1-q_1-(2+\gamma)q_0) + (1-\alpha_0)(1-q_1-(1+\gamma)q_0) = 0, \quad (8)\\ \frac{\partial U_1}{\partial q_1} &= \alpha_1 \frac{\partial \pi_1}{\partial q_1} + (1-\alpha_1)\frac{\partial CS}{\partial q_1} \\ &= \alpha_1(1-q_0-(2+\gamma)q_1) + (1-\alpha_1)(q_0+q_1) = 0. \quad (9)\end{aligned}$$

SOCs are satisfied. The reaction functions and the equilibrium pair of quantities are

$$q_0^R = \frac{1-q_1}{1+\gamma+\alpha_0}, \quad q_1^R = \frac{\alpha_1 - (2\alpha_1 - 1)q_0}{3\alpha_1 - 1 + \alpha_1\gamma}, \quad (10)$$

$$\begin{aligned} q_0^*(\alpha_0,\alpha_1) &= \frac{\alpha_1(2+\gamma)-1}{\alpha_0(3\alpha_1 - 1 + \alpha_1\gamma) + \alpha_1(1+4\gamma+\gamma^2) - \gamma}, \\ q_1^*(\alpha_0,\alpha_1) &= \frac{\alpha_1(-1+\alpha_0+\gamma)+1}{\alpha_0(3\alpha_1 - 1 + \alpha_1\gamma) + \alpha_1(1+4\gamma+\gamma^2) - \gamma}. \end{aligned} \quad (11)$$

The above equations lead to the following lemma, which confirms a well-known aggressive role of CSR and less-aggressive role of privatization.

Lemma 1.

$$\frac{\partial q_i^*(\alpha_0,\alpha_1)}{\partial \alpha_i} < 0, \quad \frac{\partial q_i^*(\alpha_0,\alpha_1)}{\partial \alpha_j} > 0 \quad for\ i=0,1,\ i \neq j. \quad (12)$$

Proof. See Appendix. □

3.2 Commitment stage

Firms solve each maximizing problem, either (6) or (7), which is subject to the constraint (11). The reaction functions in this stage are,

$$\begin{aligned} \alpha_0^R(\alpha_1) &= \max\left\{\frac{(6\alpha_1^2 - 7\alpha_1 + 2)\,\gamma}{\alpha_1\,(\alpha_1\,(\gamma^2+5\gamma+2) - 2\gamma - 1)}, 0\right\}, \\ \alpha_1^R(\alpha_0) &= \frac{\alpha_0^2 + \alpha_0(3\gamma+2) + \gamma(2\gamma+3)}{\alpha_0^2 + 3\alpha_0(\gamma+1) + 2\gamma(\gamma+2)}. \end{aligned} \quad (13)$$

Differentiating both reaction functions with respect to each argument gives strategic relationships between privatization and CSR.

Proposition 1.

1. *For $\gamma > 0$, α_0 is a strategic complement to α_1. In other words, a degree of privatization is a strategic substitute to a degree of CSR.*

2. *For $\gamma \geq 1$, α_1 is a strategic complement to α_0. In other words, a degree of CSR is a strategic substitute to a degree of privatization.*

3. *For $0 < \gamma < 1$, α_1 is a strategic substitute in a range of $0 \leq \alpha_0 < \sqrt{\gamma} - \gamma$ and complement otherwise.*

Proof. See Appendix. □

The first part of Lemma 2 can be understood in the following way. The public firm fundamentally faces a dilemma, to improve welfare, that it tries to improve CS by increasing production while at the same time to improve production cost efficiency by sharing total production with the private firm as equally as possible. Since when the private is purely own profit oriented, i.e., $\alpha_1 = 1$, q_1 is the smallest from Lemma 1, so the public needs to be moderate in production not to worsen cost inefficiency. Thus, again from Lemma 1, it privatizes the most. Yet, as the private starts CSR, it increases production, so the public has less anxiety for cost inefficiency and can increase production. Thus in this case, as CSR strengthened, the partial privatized firm become more public.

While the first part of Lemma 2 is from the public side's dilemma, the second and third part is owing to the one of the private side, a revenue and a cost. Consider infinite small increment of privatization from the point $(0, \alpha_1^R(0))$, where the public is fully nationalized and the private optimally respond to it with CSR. Due to the privatization, q_0 decreases in its first order while q_1 increases in its second order. With the first order effect only, the price would rise with other values fixed. This motivates the private firm to acquire more revenue by increasing production, so it does to promote CSR more. On the other hand, with the second order effect only, the price would drop and the cost inflate, so the private would like to withhold CSR to reduce production. The total effect is a sum of the two, thus ambiguous. However, as the marginal cost becomes heavier due to higher γ or q_1 (because of higher α_0), the second order tends to dominate the first. Therefore, beyond the threshold, CSR comes to a strategic substitute to partial privatization from a strategic complement.

Proposition 1 is about global properties of the reaction functions, and next Lemma 3 states the local property at the equilibrium.

Corollary 1.

$$\exists \gamma^* \in (0,1) \quad s.t. \quad 0 < \gamma < \gamma^* \Leftrightarrow \left.\frac{d\alpha_1}{d\alpha_0}\right|_{\alpha=\alpha^*} < 0, \quad \gamma^* \leq \gamma \Leftrightarrow \left.\frac{d\alpha_1}{d\alpha_0}\right|_{\alpha=\alpha^*} \geq 0. \tag{14}$$

Proof. See Appendix. □

The approximate value of γ^* is .8. Lemma 3 claims the reversibility of strategic relationship of CSR to privatization matters on the equilibrium, as well as off-path. As is well known, the strategic relationships of strategic values are crucial in, if any, an endogenous timing game, which is discussed in Appendix.

4 Welfare type of CSR

While up to this point the consumer surplus represents CSR, there also are papers in which social welfare plays that role[2]. Which specifications match the reality better still seems under discussion. This section is devoted to compare the welfare one to the consumer surplus one by checking if the former supports the same results as the latter. It turns out the results are different. The reasons of the difference is addressed after showing the results.

4.1 Equilibrium with SW type CSR

The game structure is exactly the same as before. The formulation is also the same with one exception. It is SW instead of CS the private takes into account in its objective function of the production stage. Accordingly, the domain of α_1 becomes to $[0, 1]$ as α_0. In order to highlight values are calculated under SW type CSR setting, overlines are added appropriately.

The FOCs, reaction functions, and equilibrium quantities in the production stage are,

$$\begin{aligned}\frac{\partial U_i}{\partial q_i} &= \overline{\alpha}_i \frac{\partial \pi_i}{\partial q_i} + (1-\overline{\alpha}_i)\frac{\partial SW}{\partial q_i} \\ &= \overline{\alpha}_i(1-\overline{q}_j-(2+\gamma)\overline{q}_i) + (1-\overline{\alpha}_i)(1-\overline{q}_j-(1+\gamma)\overline{q}_i) = 0,\end{aligned} \tag{15}$$

$$\overline{q}_i^R = \frac{1-\overline{q}_j}{1+\gamma+\overline{\alpha}_i}, \tag{16}$$

$$\overline{q}_i^*(\overline{\alpha}_i, \overline{\alpha}_j) = \frac{\overline{\alpha}_i+\gamma}{(\overline{\alpha}_i+\overline{\alpha}_j)(1+\gamma)+\overline{\alpha}_i\overline{\alpha}_j+\gamma(2+\gamma)}. \tag{17}$$

Then, the commitment stage generates the following reaction functions of CSR and privatization.

$$\overline{\alpha}_0^R(\overline{\alpha}_1) = \frac{\overline{\alpha}_1\gamma}{\gamma+(\overline{\alpha}_1+\gamma)^2}, \quad \overline{\alpha}_1^R(\overline{\alpha}_0) = \frac{\overline{\alpha}_0+\gamma}{1+\overline{\alpha}_0+\gamma}. \tag{18}$$

Given the reaction functions, the next proposition, which is a CSR-by-SW counterpart to Lemma 2 in the CSR-by-CS case, holds.

[2]For examples,Yasui and Haraguchi (2018), Ghosh and Mitra (2014) and Matsumura and Ogawa (2014)

Proposition 2.

1. *For* $0 < \gamma < \left(\sqrt{5}-1\right)/2$*,* $\overline{\alpha}_0$ *is a strategic substitute to* $\overline{\alpha}_1$ *in a range of* $0 < \overline{\alpha_1} < \sqrt{\gamma^2+\gamma}$ *and complement otherwise.*

2. *For* $\left(\sqrt{5}-1\right)/2 \leq \gamma$*,* $\overline{\alpha}_0$ *is a strategic complement to* $\overline{\alpha}_1$*. In other words, a degree of privatization is a strategic substitute to a degree of CSR.*

3. *For* $\gamma > 0$*,* $\overline{\alpha}_1$ *is a strategic complement to* $\overline{\alpha}_0$*. In other words, a degree of CSR is a strategic substitute to a degree of privatization.*

In comparison with Proposition 1, there are two main differences. The first one is that the strategic relation of privatization to CSR depends on the cost efficiency. The other one is that of CSR to privatization does not. Though the differences are two, they share one and the same root. It is expressed in the following lemma and corollary.

Lemma 2.

$$\forall \alpha_1 \in (1/2, 1) \ \ \forall \overline{\alpha}_1 \in [0, 1) \ \ \frac{\partial^2 q_1^R(q_0)}{\partial \alpha_1 \partial q_0} < 0 < \frac{\partial^2 \overline{q}_1^R(q_0)}{\partial \overline{\alpha}_1 \partial q_0}. \tag{19}$$

Proof. Though proving on the specification herein is straightforward, we provide a proof on more general assumptions. Suppose for $i, j = 0, 1$ $i \neq j$, $(\partial^2 \pi_i / \partial {q_i}^2)$, $(\partial^2 \pi_i / \partial q_i \partial q_j)$, $(\partial^2 SW / \partial {q_i}^2)$, and $(\partial^2 SW / \partial q_i \partial q_j)$ all exist and are negative and $(\partial^2 CS / \partial {q_i}^2)$ and $(\partial^2 CS / \partial q_i \partial q_j)$ exist and positive. Suppose additionally, α_i is defined in the intersectional range of $(\partial^2 U_i / \partial {q_i}^2) < 0$ and $[0, 1]$.

Under these assumptions, the two cross derivatives are,

$$\frac{\partial^2 q_1^R(q_0)}{\partial \alpha_1 \partial q_0} = \frac{-(p' - c_i'')\,(p' + p''(q_i + q_j))}{\left(\partial^2 U_i / \partial q_i^2\right)^2} = \frac{(\partial^2 SW / \partial q_i^2)(\partial^2 CS / \partial q_i^2)}{\left(\partial^2 U_i / \partial q_i^2\right)^2} < 0, \tag{20}$$

$$\frac{\partial^2 \overline{q}_1^R(q_0)}{\partial \overline{\alpha}_1 \partial q_0} = \frac{{p'}^2 + p'' q_1 c_1''}{\left(\partial^2 U_i / \partial q_i^2\right)^2} > \frac{(p' + p'' q_1)^2}{\left(\partial^2 U_i / \partial q_i^2\right)^2} > 0. \tag{21}$$

The first inequality in (21) is from strict concavity of the profit function. Q.E.D. □

Corollary 2.

$$\forall \alpha_1 \in (1/2, 1) \ \ \forall \overline{\alpha}_1 \in [0, 1) \ \ 0 > \frac{\partial \overline{q}_1^R(q_0)}{\partial q_0} > \frac{\partial q_1^R(q_0)}{\partial q_0}. \tag{22}$$

If firms try to maximize consumer surplus only, all they have to do is just producing infinite amounts regardless of competitors' actions. In contrast, if firms take care welfare only like as the fully nationalized firm, they have to more flexibly adjust own production in response to competitors' one than the pure profit maximizer does. Therefore, as the private firm doing CSR-by-CS, it pays less attention to the competitor and doing

CSR-by-SW, vice versa. It is this difference of sensitivity to competitors' action that generates a number of different results depending on the CS type CSR or SW type.

To see how the sensitivity matters in the first difference in Proposition 1 and 2, the strategic relation of privatization to CSR can be reversed under SW type CSR and not under CS type, the point of $(\alpha_0^R(1), 1)$ is important. Since, at that point, the private firm behaves as a pure private and the public responds with its optimally adjusted privatization, the difference of SW and CS has no effects yet. The initiation of CSR out of that point increases welfare through both total production and cost efficiency improvements. In response to the CSR, the public becomes able to improve welfare more than the status quo, by adjusting privatization. On its choice, there are potentially two opposite directions, either increasing or decreasing its production, i.e., nationalizing or privatizing. Since at the status quo, the public produces more than the private, increasing production improves total production but worsens the cost efficiency and vice versa. Hence, the public direction to pursue depends on the relative sizes of two effects. With CS type CSR, total production improvement dominates the cost efficiency worsening, so the public increases its production by nationalizing. With SW type CSR the opposite can hold, so the public can decrease by privatizing.

The determinant relative sizes in turn depend on the slope of the private reaction function, or sensitivity to the public production. The steeper slope, which is the case with the SW type CSR, makes the efficiency weight heavier, because small reduction of the public production induces a substantial increase of the private production. At the same time, it makes the total production harder to change, due to the higher elasticity of substitution. On the other hand, with the flatter reaction curve, as with the CS type, the opposite holds. Therefore, with SW type and the higher sensitivity, the cost efficiency improvement can dominate the worsen consumer surplus if γ and CSR are small enough. The reason of being limited with small CSR is that as CSR increases and privatization follows the production gap shrinks, so does the efficiency improvement. The reason of small γ is that with large γ, the public cannot produce large amounts in the first place, so the production gap, and hence cost inefficiency is small.

Secondly, the other difference in Proposition 1 and 2, that the strategic relation of CSR to privatization is fixed under SW type CSR but not under CS type, is also due to the sensitivity difference. As noted in the exposition after Proposition 1, the strategic relation depends on the relative sizes of the first and second order effects of privatization on the private profit. If the first order one, the decline of q_0, is large, then the private would like to promote CSR and be aggressive. Otherwise the second order, the q_1 increase, is large, the private withhold CSR and be less aggressive. Therefore, with SW type CSR, i.e., with the steeper reaction curve, the latter case is more likely to happen. This sensitive reaction is so dominant a factor in the specification for CSR to be a strategic substitute to privatization regardless of γ.

As in the CS type CSR, the next lemma characterizes the equilibrium.

Corollary 3.

$$\forall \gamma > 0 \quad \left. \frac{d\overline{\alpha}_i}{d\overline{\alpha}_j} \right|_{\overline{\alpha}=\overline{\alpha}^*} > 0 \quad i, j = 0, 1, \ i \neq j. \tag{23}$$

Proof. See Appendix. □

Corollary 3 claims that though the strategic relationship of privatization for CSR depends globally, at the equilibrium it is always a strategic substitute, as in corollary 2. That of CSR for privatization also does not depend on the marginal cost, but this is in contrast in corollary 2 owing to the difference explained above.

