Doctorate Dissertation

博士論文

Realistic Construction of Axion Model with Gauged Peccei-Quinn Symmetry

(ゲージ対称性に伴う Peccei-Quinn 対称性の現実的な構成)

A Dissertation Submitted for Degree of Doctor of Philosophy

December 2018

平成 30 年 12 月博士（理学）申請

Department of Physics, Graduate School of Science,

The University of Tokyo

東京大学大学院理学系研究科物理学専攻

Motoo Suzuki

鈴木　資生

Abstract

To date, the most elegant and intriguing solution to the Strong CP problem has been the Peccei-Quinn (PQ) mechanism. The PQ symmetry may, however, be explicitly and badly broken by the quantum gravity effect. Then, to be consistent with the measurement of the neutron electric dipole moment, it is expected that the PQ symmetry is explicitly broken by only highly suppressed non-renormalizable terms. In this thesis, to understand the origin of the PQ symmetry with such high quality, we suggest one general mechanism where the PQ symmetry is protected well by the gauge symmetry. We call this protection gauge symmetry as the gauged PQ symmetry. For one effort, we apply the mechanism to the model where the origin of the PQ symmetry breaking and the supersymmetry breaking is the same. From this work, we find a cosmological problem with extra fermions introduced to cancel the anomaly of the gauged PQ symmetry. Those fermions tend to behave as the dark radiation and eventually induce an unacceptably large number of the effective neutrino species. To resolve this problem, we propose one simple model with no un-wanted fermions. This model is the first realistic axion model with the gauged PQ symmetry.

Contents

Chapter 1

Introduction

Non-observation of the neutron electric dipole moment (nEDM) severely constrains the QCD vacuum angle $\bar{\theta}$ called the θ-parameter as $\bar{\theta} \lesssim 10^{-10}$ [1]. Why is this θ-parameter so small? This is the Strong CP problem.

The most successful solution to the strong CP problem will be the Peccei-Quinn (PQ) mechanism [2]. There, the PQ symmetry which is a global symmetry explicitly broken by the QCD anomaly provides the pseudo-Nambu-Goldstone (NG) boson, axion, after its spontaneous breaking. Below the QCD confinement scale, the axion obtains its potential due to the anomaly. Elegantly, the axion potential minimum corresponds to the $\bar{\theta} = 0$ vacuum, and thus the axion cancels the θ-term dynamically.

This axion potential may, however, be affected by quantum gravity effect [3–9]. Actually, once we assume that the global symmetries are broken by quantum gravity effect, the non-renormalizable terms may be produced *e.g.* at the lowest order,

$$\mathcal{L} = \lambda \frac{\phi^5}{M_{\rm pl}} + h.c. \tag{1.1}$$

where ϕ is the PQ breaking complex scalar field with some PQ charge, $M_{\rm pl}$ is the reduced Planck mass, and the coefficient λ is a complex coupling. Apparently, this potential breaks the PQ symmetry explicitly, and thus the axion potential is modified by

$$\mathcal{L} \simeq |\lambda| \frac{v^5}{M_{\rm pl}} \cos(a/v + \delta) \tag{1.2}$$

where $|\lambda|$ denotes the absolute value of λ, δ is the phase of λ, v is the VEV of ϕ, and the a is the axion field. This term drastically change the axion potential and generally leads the $\mathcal{O}(1)$ shift from the CP-conserving potential minimum denoted as $\Delta\theta = \mathcal{O}(1)$ for typical scale of $v \simeq 10^{9-12}\,$GeV. To match with the measurement of the nEDM, the coupling λ should be so small $\lambda \lesssim 10^{-50}$. Therefore, the Strong CP problem still remains unsolved.