5 Conclusion

In this paper we analyze strategic relationships between CSR and privatization using more general specifications than in the literature. As a result we have possibilities of strategic complementarity which has not been confirmed yet. Specifically, if the private firm which has high cost efficiency cares about consumer surplus in addition to its profit, then the CSR can be strategic complement to privatization. On the other hand, if the firm's CSR takes a form of social welfare, then privatization can be strategic complement to CSR. We also discuss the reason why the results depend on which form CSR takes, consumer surplus or social welfare. Though they are similar in terms of making firms aggressive, they are opposite with regard to the sensitivity to competitors. With consumer surplus, firms become less sensitive to competing firms, whereas become sensitive with social welfare. The difference of sensitivities leads to the different strategic relationship. One possible policy implication out of this study is that policy-makers in government sectors should pay attention to private firms' cost efficiency and sensitivity to public sector to induce more CSR with some measures. Though our calculations still rely on the specific functional forms, we believe the intuitive mechanism behind the results explained after equations contains robustness to some degree.

Appendix

Proof of Lemma 1

Differentiating two equations in (11) by each degree yields the jaccobian matrix,

$$\begin{pmatrix} \dfrac{\partial q_0^*}{\partial \alpha_0} & \dfrac{\partial q_0^*}{\partial \alpha_1} \\ \dfrac{\partial q_1^*}{\partial \alpha_0} & \dfrac{\partial q_1^*}{\partial \alpha_1} \end{pmatrix} = \frac{1}{A} \begin{pmatrix} -(\alpha_1(\gamma+2)-1)(\alpha_1(\gamma+3)-1) & \alpha_0+2\gamma+1 \\ (2\alpha_1-1)(\alpha_1(\gamma+2)-1) & -(\alpha_0^2+\alpha_0(3\gamma+2)+2\gamma^2+3\gamma+1) \end{pmatrix} \tag{24}$$

$$\textit{where} \quad A = \left(\alpha_0(\alpha_1(\gamma+3)-1)+\alpha_1\left(\gamma^2+4\gamma+1\right)-\gamma\right)^2 > 0.$$

Therefore for $\alpha_0 \in [0,1]$, $\alpha_1 \in (1/2, 1]$, and $\gamma > 0$ the statement holds. Q.E.D.

Proof of Proposition 1

Differentiating the reaction function of α_0 in (13), we have,

$$\frac{d\alpha_0}{d\alpha_1} = \begin{cases} \dfrac{\gamma\left(\alpha_1^2\left(7\gamma^2+23\gamma+8\right)-4\alpha_1\left(\gamma^2+5\gamma+2\right)+4\gamma+2\right)}{\alpha_1^2\left(-\alpha_1\left(\gamma^2+5\gamma+2\right)+2\gamma+1\right)^2}, & \alpha_1 \in [2/3,1], \\ 0 & \textit{otherwise}. \end{cases} \tag{25}$$

The sign in the range of $\alpha_1 \in [2/3,1]$ is determined by the numerator. Note that the numerator takes U-shape in terms of α_1, and its value at $\alpha_1 = 2/3$ is $2\gamma\left(2\gamma^2+4\gamma+1\right)/9 > 0$. Thus the first part of Lemma 2 is proven. Then, for the other reaction function, we have,

$$\frac{d\alpha_1}{d\alpha_0} = \frac{\alpha_0^2+2\alpha_0\gamma+(\gamma-1)\gamma}{\left(\alpha_0^2+3\alpha_0(\gamma+1)+2\gamma(\gamma+2)\right)^2} \lesseqgtr 0 \quad \Leftrightarrow \quad \alpha_0 \lesseqgtr \sqrt{\gamma}-\gamma. \tag{26}$$

Therefore when $\gamma \in (0,1)$ α_1 has a range of strategic complement to α_0 near the full nationalization, and otherwise substitute. Q.E.D.

Proof of Corollary 1

If $\gamma > 1$, the result is trivial, so we restrict our attention to the case $\gamma \in (0,1]$. In the following we compare the point $(\sqrt{\gamma}-\gamma, \alpha_1(\sqrt{\gamma}-\gamma))$ and the point $(\alpha_0(\alpha_1(\sqrt{\gamma}-\gamma)), \alpha_1(\sqrt{\gamma}-\gamma))$ along with α_0 axis. If $\sqrt{\gamma}-\gamma < \alpha_0(\alpha_1(\sqrt{\gamma}-\gamma))$, then the reaction functions intersect at the point of mutual strategic complement, and otherwise α_1 is strategic substitute to α_0.

To begin with, from the strategic complementarity of α_0 to α_1, we have,

$$0 < \alpha_0(\alpha_1(\sqrt{\gamma}-\gamma)) < \alpha_0(1) = \frac{\gamma}{1+3\gamma+\gamma^2} < \gamma. \tag{27}$$

So the public reaction point under consideration is bounded from above by γ. In turn, the private point has following property.

$$\frac{d(\sqrt{\gamma}-\gamma)}{d\gamma} = \frac{1}{2\sqrt{\gamma}} - 1 \gtreqless 0 \Leftrightarrow \gamma \lesseqgtr \frac{1}{4}. \tag{28}$$

Note that $\sqrt{\gamma}-\gamma = 1/4$ at $\gamma = 1/4$. Therefore in the range of $(0,1/4]$, $\sqrt{\gamma}-\gamma > \alpha_0(\alpha_1(\sqrt{\gamma}-\gamma))$. For $1/4 < \gamma \leq 1$, $\sqrt{\gamma}-\gamma$ decreases to 0, so if $\alpha_0(\alpha_1(\sqrt{\gamma}-\gamma))$ is increasing in this range, the existence of the unique threshold is guaranteed.

$$\begin{aligned} &\frac{d\alpha_0(\alpha_1(\sqrt{\gamma}-\gamma))}{d\gamma} \\ &= \frac{80\gamma^{\frac{3}{2}}+138\gamma^{\frac{5}{2}}+7\gamma^{\frac{7}{2}}-47\gamma^{\frac{9}{2}}-8\gamma^{\frac{11}{2}}-\gamma^6-26\gamma^5-44\gamma^4+86\gamma^3+129\gamma^2+32\gamma+6\sqrt{\gamma}}{\left(\gamma+2\sqrt{\gamma}+2\right)^2\left(6\gamma^{\frac{3}{2}}+2\gamma^{\frac{5}{2}}+\gamma^3+5\gamma^2+5\gamma+2\sqrt{\gamma}+1\right)^2} \\ &> \frac{80\gamma^{\frac{3}{2}}+138\gamma^{\frac{5}{2}}+7\gamma^{\frac{7}{2}}+86\gamma^3+3\gamma^2+32\gamma+6\sqrt{\gamma}}{\left(\gamma+2\sqrt{\gamma}+2\right)^2\left(6\gamma^{\frac{3}{2}}+2\gamma^{\frac{5}{2}}+\gamma^3+5\gamma^2+5\gamma+2\sqrt{\gamma}+1\right)^2} > 0. \end{aligned}$$

Q.E.D.

Proof of Proposition 2

Differentiating two reaction functions in (18) yields,

$$\frac{d\alpha_0^R}{d\alpha_1} = \frac{\gamma\left(-\alpha_1^2+\gamma^2+\gamma\right)}{\left(\alpha_1^2+2\alpha_1\gamma+\gamma^2+\gamma\right)^2}, \quad \frac{d\alpha_1^R}{d\alpha_0} = \frac{1}{(\alpha_0+\gamma+1)^2}. \tag{29}$$

The left is straightforward as in the proof of Lemma 2. Q.E.D.

Proof of Corollary 3

The next equation demonstrates that the private reaction curve passes under, if any, the corner of the public one.

$$For\ \gamma > 0, \quad \sqrt{\gamma^2+\gamma} - \alpha_1^R(\alpha_0^R(\sqrt{\gamma^2+\gamma})) = \frac{(3\gamma+2)\sqrt{\gamma^2+\gamma}-\gamma}{3\gamma+4} > 0. \tag{30}$$

Q.E.D.

Endogenous timing game

Here we discuss an endogenous timing game of CSR and privatization *à la* Hamilton and Slutsky (1990). The previous first stage of the game is split into two stages, thus hereafter the game consists of three stages as a whole. In the first stage, both firms have choices with respect to timing of commitment of either privatization or CSR, whether early of later. The second, ex-first, stage is played simultaneously or sequentially depending of the result of the first stage. For example, if in the first stage, say, the public firm chooses early and the private chooses later, then in the second stage, the public privatizes first and the private decides the amount of CSR following it, and vice versa. Alternatively, if both of them choose the same timing, early or later, in the first stage, the second stage is a plain simultaneous game as illustrated above. Following the two commitment stages, the third, production stage are played simultaneously.

Proposition 3. *In the endogenous timing game, where the public firm and the private choose timing of privatization of CSR commitment, the equilibrium is that,*

- *For* $0 < \gamma < \gamma^*$*, the public firm commits as a leader and the private follower.*
- *For* $\gamma^* \leq \gamma$*, both try to be a leader thus the simultaneous structure occurs.*

The resultant timing structure of Proposition 1 fundamentally owes to Lemma 3, which is new in the literature. Therefore the leader-follower structure of privatization and CSR in the range of high cost efficiency has not ever been supported until this study, at least with this type of game. However, as examples in Introduction shows, it seems the real structure with plausibility. The behind intuition is following. When the public firm knows that the private adjusts its degree of CSR in accordance with the one of privatization, the public predicts incremental CSR follows the privatization (in a relevant range) if the costs are sufficiently efficient. This is because of the private firm's rationale illustrated in Lemma 2. The incremental CSR compensates the welfare loss due to privatization. From the view point of the private also, the leader-follower structure is welcome, since the market share of the private in the production stage increases, inducing higher profits. Therefore the leader-follower structure is supported by both firms, so it is the equilibrium. The resultant amounts of CSR and privatization are summarized in the following corollary.

Corollary 4. *Comparing to the simultaneous choice version, the public leader and private follower structure induces,*

- *more CSR and privatization if costs are efficient enough,*
- *less CSR and more privatization otherwise.*

Proof. In preparation for the proof of the endogenous timing game, we put the next lemma.

Lemma 3.

$$\begin{aligned}
\frac{dSW(\alpha_0(\alpha_1),\alpha_1)}{d\alpha_1} &= \frac{\partial SW}{\partial \alpha_0}\frac{d\alpha_0}{d\alpha_1} + \frac{\partial SW}{\partial \alpha_1} = \left.\frac{\partial SW}{\partial \alpha_1}\right|_{\alpha_0=\alpha_0(\alpha_1)} \\
&= -\frac{(3\alpha_1-2)\gamma\left(2\alpha_1\gamma^2+7\alpha_1\gamma+\alpha_1-2\gamma\right)}{\left(\alpha_1^2\left(\gamma^3+7\gamma^2+15\gamma+1\right)-2\alpha_1\gamma(\gamma+5)+2\gamma\right)^2} < 0, \qquad (31) \\
\frac{d\pi_1(\alpha_0,\alpha_1(\alpha_0))}{d\alpha_0} &= \frac{\partial \pi_1}{\partial \alpha_0} + \frac{\partial \pi_1}{\partial \alpha_1}\frac{d\alpha_1}{d\alpha_0} = \left.\frac{\partial \pi_1}{\partial \alpha_0}\right|_{\alpha_1=\alpha_1(\alpha_0)} \\
&= \frac{(\alpha_0+\gamma)(\alpha_0(\gamma+1)+\gamma(\gamma+2))}{(\alpha_0+\gamma+1)^2(\alpha_0(\gamma+2)+\gamma(\gamma+3))^2} > 0. \qquad (32)
\end{aligned}$$

Second equalities in each equation are from the envelope theorem. What the lemma means is that along each reaction function each ultimate objective increases or decreases with competitor's commitment. Given this, the hypothetical leader firm picks the best point on the competitor's reaction curve. Let (α_i^L, α_j^F) denote that best point when the firm i is a leader and firm j follower. Then it necessarily satisfies the property either $\alpha_1^* \geq \alpha_1^F$ or $\alpha_0^* \leq \alpha_0^F$. For expository simplicity, take a public leader case. Suppose the leader choose the point such as $\alpha_1^F > \alpha_1^*$, then $SW(\alpha_0^L, \alpha_1^F) < SW(\alpha_0(\alpha_1^F), \alpha_1^F) < SW(\alpha_0^*, \alpha_1^*)$. The first inequality comes from the definition of reaction functions and the second one from Lemma 3. Hence the point (α_0^L, α_1^F) is strictly dominated by the simultaneous one, this is contradiction. Similar arguments hold in the private leader case. As for α_i^L to α_i^*, the relative positions depend on whether the reaction curve of firm j has a positive or negative slope.

Lastly, owing to the following equations, the two weak inequalities above are actually strict ones, except $\gamma = \gamma^*$ case.

$$sgn\left(\left.\frac{dSW(\alpha_0, \alpha_1(\alpha_0))}{d\alpha_0}\right|_{(\alpha_0^*, \alpha_1^*)}\right) = sgn\left(\frac{\partial SW}{\partial \alpha_0} + \frac{\partial SW}{\partial \alpha_1}\frac{d\alpha_1}{d\alpha_0}\right) = -sgn\left(\frac{d\alpha_1}{d\alpha_0}\right), \quad (33)$$

$$sgn\left(\left.\frac{d\pi_1(\alpha_0(\alpha_1), \alpha_1)}{d\alpha_1}\right|_{(\alpha_0^*, \alpha_1^*)}\right) = sgn\left(\frac{\partial \pi_1}{\partial \alpha_0}\frac{d\alpha_0}{d\alpha_1} + \frac{\partial \pi_1}{\partial \alpha_1}\right) = sgn\left(\frac{d\alpha_0}{d\alpha_1}\right). \quad (34)$$

Therefore if, say, the private is a leader, the point chosen is upper right to the simultaneous point in the $\alpha_0 - \alpha_1$ coordinate plane. At that point the private profit of course increases compared to the simultaneous one, but the welfare on the other hand decreases as in Lemma 3. Hence, the public deviates from its follower role. The similar results hold for the public leader case if α_1 is strategic complementary to α_0 at the simultaneous equilibrium. However, if it is strategic substitute at the intersection, then the private profit also increases, so it has no deviation incentive from the follower position. Q.E.D. □

Endogenous Timing Game with SW type CSR

Finally, we check if the SW type CSR can endogenously support the privatization leader and CSR follower structure like as the CS type one.