One solution to protect the PQ symmetry from the quantum gravity effect is using gauge symmetries. Concretely, one simple model has been suggested by Barr and Seckel in 1992 [10]. They have introduced an additional gauge symmetry $U(1)'$ and two complex scalar fields $\phi(p)$ and $\phi'(q)$ where $p,\ q$ denote the $U(1)'$ charges. For a concrete example, let us take $\phi(10)$ and $\phi'(-1)$ in the following. These scalar fields can acquire their VEV's and provide the mass of the KSVZ quarks [11,12] by the term,

$$\mathcal{L} = y\phi Q\bar{Q} + y'\phi' Q'\bar{Q}', \tag{1.3}$$

where $Q,\ \bar{Q}$ $(Q',\ \bar{Q}')$ are n (m) pair of the KSVZ quarks, y and y' are the couplings. To cancel the triangle anomalies of $U(1)'-\mathrm{SM}-\mathrm{SM}$ and $U(1)'-\mathrm{Gravity}-\mathrm{Gravity}$, we take $(n, m) = (1,\ 10)$ which is the only solution for the minimal number of the above KSVZ quarks. When the scalar fields obtain the VEV's, two NG bosons are produced. One of them is eaten by the $U(1)'$ gauge field. Then, the other one corresponds to the axion. Due to the gauge symmetry, the explicit breaking term is suppressed as large as

$$\mathcal{L} \simeq \frac{\phi^{10}\phi'}{M_{\mathrm{pl}}^7}, \tag{1.4}$$

which is sufficiently suppressed to achieve the small shift of the potential minimum $\Delta\theta < 10^{-10}$ for $\langle\phi\rangle \simeq \langle\phi'\rangle \simeq 10^{11}\,\mathrm{GeV}$.

An essential point of the Barr-Seckel model is introducing two sectors with independent PQ symmetries if the $U(1)'$ gauge coupling is switched off. There always exists an anomaly free combination of two anomalous PQ symmetries. We can gauge it, and then the gauge symmetry which is nothing but the $U(1)'$ can work to protect the axion potential if two scalar fields only couple with higher order term due to their charge relation.

In this thesis, we show that this prescription can obviously apply to the wide type of the axion models as shown in our work [13] once you notice the essential point. We call this generalized prescription as "Gauged PQ mechanism". Thanks to our mechanism, the "remained Strong CP problem" can be generally solved[1]. We also show that some extra SM singlet fermions must be added to cancel the

[1] In the literature, there have been many attempts to achieve the PQ symmetry as an accidental symmetry resulting from (discrete) gauge symmetries [10,14–28]. There have also been arguments of the origin of the axion in string theory [29–31] and in extra-dimensional setups [32–38].

self-triangle anomaly of the gauged PQ symmetry, and those fermions generally result in an unacceptably large number of effective neutrino species because as dark radiation [39]. In the face of this cosmological problem, one simple solution is to construct the model with no extra fermions for anomaly cancellations. To find out a realistic model, we have also sought the probable identification that the gauged PQ symmetry equals to the $B - L$ gauge symmetry which is the most authentic extension of the SM gauge symmetries [40]. This provides the first realistic model with our mechanism.

This thesis is organized as follows. In Chapter 2, we briefly review the $U(1)$ problem and the Strong CP Problem. In Chapter 3, we review axion models as a solution to the Strong CP problem. In Chapter 4, we also review the cosmology of axion models. In Chapter 5, we review arguments that a global symmetry is broken by quantum gravity. In Chapter 6, we show our works based on published three papers. The final Chapter 7 is devoted to the conclusion of this thesis.

Chapter 2

Strong CP problem

In this chapter, we review the $U(1)$ problem and the Strong CP problem.

2.1 $U(1)$ problem

The $U(1)$ problem is an apparent contradiction between the meson spectrum from the quantum chromodynamics (QCD) Lagrangian and the observed one.

Let us consider, the QCD Lagrangian in the massless limit of the up-quarks and the down-quarks, where there seem to be global symmetries, $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$. There, $SU(2)_L \times SU(2)_R$ is the chiral symmetry, and $U(1)_V$ is the singlet vector transformation corresponding to the baryon symmetry. The $U(1)_A$ symmetry denotes the singlet axial transformation acting on the chiral fields differently,

$$u_R \to \exp(i\alpha')u_R \quad , \quad u_L \to \exp(-i\alpha')u_L, \tag{2.1}$$

$$d_R \to \exp(i\alpha')d_R \quad , \quad d_L \to \exp(-i\alpha')d_L, \tag{2.2}$$

where we take that u_L (u_R) are the $SL(2, \mathbf{C})$ spinor fields of the 2 (2^*) representation of the up-quarks.[1] d_L (d_R) is the same for the down-quarks. α' denotes the phase of the transformation. This symmetry is broken spontaneously due to the quark condensation (*e.g.* $\left\langle u_L^\dagger u_R \right\rangle \neq 0$). Then, an "extra" pseudo-Nambu-Goldstone (NG) boson with the mass comparable to the pion is predicted. You might think that the extra one is the η meson. However, the situation is the same even if we regard the

[1] The Dirac spinor *e.g.* for the up-quark, u, is given as

$$u = \begin{pmatrix} u_R \\ u_L \end{pmatrix}. \tag{2.3}$$

strange quark mass light as well as u and d quarks, as shown by S. Weinberg in 1975 [41]. Such light meson is, of course, not observed. This is the $U(1)_A$ problem.[2]

A possible resolution to this problem is suggested by 't Hoot in 1986 [42] using the axial anomaly of the $U(1)_A$ and the vacuum structure of the QCD. Under the global transformation in Eq. (2.1), the action is affected by the term proportional to the total derivative,

$$\int d^4x \, \epsilon_{\mu\nu\rho\sigma} G_a^{\mu\nu} G_a^{\rho\sigma}, \tag{2.4}$$

where the $G_a^{\mu\nu}$ is the gluon field strength, and the $\epsilon_{\mu\nu\rho\sigma}$ is the totally asymmetric tensor with $\epsilon_{0123} = +1$. The total derivative term does not contribute to the equation of the motion. However, this term gives the non-zero contribution to the action under certain gauge configurations (*e.g.* the so-called instanton[3]) in four-dimensional non-Abelian gauge theory. What 't Hooft have shown in the QCD with the massless quarks is that there is no goldstone pole of the $U(1)_A$ breaking under the instanton background. Therefore, the structure of the QCD vacuum makes $U(1)_A$ not a true symmetry. There, the "extra" meson becomes heavy, and can be regarded as the η' meson with 957 MeV mass.

2.2 Strong CP problem

Recognizing the solution to the $U(1)_A$ problem, the QCD Lagrangian includes the so-called θ-term,

$$\mathcal{L}_\theta = -\theta \frac{g^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a\mu\nu}, \tag{2.5}$$

where θ is the parameter, the coefficient g is the QCD gauge coupling constant, and $\tilde{G}_a^{\mu\nu} \equiv \frac{1}{2} G_a^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}$. Under the parity transformation, P, and the time transverse transformation, T, the gluon field strength becomes

$$P G^a_{0i} P^{-1} = -G^a_{0i} \quad , \quad P G^a_{ij} P^{-1} = G^a_{ij}, \tag{2.6}$$

$$T G^a_{0i} T^{-1} = G^a_{0i} \quad , \quad T G^a_{ij} T^{-1} = -G^a_{ij}. \tag{2.7}$$

Thus, the $\mathcal{L}_\theta$ becomes $-\mathcal{L}_\theta$ under each P, T transformation. This fact indicates the $\mathcal{L}_\theta$ also violates CP symmetry. In the following, we call this CP violating term as the θ-term.

[2] See Appendix A for more details about the $U(1)$ problem.

[3] See Appendix C for the instanton.

To understand the physical consequence of this θ-term, let us focus on the quark mass terms and the θ-term in QCD Lagrangian,

$$\mathcal{L} = (-M_f^u \bar{u}_{Lf} u_{Rf} - M_f^d \bar{d}_{Lf} d_{Rf} + h.c.) - \theta \frac{g^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a\mu\nu} \tag{2.8}$$

where the subscripts f is the three flavor indices, $M_f^{u(d)}$ is the 3×3 complex matrix of the mass parameter of the up-(down-)type quarks, $^-$ denotes the conjugate of the spinor field. Under the redefinition of these quark fields,

$$u_{Rf} = \exp(i\alpha_f^u)\tilde{u}_{Rf} \quad , \quad u_{Lf} = \exp(-i\alpha_f^u)\tilde{u}_{Lf}, \tag{2.9}$$
$$d_{Rf} = \exp(i\alpha_f^d)\tilde{d}_{Rf} \quad , \quad d_{Lf} = \exp(-i\alpha_f^d)\tilde{d}_{Lf}, \tag{2.10}$$
$$\tag{2.11}$$