Proposition 4. *In the endogenous timing game in the commitment stage with privatization and the welfare type CSR, the equilibrium is a simultaneous structure in consequence of a common incentive for the leader.*

Proof. The counterpart of Lemma 3 is following.

$$\frac{dSW(\alpha_0^R(\alpha_1), \alpha_1)}{d\alpha_1} = \left.\frac{\partial SW}{\partial \alpha_1}\right|_{\alpha_0 = \alpha_0^R(\alpha_1)} = \frac{-\alpha_1\gamma^2(\alpha_1 + \gamma + 1)}{\left(\alpha_1^2(\gamma+1) + 2\alpha_1\gamma(\gamma+2) + \gamma\left(\gamma^2 + 3\gamma + 2\right)\right)^2} < 0, \quad (35)$$

$$\frac{d\pi_1(\alpha_0, \alpha_1^R(\alpha_0))}{d\alpha_0} = \left.\frac{\partial \pi_1}{\partial \alpha_0}\right|_{\alpha_1 = \alpha_1^R(\alpha_0)} = \frac{(\alpha_0 + \gamma)(\alpha_0(\gamma+1) + \gamma(\gamma+2))}{(\alpha_0 + \gamma + 1)^2(\alpha_0(\gamma+2) + \gamma(\gamma+3))^2} > 0. \quad (36)$$

Thus, the similar arguments as in Proposition 1 remain valid. Q.E.D. □

Because of the mutual strategic substitutability induced by SW type CSR, both try to be a leader, otherwise their objectives being impaired. Therefore, if the privatization leader CSR follower structure is imposed for some reason, it has a possibility of crowding out private CSR acts, though improving overall welfare.

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Chapter 2

Partial Privatization under Multimarket Price Competition*

Taku Masuda† and Susumu Sato‡

Abstract

We investigate the effects of multimarket contacts on the privatization policy in a mixed duopoly under price competition. There are two markets. One is served solely by a state-owned public firm, and the other is served by both public and private firms. The markets are linked by the production technology of the public firm. Using a relatively general model, we first show that privatization is never optimal in the absence of multimarket contacts (i.e., there is only one monopoly or duopoly market). However, in the presence of multimarket contacts, privatization can be optimal. We provide a parametric example for this possibility using a linear-quadratic specification. The characterization of optimal privatization demonstrates its non-monotonicity with respect to the relative sizes of the two markets and the degree of product differentiation.

JEL classification H42, L33

Keywords Multimarket contacts, partial privatization, state-owned public enterprise

1 Introduction

Since the early 1980s, we have observed a worldwide wave of privatization of state-owned public enterprises. Nevertheless, many public and semi-public enterprises (i.e., firms owned by both public and private sectors) are still active in planned and market economies in developed, developing, and transitional countries. While some public enterprises are traditional monopolists in natural monopoly markets, a considerable number of public (including semi-public) enterprises compete with private enterprises

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†Graduate School of Economics, The University of Tokyo, Email:taku121281@gmail.com

‡Graduate School of Economics, The University of Tokyo, Email:susumusato@live.jp

in a wide range of industries.[1] Optimal privatization policies in such mixed oligopolies have attracted extensive attention from economics researchers in fields such as industrial organization, public economics, financial economics, international economics, development economics, and political economy.[2]

Specifically, drawing on the result of Matsumura (1998) that full nationalization is never optimal in a Cournot mixed duopoly, many studies on mixed oligopolies investigate how economic environments affect the optimal degree of privatization.[3] Most studies on privatization policies in mixed oligopolies use the quantity competition model to characterize an optimal privatization policy. However, there are many applications where it is more plausible to assume that firms compete in price.[4] In addition, as shown by Matsumura and Ogawa (2012), when public and private enterprises can choose whether to compete in price or quantity, they choose to compete in price in the equilibrium. Therefore, discussing optimal privatization policies under price competition is also important from both practical and theoretical perspectives. That said, the literature on mixed oligopolies recognizes that a privatization policy, as a device used to change public firm's objective to one of profit maximization, does not improve welfare. The reason is that privatization increases the prices of public firms and those of private firms through their strategic interaction, both of which harm welfare.

We argue that this rationale depends on the assumption that firms compete in a single market. If the public firm operates in multiple markets, the result changes. Multimarket contacts are prevalent in realistic mixed oligopoly situations. For example, in transportation industries, there are a number of situations where the public firm provides its services in urban and rural regions, whereas private firms provide their services in urban areas only (e.g., Amtrak in the United States and Hokkaido Railway Company in Japan). These situations can happen when the public sector has the different objective from the private one or is subject to certain regulations, such as a universal service obligation. These cases can be viewed as multimarket contacts in a mixed oligopoly, where markets are geographically separated. Alternatively, firms may provide multiple products. For example, the Japanese government owns a share of major electricity providers and recently deregulated the electricity and gas markets so that electricity and gas providers could enter the other market. As another example, Japan Post provides two kinds of postal services, namely, letters and parcels, where the

[1]Examples include United States Postal Service, Deutsche Post AG, Areva, Nippon Telecom and Telecommunication, Électricité de France, and Korea Investment Corporation.

[2]The idea of a mixed oligopoly dates at least to Merrill and Schneider (1966). Recently, the literature on mixed oligopolies has become richer and more diverse. For examples of mixed oligopolies and recent developments in this field, see Ishibashi and Matsumura (2006), Ishida and Matsushima (2009), Colombo (2016), Chen (2017), Matsumura and Sunada (2013), Haraguchi and Matsumura (2018), and the papers cited therein.

[3]For example, see Lin and Matsumura (2012) for the share of foreign investors who purchase the stock of public firms, Matsumura and Kanda (2005) for free entry, and Sato and Matsumura (2017) for the shadow cost of public funds.

[4]For anslyses of price competition in mixed oligopolies, see Bárcena-Ruiz (2007), Matsumura (2012), Cremer et al. (1991), and Anderson et al. (1997).

former are provided exclusively by Japan Post. These two services can be viewed as two separated markets.[5] In these types of environments, we show an opposite result from the conventional wisdom in the literature, that a positive degree of privatization can be optimal. Our result reveals an important aspect of a privatization policy in a mixed oligopoly such as transportation or postal industries. In the presence of multimarket contacts, the privatization of the public firm can stimulate, rather than deter, the competition in urban districts through the improved production efficiency of the public firm.

Modeling the multimarket situation, this study examines a variation of the model of Kawasaki and Naito (2017) who make use of the framework of Bulow et al. (1985). Here, there are two markets, one of which is served solely by the state-owned public firm, and the other is served by both the public firm and private firm. The two markets are linked by the production cost of the public firm. As explained later, the optimality of privatization comes from the intra-firm production substitution of the public firm. An increase in the degree of privatization decreases the production of the public firm in the monopoly market. This decreases the marginal cost of production for the duopoly market, which raises the incentive to increase the production. When the degree of product differentiation between the public and private firms is small, the latter effect tends to dominate the unilateral effect of privatization to decrease the production in the duopoly market. Under price competition, this decreases the equilibrium price of the private firm through the strategic interaction and improves the welfare. This is the mechanism through which partial privatization can be optimal in the presence of multimarket contacts.

After showing that partial privatization can be optimal, the study proceeds to characterize the optimal degree of privatization with respect to the relative sizes of the markets and the degree of product differentiation. It shows that the optimal degree of privatization exhibits non-monotonicity (i.e., an inverted U-shape relation) in the relative size of the monopoly market and the degree of product substitution. Both of these parameters have two countervailing effects. On the one hand, an increase in the size of the monopoly market (or a decrease in product differentiation) makes an increase in the degree of privatization desirable owing to the competition-accelerating effect on the duopoly market. On the other hand, when the size of the monopoly market is too large (or the products in the duopoly market are almost homogeneous), the magnitudes of the competition-accelerating effects become nil, because the duopoly market is tiny relatively to the overall economy (or the competition in the duopoly is already sufficiently strong). The relative sizes of these two effects generate the non-monotonicity.

Several recent studies focus on privatization policies in multimarket mixed oligopoly settings. Bárcena-Ruiz and Garzón (2016) and Dong et al. (2018) consider the privatization policy of a state holding corporation that has plants operating in multiple markets, and show that the demand interdependence of the markets affects the optimal privatization policies. Haraguchi et al. (2018) consider the privatization of a public

[5] For other examples of multimarket contacts in a mixed oligopoly, see Kawasaki and Naito (2017).

enterprise in a mixed market in the presence of neighboring private markets. They show a non-monotone relationship between the optimal degree of privatization and the number of firms in the neighboring markets. There are two main difference between these studies and ours. First, the aforementioned studies consider quantity competition situations, whereas we consider price competition. As a result, the implication of the privatization policy on market competition in our study greatly differ from those that make use of the model of quantity competition. Second, the previous studies model the multimarket interaction by the demand interdependence between the markets, whereas we model it by the production cost of the public firm serving both markets. This modelling approach enables us to shed lights on the effects of privatization policy on the multimarket competition where markets seem to be independent in terms of demand conditions (e.g., geographically separated markets).

2 Model

The model herein is a mixed price competition version of the multimarket model *à la* Bulow et al. (1985). There are a state-owned public firm, firm 0, and a private firm, firm 1. There are two markets, A and B. Market A is solely provided by firm 0, while market B is provided by both firm 0 and firm 1. This means that the public firm serves two markets, in one of which it competes with the private firm.

The representative consumer in market A is characterized by its relative size $\phi \in [0,1]$ and the utility function $U^A(x_0^A) + y^A$, where x_0^A is the amount of the consumption of the products provided by firm 0 and y^A is the consumption of the composite goods. The representative consumer in market B is characterized by its relative size $(1-\phi)$ and the utility function $U^B(x_0^B, x_1^B) + y^B$, where x_0^B and x_1^B are the amounts of the consumption of the products provided by firm 0 and firm 1, and y^B is the consumption of the composite goods. Assuming that the representative consumer in each market has enough income and U^A and U^B are concave, its consumption is derived from the first-order conditions

$$\frac{\partial U^A}{\partial x_0^A} = p_0^A,\ \frac{\partial U^B}{\partial x_0^B} = p_0^B,\ \text{and}\ \frac{\partial U^B}{\partial x_1^B} = p_1^B, \tag{1}$$

where p_0^A, p_0^B, and p_1^B are prices of products. We denote $D^A(p_0^A)$, $D_0^B(p_0^B, p_1^B)$, and $D_0^B(p_0^B, p_1^B)$ as the demand functions and $CS^A(p_0^A)$ and $CS^B(p_0^B, p_1^B)$ as the consumer surpluses. Note that, by the envelope theorem, $\partial CS^A/\partial p_0^A = -D_0^A$ and $\partial CS^B/\partial p_i^B = -D_i^B$, $i = 0, 1$ hold.

We assume that the products in market B are substitutes, i.e., $\partial D_i^B/\partial p_j^B < 0$ for $i \neq j$. We also assume that the demands are symmetric, that is, $D_0^B(x, x) = D_1^B(x, x)$. Further, we assume that the demand functions satisfy the following regularity condition

$$\frac{\partial D_i^B}{\partial p_i^B} + \frac{\partial D_i^B}{\partial p_j^B} < 0 \text{ for } i = 0, 1, j \neq i. \tag{2}$$

This condition means that if the prices of both firms simultaneously increase, the demands for both products decrease, which is natural to assume in many applications.

The production technologies of firms are given by cost functions $C_0(q_0^A, q_0^B)$ and $C_1(q_0^B)$. Then, the profit of each firm is given by

$$\Pi_0(p_0^A, p_0^B, p_1^B) = \phi D^A(p_0^A)p_0^A + (1-\phi)D_0^B(p_0^B, p_1^B)p_0^B - C_0(\phi D^A(p_0^A), (1-\phi)D_0^B(p_0^B, p_1^B)), \tag{3}$$

and

$$\Pi_1(p_0^B, p_1^B) = (1-\phi)D_0^B(p_0^B, p_1^B) - C_1((1-\phi)D_0^B(p_0^B, p_1^B)). \tag{4}$$

Social welfare SW is given by

$$SW = \phi CS^A(p_0^A) + (1-\phi)CS^B(p_0^B, p_1^B) + \Pi_0(p_0^A, p_0^B, p_1^B) + \Pi_1(p_0^B, p_1^B). \tag{5}$$

Firm 0 maximizes the weighted average of its own profit and social welfare

$$\Omega = \alpha\Pi_0 + (1-\alpha)SW, \tag{6}$$

where $\alpha \in [0, 1]$ is the degree of privatization.

3 Equilibrium

We adopt subgame-perfect equilibrium as the solution and solve the model by backward induction. In the market stage, the first-order conditions for firm 0 are given by[6]

$$\begin{aligned} \frac{\partial \Omega}{\partial p_0^A} &= \alpha\left(\frac{\partial D_0^A}{\partial p_0^A}\left(p_0^A - \frac{\partial C_0}{\partial q_0^A}\right) + D_0^A\right) + (1-\alpha)\frac{\partial D_0^A}{\partial p_0^A}\left(p_0^A - \frac{\partial C_0}{\partial q_0^A}\right) = 0, \\ \frac{\partial \Omega}{\partial p_0^B} &= \alpha\left(\frac{\partial D_0^B}{\partial p_0^B}\left(p_0^B - \frac{\partial C_0}{\partial q_0^B}\right) + D_0^B\right) + (1-\alpha)\left(\frac{\partial D_0^B}{\partial p_0^B}\left(p_0^B - \frac{\partial C_0}{\partial q_0^B}\right) + \frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right)\right) = 0, \end{aligned} \tag{7}$$

and the first-order condition for firm 1 is given by

$$\frac{\partial D_1^B}{\partial p_1^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right) + D_1^B = 0. \tag{8}$$

We assume that the second-order conditions are satisfied, i.e., the Hessian matrix of Ω is negative definite, and $\partial^2\Pi_1/\partial {p_1^B}^2 < 0$. We also assume that the strategy of firm 1 exhibits strategic complementarity, that is,

$$\frac{\partial D_1^B}{\partial p_0^B}\left(1 - \frac{\partial D_1^B}{\partial p_1^B}\frac{\partial^2 C_1}{\partial {q_1^B}^2}\right) + \frac{\partial^2 D_1^B}{\partial p_1^B \partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right) > 0. \tag{9}$$

[6]In the model of price competition with strictly convex costs, firms may have incentives not to serve all the amount demanded. We ignore such possibilities in this model since as shown by Matsumura (2012), if the public firm faces the universal service obligation, there are no such incentives.

A sufficient condition for strategic complementarity is that $\partial^2 D_1^B/(\partial p_1^B \partial p_0^B) \geq 0$ and C_1 being weakly convex.

Further, to guarantee the uniqueness and the stability of the equilibrium, we put the following restriction. Let $R_0^A(p_1^B)$ and $R_0^B(p_1^B)$ be the best-response functions of firm 0 and $R_1^B(p_0^B)$ be the best-response function of firm 1. We assume that $|\partial R_0^A/\partial p_1^B| < 1$, $|\partial R_0^B/\partial p_1^B| < 1$, and $|\partial R_1^B/\partial p_0^B| < 1$.

Let $p_0^A(\alpha)$, $p_0^B(\alpha)$, and $p_1^B(\alpha)$ be the equilibrium prices given α.

Next, in the privatization stage the government chooses $\alpha \in [0,1]$ to maximize SW. Let α^* be the welfare-maximizing value of α. In the case of interior solution, the first-order condition is given by

$$\begin{aligned}\left.\frac{dSW}{d\alpha}\right|_{\alpha=\alpha^*} =& \frac{dp_0^A}{d\alpha}\phi\frac{\partial D_0^A}{\partial p_0^A}\left(p_0^A - \frac{\partial C_0}{\partial q_0^A}\right) + (1-\phi)\frac{dp_0^B}{d\alpha}\left(\frac{\partial D_0^B}{\partial p_0^B}\left(p_0^B - \frac{\partial C_0}{\partial q_0^B}\right) + \frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right)\right) \\ &+ \frac{dp_1^B}{d\alpha}(1-\phi)\left(\frac{\partial D_0^B}{\partial p_1^B}\left(p_0^B - \frac{\partial C_0}{\partial q_0^B}\right) + \frac{\partial D_1^B}{\partial p_1^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right)\right) = 0. \end{aligned} \tag{10}$$

We assume that the second-order condition is satisfied. In cases of corner solutions, we have either $(dSW/d\alpha)|_{\alpha=0} \leq 0$ or $(dSW/d\alpha)|_{\alpha=1} \geq 0$.

It has been recognized that, in the public monopoly or mixed oligopoly with price competition, positive degree of privatization would never be optimal. The following lemma and proposition formalize the statement.

Lemma 1. *If $\phi = 0$, $(dp_0^B/d\alpha)|_{\alpha=0} > 0$ and $(dp_1^B/d\alpha)|_{\alpha=0} > 0$.*

Proof. See Appendix. □

Proposition 1. *If $\phi = 0$ or $\phi = 1$, full nationalization is optimal.*

Proof. See Appendix. □

The reason for the above result is that an increase in the degree of privatization from full nationalization increases the public firm's price since it leans to its own profit, which also increases the price of private firms through the strategic interaction. The former change has negligible effect on the welfare since the public firm is a welfare maximizer (envelope theorem), but the latter harms the welfare.

4 Partial Privatization with Multimarket Contact

4.1 General Form

The optimal privatization policy drastically changes under multimarket contexts. It is shown that the main driving force of the difference is a cost linkage of the public firm between markets.

With multimarkets, there mainly are two possible channels of interaction, which are the demand and the cost. We restrict our study to the cost, because a prime example of interest is a transportation industry where the services in some regions are provided by a single public sector while the services in other regions are provided by both public and private enterprises. If two markets are far away like in urban and rural areas, then the demand interaction seem to play less role than the cost[7]. Note that this modeling does not exclude multiproduct competitions in a single region such as letter and package deliveries.

The next lemma sheds light on where to focus when checking necessity of (at least) partial privatization.

Lemma 2.

$$sgn\left(\left.\frac{dSW}{d\alpha}\right|_{\alpha=0}\right) = -sgn\left(\left.\frac{dp_1^B}{d\alpha}\right|_{\alpha=0}\right) = -sgn\left(\left.\frac{\partial R_0^B}{\partial\alpha}\right|_{\alpha=0}\right) \tag{11}$$

Proof. See the Appendix. □

Deviating from full nationalization to partial privatization, its effect appears through the private firm's price change in the duopoly market only, due to the envelop theorem. The direction of the price change fully depends on the strategic complementarity, thus eventually on the direction of an adjustment of the public firm's best response function in the market. On this lemma, we demonstrate the critical role of the cost linkage.

Consider a small increase in α at $\alpha = 0$. We have

$$sgn\left(\frac{\partial R_0^B}{\partial\alpha}\right) = sgn\left(\frac{\partial p_0^B}{\partial\alpha} + \frac{dp_0^B}{dp_0^A}\frac{\partial p_0^A}{\partial\alpha}\right) \tag{12}$$

The first term in the right hand side and the second factor of the second term are the first order effects of privatizing onto public firm's prices. These are positive. The remaining factor consists of relative market sizes, a price effect on a demand in the monopoly market, cross derivative of the public firm's cost function, and pass-through effect. That is,

$$\frac{dp_0^B}{dp_0^A} = \phi(1-\phi)\frac{dD^A}{dp_0^A}\frac{\partial^2 C_0}{\partial q_0^B \partial q_0^A}\frac{dp_0^B}{d(\partial C_0/\partial q_0^B)}. \tag{13}$$

If the public firm's marginal cost function exhibits economies of scope, the sign of (11) is always positive, which implies that full nationalization is optimal by Lemma 2. We note this result as another lemma.

Lemma 3. *When the marginal cost function of the state-owned firm exhibits (weak) economies of scope, full nationalization is optimal.*

[7]Conjecturally, effects of the demand interaction depend on whether it is substitute or complement between markets. If two markets are in a substitutional relationship, privatization tends to be less optimal than in the present setting, more if complement any. This is because the price decreasing in the duopoly market caused by privatization in the monopoly market become weakened if substitutional and strengthened if complement.

By contrast, if the public firm's marginal cost function has a property of diseconomies of scope, the sign of (11) can be negative. If this happens, full nationalization is suboptimal by Lemma 2. This leads to the main result of our paper.

Proposition 2. *When the marginal cost function of the state-owned firm exhibits diseconomies of scope, full nationalization is possibly suboptimal.*

It is worth discussing the plausibility of our diseconomies of scope condition. In our model, the marginal cost of the state-owned firm exhibits diseconomies of scope if an increase in the production in one markets increases the marginal cost in the other market. This is plausible if there are several fixed inputs commonly used for production in both markets (e.g., managerial resources, factories for firm-specific inputs). In such a case, an increase in the production in one market causes a congestion in fixed inputs, increasing the marginal cost of production in the other market.

It is also worth mentioning that, even if the marginal cost function exhibits the diseconomies of scope, the total cost function can exhibit the economies of scope if there is a large fixed cost, which is the case in network industries. Thus, our assumption is compatible with the characteristics of industries of our interest, such as transportation or delivery industries.

One can also argue that if there are diseconomies of scope between markets, the privatized firm may have incentive to divide itself into two. There are several factors deterring such an incentive. First, establishing another company may incur a substantial fixed cost for recruiting managers, purchasing facilities, and so on. Second, if one market is so small as to go bankrupt after the division, such activity may be forbidden by the government due to a distributional concern.

In the next subsection, we present a parametric example as a proof, where partial privatization is optimal in nonempty set of parameter values.

4.2 Parametric Form

To show an example of Proposition 2, that is, the case where (partial) privatization is optimal with the public firm's marginal cost function which exhibits diseconomies of scope, we use the following quadratic utility and cost specification. $U^A(x_0^A) = x_0^A - (x_0^A)^2/2$, $U^B(x_0^B, x_1^B) = x_0^B + x_1^B - ((x_0^B)^2 + 2\gamma x_0^B x_1^B + (x_1^B)^2)/2$ for $\gamma \in (0,1)$[8], $C_0(q_0^A, q_0^B) = (q_0^A + q_0^B)^2/2$, and $C_1(q_1^B) = (q_1^B)^2/2$.[9] There is a potential issue of corner solution for consumer choice (i.e., $x_0^B = 0$) in market B since firm 0 has cost disadvantage. To avoid this complication, we restrict our attention to the range of parameter values (γ, ϕ) such that the demand for each good is positive in the equilibrium.[10] Then we yield

[8]In the case of complementary goods, the game reduces to the one studied in Kawasaki and Naito (2017), since price competition with complements has the same structure as quantity competition with substitutes.

[9]In this specification, all the assumptions put in the general model hold.

[10]A sufficient condition is $\gamma < T^{-1}(\phi)$, where $T(\gamma)$ is defined by $T(\gamma) = \frac{\gamma^3 - \gamma^2 - 2\gamma + 3}{-\gamma^3 + 2\gamma + 1}$, which decreases in γ.

$$\Pi_1 = p_1^B(1-\phi)\left(\frac{1-\gamma-p_1^B+\gamma * p_0^B}{1-\gamma^2}\right) - \frac{(1-\phi)^2}{2}\left(\frac{1-\gamma-p_1^B+\gamma p_0^B}{1-\gamma^2}\right)^2, \quad (14)$$

$$\Pi_o = p_0^A\phi(1-p_0^A) + p_0^B(1-\phi)\left(\frac{1-\gamma-p_0^B+\gamma p_1^B}{1-\gamma^2}\right) - \frac{1}{2}\left(\frac{(1-\phi)(1-\gamma-p_0^B+\gamma p_1^B)}{1-\gamma^2} + \phi(1-p)\right)^2,$$

$$CS^A = \frac{(1-p_0^A)^2}{2}, \quad (15)$$

and

$$CS^B = \frac{{p_0^B}^2 + {p_1^B}^2 + 2(1-p_0^B-p_1^B) - 2\gamma(1-p_1^B)(1-p_0^B)}{2(1-\gamma^2)}. \quad (16)$$

In this specification, we obtain the following lemma.

Lemma 4. *If $\phi \in (0,1)$ then,*

$$\exists\gamma^*(\phi) \;\; s.t. \;\; \forall\gamma \in (\gamma^*, \min\{T^{-1}(\phi), 1\}) \;\; \left.\frac{dp_0^B}{d\alpha}\right|_{\alpha=0} < 0 \; and \; \left.\frac{dp_1^B}{d\alpha}\right|_{\alpha=0} < 0, \quad (17)$$

$$and \; \forall\gamma \in (0, \gamma^*] \;\; \left.\frac{dp_0^B}{d\alpha}\right|_{\alpha=0} \geq 0 \; and \; \left.\frac{dp_1^B}{d\alpha}\right|_{\alpha=0} \geq 0. \quad (18)$$

Proof. See Appendix. □

The mechanism behind Lemma 4 is following. Departing from full nationalization to partial privatization makes a public enterprise lean to its own profit, and that basically marks up its prices in both markets. In a market solely supplied by the public firm especially, it leads to less production. Because of the less production in the one market, the public firm can have a room in its cost function to cut down the price in the other market. This pass-through effect gets stronger as their products being similar, and beyond some threshold it dominates the first mark-up effect. Finally, the dominant pass-through effect pulls down the competitor's price as well through strategic complement relationship.

Lemma 4 immediately yields a parametric version of our main proposition stating an optimality of the partial privatization in price competition, which never be optimal without multimarket contacts.

Proposition 3. *If $\phi \in (0,1)$ then for $\gamma^*(\phi)$ defined in Lemma 4,*

$$\forall\gamma \in (\gamma^*(\phi), \min\{T^{-1}(\phi), 1\}) \;\; \left.\frac{dSW}{d\alpha}\right|_{\alpha=0} > 0 \quad (19)$$

Proof. See Appendix. □

Figure 1 shows the set of parameter values (γ, ϕ) where partial privatization is optimal. This figure illustrates that partial privatization is optimal for a substantial range of parameter values.

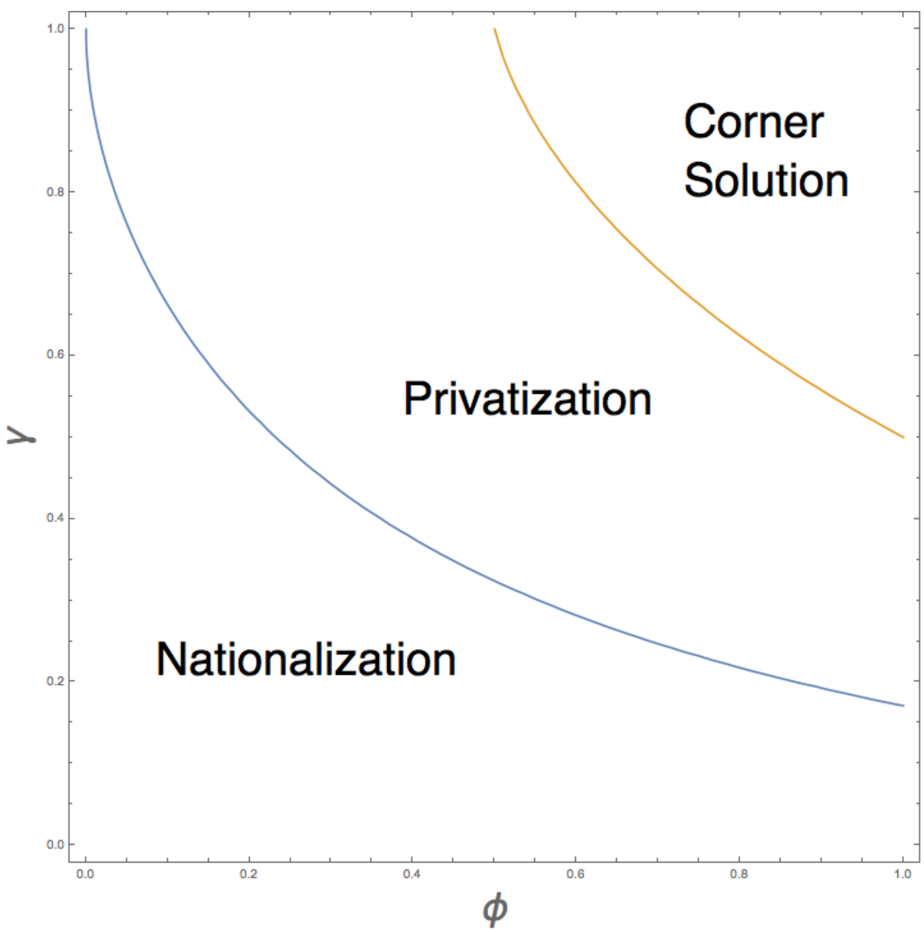

Figure 1: In the lower-left area, full-nationalization is optimal. The middle area is the area where (partial) privatization is optimal and the equilibrium consists of interior solutions. In the upper-right area, zero production by the public firm in market B is optimal with full nationalization.The border between middle and upper-right areas is $T(\gamma)$.

4.3 Comparative Statics

The optimal degree of privatization in an explicit form is over-complicated, but its relationship with two parameters γ and ϕ remain manageable and it turns out that it is non-monotonic.

Proposition 4. *The relationship of the optimal degree of privatization and the degree of product differentiation is non-monotonic. Precisely,*

$$\forall \phi \in (0, \lim_{\gamma \to 1} T(\gamma)) \quad \left.\frac{d^2 SW}{d\gamma d\alpha}\right|_{\alpha=0,\gamma=\gamma^*} > 0, \lim_{\gamma \to 1}\left(\left.\frac{d^2 SW}{d\gamma d\alpha}\right|_{\alpha=0}\right) < 0.$$

Proof. See Appendix. □

When γ is near γ^*, competition in the duopoly market is low, allowing enough a room for welfare improvement. Thus, as γ grows higher, or products similar, the pass-through effect from the monopoly market to the duopoly market caused by privatization becomes larger, enhancing the welfare improvement. When γ is near 1, however, the competition is already cut-throat enough that privatization cannot make significant welfare gain.

The next proposition is on the relative market size ϕ.

Proposition 5. *The relationship of the optimal degree of privatization and the relative sizes of markets is non-monotonic. Precisely,*

$$\forall \gamma \in [\gamma^*(1), T^{-1}(1)) \quad \left.\frac{d^2 SW}{d\phi d\alpha}\right|_{\alpha=0,\phi=\phi^*} > 0, \left.\frac{d^2 SW}{d\phi d\alpha}\right|_{\alpha=0,\phi=1} < 0$$

where $f(\gamma^*(1), 1) = 0$ *and* $f(\gamma, \phi) \gtreqless 0 \Leftrightarrow \phi \gtreqless \phi^*$.