The Lagrangian becomes,

$$\mathcal{L} = -(M_f^u e^{2i\alpha_f^u} \bar{\tilde{u}}_{Lf} \tilde{u}_{Rf} + M_f^d e^{2i\alpha_f^d} \bar{\tilde{d}}_{Lf} \tilde{d}_{Rf} + h.c.) \tag{2.12}$$
$$- (\theta + 2\sum_f (\alpha_f^u + \alpha_f^d)) \frac{g^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a\mu\nu}, \tag{2.13}$$

where the θ-term is changed from the effect on the measure for the path integral. We can always redefine the fermion fields as all M_f^u and M_f^d are the real, and we define the θ-parameter in this basis as $\bar{\theta}$ for convenience.

The $\bar{\theta}$ is upper-bounded by the measurement of the neutron electric dipole moment (d_n) [1],

$$|\bar{\theta}| \lesssim 10^{-10}. \tag{2.14}$$

Why should this parameter $\bar{\theta}$ be so small? This is the Strong CP problem.

One may think that, even if we take the mass parameters to be real, the $|\bar{\theta}| \simeq \pi$ seems to be another possible solution because the $|\bar{\theta}| = \pi$ does not produce the CP violation effect. However, the relation between the mass parameter of the up-quark (m_u) and the one of the down-quark (m_d),[4]

$$\frac{m_d - m_u}{m_d + m_u} = \frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_{\pi^0}^2} \tag{2.15}$$
$$\simeq 0.3, \tag{2.16}$$

and $|m_d| > |m_u|$ denotes the same sign between m_u and m_d. From the basis of the real mass parameter, we can always change the basis by the redefinition of the d_R to cancel the θ-term,

$$d_{Rf} = \exp(-i\bar{\theta})\tilde{d}_{Rf}. \tag{2.17}$$

[4] See Appendix A for the derivation of the relation.

Now, the m_u and the m_d have the opposite sign, which contradicts with the meson mass relation. Therefore, the $\bar{\theta}$ must be near $\bar{\theta} = 0$. It should be also noted that the θ-parameter is unphysical if there exist one or more massless quarks because the θ-term can always be shifted away by the redefinition of the massless quark fields.

Chapter 3

Peccei-Quinn mechanism

One idea to solve the Strong CP problem has been suggested by Peccei and Quinn in 1977 [2, 43]. They have proposed a theory where the θ becomes the dynamical variable *i.e.* the field, and then a minimum of the effective potential of that field conserves the CP and P symmetry. Then, by Weinberg and Wilczek [44, 45], it has been shown that such field corresponds to a pseudo-NG boson called the axion of the global symmetry, and is excluded by the measurement of the charge Kaon decay. We call that global symmetry as the Peccei-Quinn symmetry or $U(1)_{PQ}$. The original idea is applied to the model with the axion couples too weak to the SM particles.

3.1 PQWW model

Here, let us discuss the original PQWW model [2, 43–45]. They introduced two $SU(2)_L$ doublet Higgs field, H_u and H_d, which couples only to the up-type quarks or the down-type quarks, respectively,

$$\mathcal{L} = -(\bar{q}_L u_R H_u + \bar{q}_L d_R H_d + h.c.) - V(H_u, H_d) - \theta \frac{g^2}{32\pi^2} G\tilde{G}, \tag{3.1}$$

where the hypercharges of the Higgs fields are $H_u(+1/2)$ and $H_d(-1/2)$, q_L is the $SU(2)_L$ doublet quarks in SM. The indices of the flavor, the gauge group, the Lorentz group, and the Yukawa coefficients are omitted for simplicity. The difference between the H_u and H_d is guaranteed by two discrete Z_2 symmetry,

$$H_u \to -H_u, \; u_R \to -u_R, \tag{3.2}$$

and

$$H_d \to -H_d, \; d_R \to -d_R. \tag{3.3}$$

The Higgs potential $V(H_u, H_d)$ should not allow the term $(H_u H_d)^2$. In this setup, the Lagrangian is invariant under the following chiral rotation,

$$u_R \to e^{i\alpha} u_R,\ d_R \to e^{i\alpha} d_R,\ H_u \to e^{-i\alpha} H_u,\ H_d \to e^{-i\alpha} H_d. \tag{3.4}$$