Proof. See Appendix. □

Increasing ϕ has two effects on the welfare result of privatization. One is since the monopoly market swells, the amount of marginal cost reduction induced by privatization heightens. The other is since the duopoly market shrinks, welfare gain there by the marginal cost reduction shrinks as well. The net result of these two flips as ϕ varies from 0 to 1.

From Propositions 4 and 5, it is natural to expect that the optimal degree of privatization has an inverted U-shape, that is, increasing in γ and ϕ as long as these are below some threshold and decreasing above them. We confirm this conjecture by numerical examples shown in Figure 2.

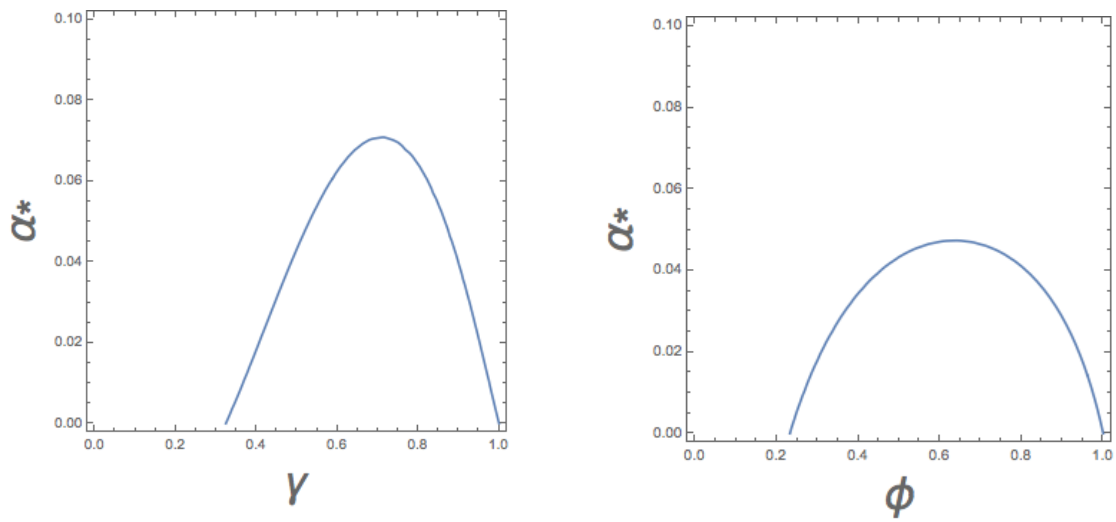

Figure 2: Numerical examples of an optimal degree of privatization(Left:$\phi = 1/2$, Right:$\gamma = 1/2$).

5 Discussion

5.1 Separate Privatization

- Issue: how to share the cost?
- One alternative: set λ as a sharing rule of common cost component.
- Result: privatization only in monopoly market
- Another alternative: separate operation incurs another unit of fixed cost K.
- Then the separate privatization incurs large fixed cost and infeasible.

5.2 Duopoly in Both Markets

- Important element is the asymmetry between market conditions.
- As long as markets are asymmetric in competition/size/something, partial privatization can be optimal.

6 Conclusion

In this study, we formalize the well-known fact that under the price competition, the privatization of public enterprises never improves welfare if they serve a single market. We then show that partial privatization can be optimal if the public firm faces multimarket contacts. In addition, our comparative statics analysis demonstrates that the optimal degree of privatization depends non-monotonically on the relative sizes of the markets and on the degree of competition in the duopoly market. These results have policy implications for privatization policy in sectors such as transportation, in which public enterprises often serve rural areas on their own, but compete with private enterprises in urban areas. In summary, our analysis indicates the importance of multimarket interactions when analyzing optimal privatization policies.

Several important components of reality are left untouched in the model. First, the state-owned firm may have multiple competitors. In such cases, the number of private operating markets need not be one. Accordingly, the number of ways of in which we can set the heterogeneity of goods theoretically increases more than proportionately. Thus, finding and focusing on a relevant configuration depending on interest, in which case, we recommend disentangling the interactions. The possibility of foreign competitors provides a natural and frequently analyzed direction to extend this research. In contrast to multiple private firms, the case of multiple public firms may attract attention, especially for nationwide municipal public-firm competition, for example. Finally, privatization policies can take various forms, including market-by-market degrees of privatization or sequential privatization, among others.

Appendix

Equilibrium Prices in Section 4

The equilibrium prices given α under the specification in Section 4 are as follows:

$$p_0^A(\alpha) = \frac{1}{\delta}[\alpha^2\left(\gamma^2-1\right)(\phi-3)+\alpha\left(\gamma\left(\gamma\left(\gamma^2+\gamma(\phi-1)+(\phi-2)\phi-4\right)-\phi+1\right)-2\phi+6\right)$$
$$+(\gamma-1)\gamma\left(\left(\gamma^2-2\right)\phi+\gamma\right)-2\gamma-\phi+3] \quad (20)$$

$$p_0^B(\alpha) = \frac{1}{\delta}[\alpha^2(\gamma^2-1)(2\gamma+\phi-3)+\alpha(\gamma(\gamma(\gamma^2+2\gamma\phi+\gamma+(\phi-1)\phi-5)-\phi-2)-2\phi+6)$$
$$+\gamma((\gamma-2)\gamma(\gamma+1)(\phi+1)+2\phi)-\phi+3] \quad (21)$$

$$p_1^B(\alpha) = \frac{(\gamma^2+\phi-2)\left(\alpha^2\left(\gamma^2-1\right)+\alpha\left(\gamma^2(\phi+1)+\gamma-3\right)+\gamma-2\right)}{\delta}, \quad (22)$$

where

$$\delta \equiv \alpha^2\left(\gamma^2-1\right)(\phi-3)+\alpha\left(\gamma^4+\gamma^2\left(\phi^2-\phi-7\right)-3\phi+9\right)+\gamma^4(\phi+1)-2\gamma^2(\phi+2)-2\phi+6. \quad (23)$$

Proof of Lemma 1

In the case where $\phi=0$, the equilibrium prices given α is characterized by $\partial\Omega/\partial p_0^B=0$ and $\partial\Pi_1/\partial p_1^B=0$. Using the implicit function theorem, we have

$$H\begin{pmatrix} \frac{dp_0^B}{d\alpha} \\ \frac{dp_1^B}{d\alpha} \end{pmatrix} = -\begin{pmatrix} D_0^B-\frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B-\frac{\partial C_1}{\partial q_1^B}\right) \\ 0 \end{pmatrix} \quad (24)$$

where

$$H = \begin{pmatrix} \frac{\partial^2 \Omega}{\partial {p_0^B}^2} & \frac{\partial^2 \Omega}{\partial p_0^B \partial p_1^B} \\ \frac{\partial^2 \Pi_1}{\partial p_1^B \partial p_0^B} & \frac{\partial^2 \Pi_1}{\partial {p_1^B}^2} \end{pmatrix} \tag{25}$$

At $\alpha = 0$, we have

$$\begin{aligned} & D_0^B - \frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right) \\ =& D_0^B + \frac{\partial D_1^B / \partial p_0^B}{\partial D_1^B / \partial p_1^B} D_1^B > 0, \end{aligned} \tag{26}$$

which follows from the regularity condition, $p_0^B < p_1^B$, and symmetry of the demand function.

Then, using Cramer's rule, we have

$$\left.\frac{dp_0^B}{d\alpha}\right|_{\alpha=0} = \frac{-\left(D_0^B + \frac{\partial D_1^B/\partial p_0^B}{\partial D_1^B/\partial p_1^B} D_1^B\right)\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}}{\det H} > 0$$

since

$$\begin{aligned} \det H &= \frac{\partial^2 \Omega}{\partial {p_0^B}^2}\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2} - \frac{\partial^2 \Omega}{\partial p_0^B \partial p_1^B}\frac{\partial^2 \Pi_1}{\partial p_1^B \partial p_0^B} \\ &= \frac{\partial^2 \Omega}{\partial {p_0^B}^2}\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}\left(1 - \frac{\partial R_0^B}{\partial p_1^B}\frac{\partial R_1^B}{\partial p_0^B}\right) > 0 \end{aligned} \tag{27}$$

from the stability condition.

Finally, the equation

$$\frac{\partial^2 \Pi_1}{\partial p_1^B \partial p_0^B}\frac{dp_0^B}{d\alpha} + \frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}\frac{dp_1^B}{d\alpha} = 0$$

implies that $dp_1^B/d\alpha > 0$. This completes the proof. Q.E.D.

Proof of Proposition 1

For any $\phi \in [0, 1]$, we have

$$\left.\frac{dSW}{d\alpha}\right|_{\alpha=0} = (1-\phi)\frac{dp_1^B}{d\alpha}\frac{1}{\partial D_0^B/\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right)\left(\frac{\partial D_1^B}{\partial p_1^B}\frac{\partial D_0^B}{\partial p_0^B} - \frac{\partial D_1^B}{\partial p_0^B}\frac{\partial D_0^B}{\partial p_1^B}\right). \tag{28}$$

When $\phi = 1$, this equals zero, which implies $\alpha^* = 0$. When $\phi = 0$, Lemma 1 implies that $dp_1^B/d\alpha|_{\alpha=0} > 0$. Since the term other than $dp_1^B/d\alpha|_{\alpha=0} > 0$, say dSW/dp_1^B, is negative from the stability condition and the first-order condition of firm 1, we have $(dp_1^B/d\alpha|_{\alpha=0})(dSW/dp_1^B) < 0$. Thus, we have $\alpha^* = 0$ in both cases. Q.E.D.

Proof of Lemma 2

Given FOCs (7) and (8), define following matrices H_3 and H_2 respectively,

$$H_3 = \begin{pmatrix} \frac{\partial^2 \Omega}{\partial {p_0^A}^2} & \frac{\partial^2 \Omega}{\partial p_0^B \partial p_0^A} & \frac{\partial^2 \Omega}{\partial p_1^B \partial p_0^A} \\ \frac{\partial^2 \Omega}{\partial p_0^B \partial p_0^A} & \frac{\partial^2 \Omega}{\partial {p_0^B}^2} & \frac{\partial^2 \Omega}{\partial p_1^B \partial p_0^B} \\ \frac{\partial^2 \Pi_1}{\partial p_0^A \partial p_1^B} & \frac{\partial^2 \Pi_1}{\partial p_0^B \partial p_1^B} & \frac{\partial^2 \Pi_1}{\partial {p_1^B}^2} \end{pmatrix}, \; H_2 = \begin{pmatrix} \frac{\partial^2 \Omega}{\partial {p_0^A}^2} & \frac{\partial^2 \Omega}{\partial p_0^B \partial p_0^A} \\ \frac{\partial^2 \Omega}{\partial p_0^B \partial p_0^A} & \frac{\partial^2 \Omega}{\partial {p_0^B}^2} \end{pmatrix}. \tag{29}$$

Assume H_3 and H_2 are negative definite. Note that $\partial^2 \Omega / \partial p_1^B \partial p_0^A = \partial^2 \Pi_1 / \partial p_0^A \partial p_1^B = 0$ because of no demand interaction. First equality in the statement comes from the envelop theorem and $(dSW/dp_1^B)|_{\alpha=0} < 0$. For the second one, by Cramer's rule,

$$\frac{dp_1^B}{d\alpha} = \frac{dp_0^B}{d\alpha} \frac{dR_1^B}{dp_0^B}, \tag{30}$$

and the second factor on the right hand side is positive reflecting strategic complementarity. Furthermore,

$$\frac{dp_0^B}{d\alpha} = \frac{det(H_2)}{det(H_3)} \frac{\partial^2 \Pi_1}{\partial {p_1^B}^2} \frac{\partial R_0^B}{\partial \alpha} \tag{31}$$

where the former two factors are negative. Q.E.D.

Proof of Lemma 4

$$\left. \frac{dp_0^B}{d\alpha} \right|_{\alpha=0} = \frac{(\gamma+1)\left(2\gamma^2+\phi-3\right) f(\gamma,\phi)}{\left(\gamma^4(\phi+1) - 2\gamma^2(\phi+2) - 2\phi + 6\right)^2} \tag{32}$$

$$\left. \frac{dp_1^B}{d\alpha} \right|_{\alpha=0} = \frac{\gamma(\gamma+1)\left(\gamma^2+\phi-2\right) f(\gamma,\phi)}{\left(\gamma^4(\phi+1) - 2\gamma^2(\phi+2) - 2\phi + 6\right)^2} \tag{33}$$

where

$$f(\gamma,\phi) \equiv \gamma^4(\phi+1)^2 - \gamma^3\left(\phi^2+\phi+1\right) - \gamma^2\left(\phi^2+8\phi+4\right) + 7\gamma(\phi+1) + \phi - 3. \tag{34}$$

Since $\gamma, \ \phi \in (0,1)$, the signs of the derivatives are the opposite from that of f. We have $f(0,\phi) < 0, \ f(1,\phi) > 0$ and $f(\gamma,\phi)$ belongs to C^∞ class. Then showing $f(\cdot,\phi)$ has at most one extremum in $\gamma \in (0,1)$ for any $\phi \in (0,1)$ proves Lemma 2.[11]

[11]Suppose that $f(\cdot,\phi)$ has at most one minimum or maximum. If the extremum is minimum at $\underline{\gamma}$, then $f_\gamma(\gamma,\phi) < 0$ for all $\gamma < \underline{\gamma}$, since otherwise there is some point $\gamma' \in (0,\underline{\gamma})$ such that $f_\gamma(\gamma',\phi) = 0$, contradicting the assumption that $f(\cdot,\phi)$ has at most one extremum. Similarly, $f_\gamma(\gamma,\phi) > 0$ for all $\gamma \in (\underline{\gamma},1)$. These imply that there exists γ^* such that $f(\gamma,\phi) < 0$ for any $\gamma \in [0,\gamma^*)$, $f(\gamma^*,\phi) = 0$, and $f(\gamma,\phi) > 0$ for any $\gamma \in (\gamma^*,1]$. The case where the extremum is maximum is analogous.

Let $f_\gamma(\gamma,\phi)$ and $f_{\gamma\gamma}(\gamma,\phi)$ be the first and second partial derivatives with respect to γ. Then,

$$f_\gamma(\gamma,\phi) = 4\gamma^3(1+\phi)^2 - 3\gamma^2(1+\phi+\phi^2) - 2\gamma(4+8\phi+\phi^2) + 7(1+\phi) \quad (35)$$
$$f_{\gamma\gamma}(\gamma,\phi) = 12\gamma^2(1+\phi)^2 - 6\gamma(1+\phi+\phi^2) - 2(4+8\phi+\phi^2)\ . \quad (36)$$

Since $f_{\gamma\gamma}(0,\phi) < 0$ and $f_{\gamma\gamma}$ is convex, f_γ has at most one extremum. In addition to it, $f_\gamma(0,\phi) > 0$ and $f_\gamma(1,\phi) < 0$ together show the solution of $f_\gamma(\cdot,\phi) = 0$ with respect to γ is unique, which implies the uniqueness of the extremum of $f(\cdot)$. Q.E.D.

Proof of Proposition 3

Suppose that $T^{-1}(\phi) \geq \gamma > \gamma^*$.

$$\frac{dSW}{d\alpha} = \frac{dp_0^A}{d\alpha}\frac{dSW}{dp_0^A} + \frac{dp_0^B}{d\alpha}\frac{dSW}{dp_0^B} + \frac{dp_1^B}{d\alpha}\frac{dSW}{dp_1^B} \quad (37)$$

At $\alpha = 0$, the first and second term of the right hand side are zero from the envelope theorem. The first factor of the third term is negative from Lemma 2, and the second factor is negative as shown in the proof of Proposition 1 in the Appendix. Thus the sign of the whole derivative is positive. Q.E.D.

Proof of Proposition 4

When $\phi \in (0, \lim_{\gamma\to 1} T(\gamma))$, the equilibrium demand for each product is positive for all $\gamma \in (0,1)$, and we can conduct comparative statics by differentiation. First, for the case of $\gamma = \gamma^*$, from the envelope theorem and the definition of γ^* we have,

$$sgn\left(\left.\frac{d^2SW}{d\gamma d\alpha}\right|_{\alpha=0,\gamma=\gamma^*}\right) = -sgn\left(\left.\frac{d^2p_1^B}{d\gamma d\alpha}\right|_{\alpha=0,\gamma=\gamma^*}\right) = sgn\left(f_\gamma(\gamma^*,\phi)\right) \quad (38)$$

where f_γ is defined in the proof of Lemma 2. By the definition of γ^*, $f_\gamma(\gamma^*,\phi) \geq 0$. From the facts shown in the proof of Lemma 2, that $f_\gamma(0,\phi) >, f_\gamma(1,\phi) < 0$, and $f_\gamma = 0$ has a unique solution in terms of γ in $[0,1]$ show $f_\gamma(\gamma^*,\phi) \neq 0$.
Next, when $\gamma = 1$, note full-nationalization is optimal, since,

$$\left.\frac{dSW}{d\alpha}\right|_{\gamma=1} = \frac{-4\alpha(3-\phi)\phi}{(3+\alpha(3-\phi))^3}.$$

Then,

$$\lim_{\gamma\to 1}\left(\left.\frac{d^2SW}{d\gamma d\alpha}\right|_{\alpha=0}\right) = \frac{-4\phi}{27} < 0.$$

Q.E.D.

Proof of Proposition 5

When $\gamma \in (0, T^{-1}(1))$, the equilibrium demand for each product is positive for all $\phi \in (0,1)$, and we can conduct comparative statics by differentiation. For the case of $\phi = \phi^*$, as in the γ case, we have,

$$sgn\left(\left.\frac{d^2SW}{d\phi d\alpha}\right|_{\alpha=0,\phi=\phi^*}\right) = sgn\left(f_\phi(\gamma,\phi^*)\right) \tag{39}$$

where f_ϕ is a partial derivative with respect to ϕ. Since $f(\gamma,0) < 0$ and $\forall\gamma > \gamma^*(1)$, $f(\gamma,1) > 0$ and $f(\gamma,\phi)$ is strictly increasing in ϕ, $f_\phi(\gamma,\phi^*) > 0$. For the other case, we have

$$\left.\frac{d^2SW}{d\phi d\alpha}\right|_{\alpha=0,\phi=1} = \frac{-\gamma(2-\gamma)f(\gamma,1)}{8(1-\gamma)(2-\gamma^2)^3}.$$

This is negative $\forall\gamma \in [\gamma^*(1), 1)$. Q.E.D.

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Chapter 3

Altruism, Malice and Tax Competition

Satoshi Kasamatsu*, Taku Masuda†, Hikaru Ogawa‡

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1 Introduction

When considering policies that affect neighboring countries, such as refugee relief, trade agreements and measures against global warming, most citizens prefer a policy that would benefit themselves, but some citizens may have different preferences. Altruistic citizens may wish policies not only for their own country's gains but also on the merits in other countries affected by such policies. In contrast, some citizens are concerned about their own country's relative position compared to other countries. They may emotionally feel malice or envy of the state of other countries. Specifically, it is sometimes observed that the masses of a country have hostilities towards neighboring country, which brings economic conflicts, and such are particularly likely to arise between neighboring countries with negative history. We see such political emotions when one country imposes a punitive trade policy and the movement to boycott the products of target country, even though both countries suffer from it.

This paper proposes a model of tax competition led by policy makers selected from citizens with different stance to other countries. When countries in the world are isolated and there is no mutual relationship between them, the individual with moderate preference will be chosen as a representative of diverse citizens. In most cases, such a moderate representative is self-interested without having strong malice or altruism against other countries. However, circumstances change when the home country links to other countries through spillovers, trade and factor mobility. The citizens will choose a political leader who makes them expect to bring the greatest benefit in relation to other countries. Our main concern in this paper is the type of policy-maker elected under the indirect democracy when the interactions among countries are present in the globalized market. The globalization is captured by the increased mobility of capital

*Graduate School of Economics, The University of Tokyo.

†Corresponding author. Graduate School of Economics, The University of Tokyo. Email. taku121281@gmail.com.

‡Graduate School of Economics and Graduate School of Public Policy, The University of Tokyo.

accompanied by capital market integration, which leads us to use the canonical models of tax competition [Zodrow and Miezskowski (1986) and DePater and Myers (1994)]. The tax competition model is, of course, not a sole, but one of the useful methods that has been widely used as an analytical tool to describe the consequences of market integration. With this approach, our interest is on which of the altruistic or malicious, in other words hostile politician towards neighboring countries will be more likely to be elected as the policy-maker who represents the diverse citizens when countries have become connected in a single market.

Although most of previous tax competition studies assume the individuals having homogeneous preferences, we depart from this setting to analyze the emergence of either altruistic or malicious policy-maker (politician) in the tax competition environment. When citizens in a country are in some way heterogeneous, there arises a question that whose preference is reflected in the policy-making. A stylized model of representative democracy with citizen candidates is effective for analyzing such environment and has been used to reveal the type of policy-maker elected by the majority voting. Assuming residents differ in their endowment (stock of capital), Persson and Tabellini (1992) and Ihori and Yang (2009) showed that a decisive voter in the election tends to delegate the authority to set the tax rate to a poor citizen, or a citizen whose capital share is lower than that of the decisive voter. Extending the model to asymmetric tax competition, Ogawa and Susa (2017) show the appearance of equilibrium, in which a decisive voter delegates the authority to tax to a rich citizen in some countries and to a poor citizen in other countries.[1] These studies construct the two-stage game model in which a policy-maker is elected from the heterogeneous citizens through majority voting, and then the elected policy-maker chooses the tax rate to compete for mobile factors. We follow this standard approach to find whether the citizens choose altruist as their representatives or choose malicious representative.

Studies that assumed individuals with altruism or malice are nothing new. In the context of rent-seeking contests pioneered by Tullock (1980), there are a great number of studies that assumed individuals with envy, malice and altruism.[2] There are several reasons why these have been extensively studied. First, non self-interested behavior has long been observed in the experimental nonmarket like situations. Specifically, the subjects are often act as if altruism or fairness is an element in their preferences.[3] Second, it is known that a particular mixture of altruists and envious individuals in the society is evolutionarily stable. For instance, Konrad (2004) formally proved that altruists and envious people do better than a population of narrowly rational individuals in

[1] Another approach is to use a simple median voter model, which has been taken by Fuest and Huber (2001), Borck (2003), Grazzini and van Ypersele (2003), Lockwood and Makris (2006), and Ogawa and Susa (2017). This approach reflects the policy setting in a direct democracy. In contrast, our approach reflects the delegation of power to policy-making under representative democracy, which is effective in studying the type of political leader emerged under the democratic institution.

[2] See, for example, Frech (1978), Konrad (2004), Shaffer (2006), and Schmidt (2009), among others. See also Sheremeta (2018), especially, for the review of experimental researches.

[3] For earlier researches, see Marwell and Ames (1979) and Miller and Oppenheimer (1982).

the rent-seeking game, and that it is evolutionary stable that they exist in society at a certain rate. Third, it is recognized that the existence of altruistic and envious individuals affects the efficiency and policy implication significantly. For instance, Beckman et al. (2002) show in their experiments that envy and malice account for 60% of the reasons for rejecting Pareto gains.[4] It has also long been known that altruism and envy influence the optimal policy. At an early stage, for instance, Oswald (1983) derives the Mirrelsian optimal tax structure, i.e., Mirrlees (1971), when there is altruism and jealousy and show that all of optimal tax theory's general results no longer hold.[5]

Our paper extends the model of interest to neighbors to situations of interest to neighboring countries and clarifies the type of the policy-maker chosen as the representative of the citizens having diverse preferences in tax competition model. The analysis aims presenting a possible explanation of the appearance of benevolent or discriminative political leaders in a globalized society. This framework is more or less similar to Konrad and Morath (2012), which study conflict between two groups of individuals. Their model contains in-group altruism and spiteful behavior towards members of the out-group. Our paper also analyzes the situation where the two groups, i.e., two independent countries, conflict in attracting capital investment. Their focus is on the evolutionarily stable combinations of in-group favoritism and out-group spite, but our focus is not on there and on the emergence of either altruistic or malicious political leader in a country.

The definition of altruism in this paper is very standard. We define individuals as altruistic when they receive benefit not only in the utility obtained in his own country but also have a feeling of concern for the utility of the residents in other countries. In contrast, our strategy to capture malice or hostile preferences is based on the "difference-maximizing behavior" proposed in Frohlich et al. (1984). This can be supported by the widely acknowledged phenomenon of *keeping up with the Joneses*. That is, the utility in particular country depend on the relative standing, and thus the residents' utility in the neighboring country negatively affects residents in the home country.[6] By expressing the difference in preferences to the rival country like this, we first show in the baseline model of symmetric tax competition that the citizen with altruistic preference will be more likely to be elected as the representative of the diverse residents in the

[4]It is confirmed by many experiments and empirical studies that individuals care about their relative positions with their neighbors. For instance, Zizzo and Oswald (2001) conduct the experiment in which subjects can pay to reduce other subjects' money and find that most of them do. Luttmer (2005) shows the evidence that the negative effect of increases in neighbors' earnings on own well-being is most likely caused by interpersonal preferences. These suggest that individuals have utility functions that depend on relative consumption in addition to absolute consumption.

[5]Among others, Johansson (1997) studies how different kinds of altruistic behavior would affect optimal externality-correcting taxes and Aronsson and Johansson-Stenman (2008) modified optimal tax formulas when individuals concern relative utilities.

[6]See Ok and Kockesen (2000) that models negatively interdependent preferences. See also Risse (2011) which applies the concept of negatively interdependent preferences to the group contest game. The "difference-maximizing behavior" is also applied to firms' strategy in the oligopolistic market [Lundgren (1996) and Matsumura et al. (2013)].

country. By extending the model to asymmetric tax competition. in which one country imports and the other exports capital, we secondly show the emergence of malicious policy-maker.

The remainder of this paper is organized as follows. In section 2, we present a basic model. The equilibrium properties are presented in section 3 along with the main results. Section 4 presents the discussion of the model, which is mainly extended to the case of asymmetric tax competition. Here, the asymmetry is captured by the political institution: One country chooses the tax policy for competing mobile capital under the indirect democracy and the other country under the direct democracy. Section 5 offers conclusions.

2 Model

Environment. The economy consists of two symmetric countries ($i = 1, 2$), and the two countries compete for investment by residents in two countries and overseas investors. We assume that residents in each country have a high attachment to where they live, and so do not migrate between countries. For simplicity, we here assume the population in each country is normalized to 1. A resident in each country is endowed κ units of capital, and thus, the initial endowment of capital in the two countries is 2κ. Capital is invested in one of the two countries, depending on the net return to the investment, which leads the countries to compete for attracting the investment.

Production. The production of private goods in country i requires capital and labor, along with technology that exhibits constant returns to scale. The per capita production function in country i is given by $y_i = f(k_i)$, where k_i is the amount of capital per capita used in country i. A firm's profit is given by $\pi_i = f(k_i) - rk_i - t_i k_i - w_i$, where r is the price of capital, t_i is the unit tax on capital employment imposed by the government in country i, and w_i is the wage. Profit maximization gives $w_i = f(k_i) - k_i f_k(k_i)$ and $r = f_k(k_i) - t_i$, which yield the capital demand function, $k_i = k(t_i, r)$.

Domestic capital market. The supply function of capital in the country is expressed in reduced form, and is assumed to be linear in the interest rate: $2\kappa + br$, where r is the (equalized) interest rate the two country can offer, and $b(\geq 0)$ is the responsiveness of the capital supply to the interest rate, which has a positive relation with the capital supply elasticity with respect to the interest rates [Eichner and Runkel (2012) and Wang and Ogawa (2018)].[7]

This setting allows us to deal with the endogenous supply of capital without modelling savings: If $b = 0$, the model reduces to the standard capital tax competition

[7] Although we assumed the linear supply function for simplicity, this assumption might be justified for two reasons. First, the supply function of each investor living outside the country can be convex or concave. If there is a sufficiently large number of investor, then the total supply function would be approximately represented by the linear supply function. Second, the linear supply function of capital would be obtained if the external investors incur a convex sunk cost when investing abroad, which has been widely assumed in the models of tax competition (Bacchetta and Espinosa, 1995).

model, in which the capital market of the two countries is closed to outside the two countries and, thus, the total amount of capital in the two countries is constant. As the parameter b increases, the capital supply responds sensitively to the interest rate. Here, b can be interpreted as a proxy for the connection between the countries to be analyzed and the outside the two countries. The larger the value of b, the more the country's market is open to the overseas investors.

The capital market equilibrium for the two-country economy is reached when the sum of the capital demand in the two countries is equal to the total capital supply:

$$k_1(t_1, r) + k_2(t_2, r) = 2\kappa + br. \tag{1}$$

To obtain a clear solution, we specify the form of production as $f(k_i) = (A - k_i)k_i$, where A represents the productive efficiency.[8] Under this specification, we have

$$r = \frac{A}{1+b} - \frac{4\kappa + t_1 + t_2}{2(1+b)}, \tag{2}$$

$$k_i = \frac{k}{1+b} + \frac{t_j - t_i + 2(A - t_i)b}{4(1+b)}. \tag{3}$$

Individuals. The citizens in each country are homogeneous in terms of endowments of labor and capital, but their preferences are different. That is, the residents are assumed to be heterogeneous in standpoints to neighboring countries. The residents in country i are indexed by a_i, an index that is uniformly distributed over $[-1, 1]$, and the resident's utility of type a_i is given by

$$U_i = u_i + a_i u_j, \tag{4}$$

where $u_i (= x_i)$ is the utility (consumption of a private numeraire good) in country i.[9] An individual with $a_i = 0$ is interested only in her utility. Individuals having positive value of a_i are altruistic, and an individual with $a_i = 1$ means a completely benevolent. Conversely, the individuals who have negative values of a_i have malice to neighboring countries, and the individual of type $a_i = -1$ corresponds to the "difference-maximizer" identified in Frohlich et al. (1984). Since a_i distributes uniformly along the interval of $[-1, 1]$, the median voter is an individual with $a_i = 0$, implying that she does not have altruism or malice in the neighboring countries.

A resident in country i receives labor income $w_i (= f^i(k_i) - k_i f^i_k(k_i))$, a return on her capital investment $r\kappa$, and a lump-sum transfer from the government g_i. Hence, the budget constraint of the resident is given as follows:

[8] The production function is assumed to be quadratic, so that the marginal product of capital is a linear function of the capital labor ratio. Although this is a strong assumption, it has been often used in the literature to get analytical solutions. See Keen and Konrad (2013, p.270). We also assume $A - 2\kappa > 0$ to ensure to ensure $k_i > 0$.

[9] The assumption of uniform distribution is not crucial and all results are maintained as long as the type of medium voter is represented by $a_i = 0$.

$$x_i = w_i + r\kappa + g_i. \tag{5}$$

The government in each country can only use a unit tax on mobile capital. Thus, the government budget constraint in country i becomes

$$g_i = t_i k_i. \tag{6}$$

3 Equilibrium

The model constructed here follows that of Person and Tabellini (1992) which originates from the citizen-candidate models presented by Osborne and Slivinski (1996) and Besley and Coate (1997). The timing of the decision-making is as follows. In the first stage, a simple-majority election takes place in each country to pick a citizen as the policymaker. This policymaker governs the country and determines a tax rate in the next stage. In the second stage, tax policies are selected simultaneously in both countries by the individuals elected as policymakers.

Because the concept of a sub-game perfect Nash equilibrium is applied, we solve the model backward.

3.1 Second stage

We denote the type of the policymaker in country i elected in the first stage as a_{iP}. Given the tax rate in the other country j, t_j, the policymaker determines the tax rate in her country by solving the following maximization problem:

$$\begin{array}{ll} \max_{t_i} & U_{iP} = u_i + a_{iP} u_j, \\ \text{s.t.} & (2) \text{ and } (3), \end{array}$$

where $u_i = w_i + r\kappa + g_i = (A_i - k_i)k_i + r(\kappa - k_i)$. The first-order condition gives us the following reaction function for country i:

$$t_i^R = \frac{1 + a_{iP}}{4b^2 + 8b + 3 - a_{iP}} t_j + \frac{2b(1 + a_{iP})(A - 2\kappa)}{8b + 4b^2 + 3 - a_{iP}} \tag{7}$$

Taxes are strategic complementary relationship except for $a_i = -1$. From (7), we have

$$\frac{\partial t_i^R}{\partial a_i} = \frac{4(1 + b)^2[2b(A - 2\kappa) + t_j]}{(3 + 8b + 4b^2 - a_i)^2} > 0, \tag{8}$$

suggesting that the more an altruistic individual is elected as the policy-maker, the higher tax is chosen to mitigate tax competition. Solving (7) for $i = 1, 2$, we obtain the tax rate of country i in the equilibrium of the following sub-game:

$$t_i^* = \frac{2b\,(1+a_{iP})\,(A-2\kappa)}{(8b+4b^2+2-a_{iP}-a_{jP})}, \tag{9}$$

which gives the following results:

Lemma 1. *Under* $a_i \in [-1, 1]$,

$$\frac{\partial t_i^*}{\partial a_i} = \frac{2b(A-2k)(3+8b+4b^2-a_j)}{(2+8b+4b^2-a_i-a_j)^2} > 0 \tag{10}$$

$$\frac{\partial t_i^*}{\partial a_j} = \frac{2b(A-2k)(1+a_i)}{(2+8b+4b^2-a_i-a_j)^2} \geq 0. \tag{11}$$

The equality in (11) holds if $a_i = -1$. It is straightforward, from (8) and the feature of tax complementarity, that the own effect of a change in a_i is positive. The flip side of this is that by electing a more altruistic policy-maker the residents in country i can expect a tax increase in other country j.

Substituting (9) for $i = 1, 2$ into (2) and (3) yields the equilibrium values: $k_i^* = k_i(a_{iP}, a_{jP})$ and $r^* = r(a_{iP}, a_{jP})$. Using these values with (9), we obtain the utility of the median in country i as $u_i(a_{iP}, a_{jP})$, which is maximized in the first stage.

3.2 Frist stage

The simple median voter theorem leads us to set the first-stage optimization problem as

$$\max_{a_{iP}} \;\; u_i = (A_i - k_i^*)k_i^* + r^*(\kappa - k_i^*),$$

where $k_i^* = k_i(a_{iP}, a_{jP})$ and $r^* = r(a_{iP}, a_{jP})$. The first-order condition of each country's decisive voter yields the following reaction function:

$$a_{iP}^R = \frac{a_{jP}+1}{8b+4b^2+3-a_{jP}}. \tag{12}$$

From (12), we find that preferences of elected policy makers are strategic complements,

$$\frac{da_{iP}^R}{da_{jP}} = \frac{4(1+b)^2}{(3+8b+4b^2-a_{jp})^2} > 0. \tag{13}$$

Furthermore, from (12), in the sub-game perfect Nash equilibrium of this game, the policymaker of each country, selected by the median voter, is characterized by[10]

$$a^* \equiv a_{iP} = a_{jP} = 2b\,(b+2) + 1 - 2(1+b)\sqrt{b\,(b+2)} \in [0, 1], \tag{14}$$

[10]The other solution does not satisfy the condition that $a_i \in [-1, 1]$ and the second-order condition for the optimum choice of a_i. See Appendix A.

where

$$
\begin{aligned}
\frac{da^*}{db} &= -\frac{2\sqrt{b\,(b+2)}\left(1+b-\sqrt{b\,(b+2)}\right)^2}{b(b+2)} < 0,\\
\frac{d^2a^*}{db^2} &= \frac{2\sqrt{b\,(b+2)}\left(1+b-\sqrt{b\,(b+2)}\right)^2\left(1+b+2\sqrt{b\,(b+2)}\right)}{b^2\,(b+2)^2} > 0,
\end{aligned}
$$

with $\lim_{b\to 0} a^* = 1$ and $\lim_{b\to\infty} a^* = 0$. Since $a^* \in [0,1]$ for $b \geq 0$, our main finding can be summarized as follows.

Proposition 1.

In a symmetric tax competition, citizens choose partially altruistic individual as the representative of the state, $a_{iP} > 0$. As b increases, more self-interested individual is elected as the representative, $\partial a_{iP}/\partial b < 0$.

The mechanism behind the result is as follows. To see the median voter's incentive to delegate the authority to either an individual with malicious preference located to the left of the median's position or an altruistic individual who locates to the right of her, we first take the first-order derivative of the utility at the point of $a_{iP} = a_{jP} = 0$:

$$
\left.\frac{\partial u_i}{\partial a_{iP}}\right|_{a_{iP}=a_{jP}=0} = \underbrace{\frac{\partial u_i}{\partial t_i}}_{=0}\frac{\partial t_i*}{\partial a_{iP}} + \frac{\partial u_i}{\partial t_j}\underbrace{\frac{\partial t_j*}{\partial a_{iP}}}_{>0} \tag{15}
$$

The first term is zero because of the envelope theorem from the second stage, and the last term is positive from (8). Therefore, the sign of whole effect is the same as the sign of $\partial u_i/\partial t_j$:

$$
\frac{\partial u_i}{\partial t_j} = (f_k - r)\frac{\partial k_i}{\partial t_j} + (\kappa - k_i)\frac{\partial r}{\partial t_j}. \tag{16}
$$

Although the sign of $f_k - r$ is always positive in the symmetric equilibrium, the sign of the derivative in the first term of (16), $\partial k_i/\partial t_j$, is technically ambiguous because there are two paths from t_j to k_i. A first-order effect is that the country j loses its attraction for investment if it increases t_j, so capital moves to country i. This makes the sign of $\partial k_i/\partial t_j$ positive. However, there is a second-order effect: An increase in t_j decreases the price of capital r and it makes the entire market unattractive for outside investors. This lowers the total supply of capital in two countries, and therefore investment to country i is decreased. In our specification, however, the first-order effect dominates the second one, and that the sign is positive; $\partial k_i/\partial t_j = 1/4(1+b) > 0$.

The second term is also positive, because of, so called, the terms of trade effect. Since two countries import capital from outside the two countries, the sign of $\kappa - k_i$ is negative. In addition, an increase in tax rate lowers the capital price, which makes capital importing countries better off. Therefore,

$$\frac{\partial u_i}{\partial t_j} > 0 \rightarrow \left.\frac{\partial u_i}{\partial a_{iP}}\right|_{a_{iP}=a_{jP}=0} > 0,$$

suggesting that the median voter in each country denoted by $a_i = 0$ does not select herself as the policy-maker, but selects the altruistic politician who has positive value of a_i.

Summarizing the above, the entire mechanism is three-fold: (i)By electing a bit altruistic policy maker in country i, country i can commit to a higher tax rate in the second stage; (ii)When country i sets a high tax rate, the rival country j also sets a high tax rate because the tax policies are in a strategic complementary relationship; (iii)The tax increase of rival country j benefit country i by two routes: a declining interest rate and a capital inflow. (i) and (ii) ease pressures of tax-cut competition and (iii) directly benefits the country. Anticipating the mechanism above, the median voter in the country delegates the power to tax to the citizen with the altruistic preference, $a_{iP} > 0$.

4 Asymmetric countries

In this section, we discuss an asymmetric the tax competition. Basically, the asymmetry is captured by the different political institutions. One country ($i = 1$) is governed by an indirect democracy, so the median voter in the country delegates its authority to set a tax rate to a policy maker as before. In contrast, in the other country ($i = 2$), all citizens vote to decide the tax rate directly. In this case, the simple median voter theorem is applied, and thus, the tax rate in country 2 reflects the preference of the median voter. In the second part, we add another element that generates regional differences to show the emergence of malicious policy-maker.

4.1 Different political institutions

In country 2, citizens directly vote to express their opinions on tax policy, and then the tax policy of the Condorcet winner among the citizens comes into force. Since the median voter theorem holds, the citizen located at the median of the preference distribution becomes the decisive median voter in this game, suggesting that country 2 is virtually committed to $a_{2P} = 0$ before the country 1's election. To respond to it optimally, the median voter in country 1 chooses the policy maker in accordance with (12),

$$a_{1P}^{**} \equiv \frac{1}{8b + 4b^2 + 3}. \tag{17}$$

Note that $a_{1P}^{**} < a^*$. This means that the median voter in country 1 still delegates authority to those who are more altruistic than herself, but the type of policy-maker chosen approaches more self-interested one. Under this setting, we can compare the equilibrium values between two countries as follows:

$$t_1(a_{1P}^{**}, 0) > t_2(a_{1P}^{**}, 0), \tag{18}$$
$$k_2(a_{1P}^{**}, 0) > k_1(a_{1P}^{**}, 0), \tag{19}$$
$$u_2(a_{1P}^{**}, 0) > u_1(a_{1P}^{**}, 0). \tag{20}$$

The tax rate in country 1 is higher than in country 2, so capital and utility in country 1 are less than in country 2. These results are because of the strategic complement property of tax rates. As explained in the previous section, country 1 increases its tax rate to induce country 2 to increase the tax rate. The adjustment is done along with a country 2's reaction curve which has a slope less than 1. Therefore, tax increase is larger for country 1 than for country 2.

4.2 Productivity differences

Now, we add another element representing the regional differences, keeping $a_{2P} = 0$. We here assume that the productivity between countries captured by A_i differs between two countries. Having $a_{2P} = 0$ makes equations herein simpler and analyses of the productivity asymmetry possible.

According to the same procedure so far, the reaction function of country 1 in the second stage is given by

$$t_1^R = \frac{1 + a_{1P}}{3 + 8b + 4b^2 - a_{1P}} t_2 + \frac{2b[A_1 + A_2 a_{1P} - 2\kappa(1 + a_{1P})] + (A_1 - A_2)(1 - a_{1P})}{3 + 8b + 4b^2 - a_{1P}}. \tag{21}$$

Then, we have,

$$\frac{\partial t_1^R}{\partial a_{1P}} = \frac{2(1+b)\{2(1+b)[2b(A_2 - 2\kappa) + t_2] - (1+2b)(A_1 - A_2)\}}{(3 + 8b + 4b^2 - a_{1P})^2}. \tag{22}$$

(22) shows the difference from the case of symmetric equilibrium, i.e., (8): The sign of (22) is ambiguous, and thus it is not clear whether an altruistic policy maker raises a tax rate, depending mainly on the productivity gap and tax rate in country 2.

The equilibrium tax rates are,

$$t_1^{**} = \frac{4b(1+b)(A_1-2\kappa)+(1+2b)(A_1-A_2)-2(1+b)a_{1P}[A_1-A_2-2b(A_2-2\kappa)]}{2(1+b)(2+8b+4b^2-a_{1P})}, \tag{23}$$

$$t_2^{**} = \frac{4b(1+b)(A_2-2\kappa)-(1+2b)(A_1-A_2)}{2(1+b)(2+8b+4b^2-a_{1P})}. \tag{24}$$

Differentiating the equilibrium tax rates by a_{iP} provides,

$$\frac{\partial t_1^{**}}{\partial a_{1P}} = \frac{3+8b+4b^2}{2+8b+4b^2-a_{1P}} t_2^{**}, \tag{25}$$

$$\frac{\partial t_2^{**}}{\partial a_{1P}} = \frac{1}{2+8b+4b^2-a_{1P}} t_2^{**}. \tag{26}$$

Substitution of the equilibrium tax into k_2 gives

$$k_2^{**}-\kappa = \frac{(1+2b)[4b(1+b)(A_2-2\kappa)-(1+2b)(A_1-A_2)]}{4(1+b)(2+8b+4b^2-a_{1P})} = \frac{1+2b}{2} t_2^{**}. \tag{27}$$

Therefore the following lemma holds.

Lemma 2.

$$sgn(t_2^{**}) = sgn(k_2^{**}-\kappa) = sgn\left(\frac{\partial t_1^{**}}{\partial a_{1P}}\right) = sgn\left(\frac{\partial t_2^{**}}{\partial a_{1P}}\right) \tag{28}$$

Lemma 2 can be interpreted clearly. Suppose that country 2 imports capital, $k_2^{**} > \kappa$, as before. Lemma 2 suggests that the stronger the altruism of policy maker in country 1, the higher the tax rate she chooses. This is simply because, by increasing tax rate, the policy maker with strong altruism expects she can lower the capital price, and thereby reduce borrowing costs of country 2. A raise of tax rate in country 1 increases the tax rate of country 2 as well because of the strategic complement property. In contrast, suppose country 2 exports capital, $k_2^{**} < \kappa$, and so it subsidizes rather taxes capitals, $t_2^{**} < 0$. In this, the stronger the altruism of policy maker in country 1, the lower the tax rate he/she chooses. By lowering the tax rate, altruistic policy-maker in country 1 induces a high interest rate which increases the capital income in country 2.

Anticipating the responses in the second stage, the median voter in country 1 chooses the type of policy maker who has a following preference[11],

$$a_{1P}^{**} = \frac{\Phi_1}{2(1+b)\chi_1}, \tag{29}$$

[11] We further assume a regulatory condition for the solution to be interim, $\frac{\Phi_1}{2(1+b)\chi_1} < 2+8b+4b^2$.

where

$$\begin{aligned}\Phi_1 &\equiv 4b(1+b)(A_1-2\kappa)+(1+2b)(A_1-A_2),\\ \chi_1 &\equiv 2b(5+10b+4b^2)(A_2-2\kappa)-\Phi_1.\end{aligned}$$

Depending on parameter values, both Φ_1 and χ_1 can be positive or negative. This means that whether the type of policy-maker chosen will deviate to the left or right from the median voter's preference is ambiguous. To address what determines it, we see the next equation.

$$\begin{aligned}\left.\frac{\partial u_1}{\partial a_{1P}}\right|_{a_{1P}=0} &= \frac{\partial u_1}{\partial t_1}\frac{\partial t_1^{**}}{\partial a_{1P}}+\frac{\partial u_1}{\partial t_2}\frac{\partial t_2^{**}}{\partial a_{1P}}\\ &= \frac{\partial u_1}{\partial t_2}\frac{\partial t_2^{**}}{\partial a_{1P}}\\ &= \left[t_1^{**}\frac{\partial k_1}{\partial t_2}+\frac{\partial r}{\partial t_2}(\kappa-k_1^{**})\right]\frac{t_2^{**}}{2+8b+4b^2}\\ &= \frac{t_1^{**}}{2}\frac{t_2^{**}}{2+8b+4b^2}\\ &= \frac{\Phi_1\Phi_2}{64(1+b)^2(1+4b+2b^2)^3}\end{aligned}$$

where $\Phi_2 \equiv 4b(1+b)(A_2-2\kappa)+(1+2b)(A_2-A_1)$. The direction the median voter ($a_i = 0$) delegates authority depends on the signs of Φ_1 and Φ_2, which indicate the capital position of two countries.

Lemma 3. *If $\Phi_i > (<)0$, country i imports (exports) capital and chooses a positive (negative) tax rate to lower (increase) interest rates.*

Proof. The numerator of equation (23) and (24) are, respectively, equal to Φ_1 and Φ_2 when $a_{1P} = a_{2P} = 0$. Hence, $sgn(\Phi_i) = sgn(t_i^{**}) = sgn(k_i - \kappa)$. Q.E.D.

Lemma 3 shows that the sign of Φ_1 represents whether country 1 taxes or subsidizes capitals, or equivalently, it is a capital importer or exporter when $a_{1P} = 0$. Φ_2 is its counterpart in country 2 and has the same sign as χ_1 under the regularity conditions in use. That is, country i imports capital when $\Phi_i > 0$ and $a_{iP} = 0$, and it exports capital when $\Phi_i < 0$ and $a_{iP} = 0$. Therefore, which type of individual is elected as the representative in country 1 depends on the capital position of the country when the median voter chooses the tax rate of the country. We summarize it as follows:

Proposition 2. *Under the regularity conditions with $a_{2P} = 0$,*

1. *assume that both countries import capital ($\Phi_1 > 0$ and $\Phi_2 > 0$) when the median voter chooses the tax rate of the country.*[12] *In such countries, altruistic individual is chosen as the representative in country 1, $a^{**}_{1P} > 0$.*

2. *assume that one of the two countries imports and the other exports capital ($\Phi_i > 0$ and $\Phi_j < 0$ for $i \neq j$) if the median voter chooses the tax rate of the country. In such countries, malicious individual is chosen as the representative in country 1, $a^{**}_{1P} < 0$.*

The mechanism behind this result can be explained by the terms of trade effect and strategic complement property of tax rates in the second stage. Suppose that country 1 imports but country 2 exports capital when $a_{1P} = a_{2P} = 0$, $\Phi_1 > 0$ and $\Phi_2 < 0$. In this case, if the median voter in country 1 selects a bit more altruistic individual as the policy maker in her country, he/she expects a tax reduction in own country because the altruistic policy-maker aims increasing capital income of country 2 by raising the capital price. In addition, the tax reduction in country 1 induces country 2 to reduce the tax rate because of strategic complement property. These result in the increase in capital price which harms country 1 since it imports capital. Therefore the median voter in country 1 never delegates authority to those who are on the right of herself but rather delegates authority to an individual with malicious preference located to the left of herself.

The intuitive mechanism can be explained in a same manner when country 1 exports but country 2 imports capital, $\Phi_1 < 0$ and $\Phi_2 > 0$. If the median voter in country 1 selects a bit more malicious individual as the policy maker in her country, he/she expects a tax reduction in own country because the malicious policy-maker aims increasing capital borrowing costs of country 2 by raising the capital price. The increase in capital price benefits country 1 since it exports capital to country 2. Therefore, the median voter in country 1 will delegates authority to those who are on the left of herself who has malicious preference.

The above mechanism has been explained by assuming the type of policy maker in country 2 is constant, $a_{2P} = 0$. However, this mechanism works in the same way even if we allow two countries freely choose the type of policymaker (see Appendix B). This means that whether the citizens choose altruistic or envious individual as their representative depends on the capital position of the countries. The citizens in a country which imports capital are likely to choose the altruistic policy-maker if the neighboring countries import capital, while they will choose the individual with malicious preference as the policy-maker if the neighboring countries export capital.

In closing the section, we refer on the effect of growth in productivity and the endowments on the type of policy-maker elected. The simple comparative statics, using (29), shows the following results:

[12] We can ignore the case of $\Phi_1 < 0$ and $\Phi_2 < 0$ since the two countries will never become capital exporters at the same time as long as $r \geq 0$.

Corollary 1.

$$\frac{\partial a_{1P}^{**}}{\partial A_1} > 0, \quad \frac{\partial a_{1P}^{**}}{\partial A_2} < 0, \quad \frac{\partial a_{1P}^{**}}{\partial \kappa} \gtreqless 0 \Leftrightarrow A_1 \gtreqless A_2.$$

Corollary 1 shows that technological progress and the increase in capital endowments will have different impacts from country to country. That is, the productivity growth in country i leads country i to have altruistic policy-maker while it leads country j to have malicious policy-maker. In addition, the increase in capital endowment leads high-productivity country to have altruistic policy-maker, but it leads low-productivity country to have malicious policy-maker.

5 Conclusion

Studies on tax competition with delegation have modeled the residents having homogeneous preferences. This paper has treated the individuals having heterogeneous stance to the neighboring countries in the citizen candidate model in which the policy-maker is selected under the majority voting. The results in the symmetric tax competition show that there exists an incentive for the median voter of the country to delegate the power to decide the tax policy to the altruistic individual. This finding questions the assumption made in the literature that the government maximizes the utility of the resident in the country. However, the result that altruistic individual is elected as the representative of the country is not necessarily robust. In our study allowing the asymmetries across the countries, it was also shown that the individual having malicious preferences could become representatives of the country. Specifically, such a situation is likely to occur when the two countries are divided into a position to export and to import capital. Specifically, In countries importing capital, it is likely to choose individual with malicious preference as the representative of the country, whereas citizens in countries exporting capital are more likely to choose altruistic individual as the representative. In any cases, however, our results imply that policy-maker elected by the diverse citizens considers not only the utility of the residents in the home country but also cares for the utility of neighboring countries.

The basic model can be extended in several directions. Firstly, one of the promising extensions is to formulate the model with different form of fiscal competition. Although we have found that the capital position of two countries critically effects on the resulting type of policy-maker, there is another source that determines the equilibrium characteristics. Specifically, since the results depend on the strategic complementarity between two countries, our findings would be changed if we model the game of public investment in which countries are in the strategic substitution. Following the model of investment competition developed by Hindriks et al. (2008), we have confirmed that the median voter will delegate the authority to decide the investment policy to the individual with malicious preferences if countries are symmetric. The analysis on asymmetric investment competition will add new insights into the type of policy-maker in the fiscal competition models. Secondly, the generalization of the model might contribute to check the robustness of the results. In our paper, we rely on several strong assumptions, which should be relaxed. For instance, we assumed that all tax revenue is returned to the residents in the lump-sum manner, implying that there are no public goods that directly benefit resident's utility. We also specify the production function, which is made for tractability to derive closed-form solutions. More general formulations complicate our analysis, but are left for future research.

Appendices

Appendix A. In equation (14), we define a^* as the unique solution of the game. Yet there also exists the other solution that satisfies (12). Let a° denote the other one,

$$a^\circ \equiv a_{iP} = a_{jP} = 2b\,(b+2) + 1 + 2(1+b)\sqrt{b\,(b+2)}.$$

The followings show that a° does not satisfy the second-order condition.

$$\begin{aligned}\left.\frac{\partial^2 u_i}{\partial {a_{iP}}^2}\right|_{a=a^\circ} &= -\frac{\left(2b+b^2-\sqrt{b(2+b)(1+b)^2}\right)(A-2\kappa)^2}{16(1+b)^2(2+b)^2}\\ &= -\frac{\sqrt{b(2+b)}\left(\sqrt{b(2+b)}-\sqrt{(1+b)^2}\right)(A-2\kappa)^2}{16(1+b)^2(2+b)^2}\\ &= -\frac{\sqrt{b(2+b)}\left(\sqrt{b(2+b)}-\sqrt{1+b(2+b)}\right)(A-2\kappa)^2}{16(1+b)^2(2+b)^2} > 0.\end{aligned}$$

Appendix B. In Section 4.2, we only study the one-sided delegation using parametric formula. This made the comparative statics possible, and as a result we were able to obtain Corollary 1. However, if it is only for leading Lemma 2 and Proposition 2, there is no need to restrict the analysis to the one-sided delegation. We here show that Lemma 2 and Proposition 2 hold with weaker assumptions and two-sided delegation setting, keeping the assumption of productivity difference between countries.

Objective function of country i in the second stage is

$$U_i = u_i + a_i u_j.$$

The first-order condition is given by

$$\frac{\partial U_i}{\partial t_i} = \frac{\partial u_i}{\partial t_i} + a_i \frac{\partial u_j}{\partial t_i} = 0.$$

The implicit function theorem, with the first-order condition, gives

$$H\begin{pmatrix} \frac{\partial t_i^*}{\partial a_i} \\ \frac{\partial t_j^*}{\partial a_i} \end{pmatrix} = -\begin{pmatrix} \frac{\partial u_j}{\partial t_i} \\ 0 \end{pmatrix} \quad \text{where} \quad H \equiv \begin{pmatrix} \frac{\partial^2 U_i}{\partial {t_i}^2} & \frac{\partial^2 U_i}{\partial t_i \partial t_j} \\ \frac{\partial^2 U_j}{\partial t_j \partial t_i} & \frac{\partial^2 U_j}{\partial {t_j}^2} \end{pmatrix}.$$

Applying the Cramer's rule, we have[13],

$$\begin{pmatrix} \frac{\partial t_i^*}{\partial a_i} \\ \frac{\partial t_j^*}{\partial a_i} \end{pmatrix} = -\frac{1}{det(H)} \begin{pmatrix} \frac{\partial^2 U_j}{\partial {t_j}^2} & -\frac{\partial^2 U_i}{\partial t_i \partial t_j} \\ -\frac{\partial^2 U_j}{\partial t_j \partial t_i} & \frac{\partial^2 U_i}{\partial {t_i}^2} \end{pmatrix} \begin{pmatrix} \frac{\partial u_j}{\partial t_i} \\ 0 \end{pmatrix}$$
$$= -\frac{1}{det(H)} \begin{pmatrix} \frac{\partial^2 U_j}{\partial {t_j}^2} \\ -\frac{\partial^2 U_j}{\partial t_j \partial t_i} \end{pmatrix} \frac{\partial u_j}{\partial t_i}$$

With the concavity ($\partial^2 U_i / \partial {t_i}^2 < 0$) and strategic complementarity ($\partial^2 U_j / \partial t_j \partial t_i > 0$) assumptions,

$$sgn\left(\frac{\partial t_i^*}{\partial a_i}\right) = sgn\left(\frac{\partial t_j^*}{\partial a_i}\right) = sgn\left(\frac{\partial u_j}{\partial t_i}\right). \tag{30}$$

The last term of (30) can be decomposed as

$$\frac{\partial u_j}{\partial t_i} = (f_k - r)\frac{\partial k_j}{\partial t_i} + \frac{\partial r}{\partial t_i}(\kappa - k_j)$$
$$= t_j \frac{\partial k_j}{\partial t_i} + \frac{\partial r}{\partial t_i}(\kappa - k_j).$$

We additionally make natural assumptions that $\partial k_i / \partial t_i < 0$, $\partial k_j / \partial t_i > 0$, and $\partial r / \partial t_i < 0$. Under these assumptions, we obtain the same properties as in Section 4.2:

$$\left.\frac{\partial U_j}{\partial t_j}\right|_{a_j=0} = \frac{\partial u_j}{\partial t_j} = 0$$
$$\Leftrightarrow t_j^* \frac{\partial k_j}{\partial t_j} + \frac{\partial r}{\partial t_j^*}(\kappa - k_j) = 0$$
$$\Leftrightarrow t_j^* = -\frac{\partial r / \partial t_j^*}{\partial k_j / \partial t_j}(\kappa - k_j), \tag{31}$$

suggesting that the sign of the last term of (30) equals that of t_j at $a_j = 0$ because t_j and $\kappa - k_j$ have opposite signs. Therefore, we obtain the following result, which corresponds to Lemma 2.

Lemma 4. *At* $a_j = 0$,

$$sgn(t_j^*) = sgn(k_j^* - \kappa) = sgn\left(\frac{\partial t_i^*}{\partial a_i}\right) = sgn\left(\frac{\partial t_j^*}{\partial a_i}\right) = sgn\left(\frac{\partial u_j}{\partial t_i}\right).$$

Finally, using Lemma 2, the general version of Proposition 2 is straightforward, which can be summarized as follows. .

Proposition 3.

$$sgn\left(\left.\frac{\partial u_i}{\partial a_i}\right|_{a=0}\right) = sgn\left(\frac{\partial u_i}{\partial t_j}\right) sgn\left(\frac{\partial t_j^*}{\partial a_i}\right) = sgn(k_i^* - \kappa) sgn(k_j^* - \kappa).$$

[13]For inversion, we assume $det(H) > 0 \Leftrightarrow \frac{\partial^2 U_i}{\partial {t_i}^2}\frac{\partial^2 U_j}{\partial {t_j}^2} - \frac{\partial^2 U_i}{\partial t_i \partial t_j}\frac{\partial^2 U_j}{\partial t_j \partial t_i} > 0$.

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