This symmetry is the $U(1)_{PQ}$ symmetry. To find the axion components explicitly, let us write down two doublet Higgs as

$$H_u = \begin{pmatrix} H_u^0 \\ H_u^- \end{pmatrix},\ H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}, \tag{3.5}$$

, and the neutral components H_u^0 and H_d^0 obtain the vacuum expectation value $v_u/\sqrt{2},\ v_d/\sqrt{2}$,

$$H_u^0 = \frac{v_u + \rho_u}{\sqrt{2}} \exp(i x_u / v_u), \tag{3.6}$$

$$H_d^0 = \frac{v_d + \rho_d}{\sqrt{2}} \exp(i x_d / v_d), \tag{3.7}$$

where $\rho_{u,d}$ is the real scalar field to the radial direction, and $x_{u,d}$ is the real scalar field to the phase direction. The one linear combination of the $x_{u,d}$ is absorbed by the Z-boson and the other one remains as the axion. From the mass term of the Z-boson, the absorbed components, h_z, are given by

$$h_z = -x_u \frac{v_u}{\sqrt{v_u^2 + v_d^2}} + x_d \frac{v_d}{\sqrt{v_u^2 + v_d^2}}. \tag{3.8}$$

and $\sqrt{v_u^2 + v_d^2} \equiv v = 246\,\mathrm{GeV}$. Therefore, the axion component, a, is obtained by the following unitary transformation,

$$\begin{pmatrix} a \\ h_z \end{pmatrix} = \frac{1}{\sqrt{v_u^2 + v_d^2}} \begin{pmatrix} v_d & v_u \\ -v_u & v_d \end{pmatrix} \begin{pmatrix} x_u \\ x_d \end{pmatrix}. \tag{3.9}$$

Now, let us focus on only the axion component,

$$H_u^0 = \frac{v_u}{\sqrt{2}} \exp\left(i \frac{v_d}{v_u} \frac{a}{v}\right),\ H_d^0 = \frac{v_d}{\sqrt{2}} \exp\left(i \frac{v_u}{v_d} \frac{a}{v}\right). \tag{3.10}$$

By the redefinition of the quark fields,

$$u_R = \exp\left(-i \frac{v_d}{v_u} \frac{a}{v}\right) \tilde{u}_R,\ d_R = \exp\left(-i \frac{v_u}{v_d} \frac{a}{v}\right) \tilde{d}_R, \tag{3.11}$$

the axion field can be erased from the up and down type Yukawa sector, but emerges in the coupling to the gluon as in θ-term

$$\mathcal{L}_{agg} = \frac{g^2}{32\pi^2} \frac{3}{v} \left(\frac{v_u}{v_d} + \frac{v_d}{v_u}\right) \left(a - v\theta'\right) G\tilde{G}, \tag{3.12}$$

$$\tag{3.13}$$

where using,

$$\theta' = \theta \left[3 \left(\frac{v_u}{v_d} + \frac{v_d}{v_u} \right) \right]^{-1} . \tag{3.14}$$

For the later convenience, let us redefine the axion field,

$$a - v\theta' \to a, \tag{3.15}$$

i.e,

$$\mathcal{L}_{agg} = + \frac{g^2}{32\pi^2} 3 \left(\frac{v_u}{v_d} + \frac{v_d}{v_u} \right) \frac{a}{v} G\tilde{G}. \tag{3.16}$$

To find the axion solve the Strong CP problem dynamically, let us redefine the quark fields to erase the axion field from the $G\tilde{G}$ coupling which corresponds to the inverse transformation which is similar to Eq. (3.11) but only using u- and d- quark,

$$\tilde{u}_R = \exp\left(c_u i \frac{a}{v} \right) u_R \quad , \quad \tilde{d}_R = \exp\left(c_d i \frac{a}{v} \right) d_R, \tag{3.17}$$

$$c_u + c_d = 3 \left(\frac{v_u}{v_d} + \frac{v_d}{v_u} \right). \tag{3.18}$$

Then, the Lagrangian of the axion after the decoupling of the heavy quarks except for the u- and d- quarks is

$$\mathcal{L} = \frac{1}{2} (\partial a)^2 - \left[m_u \exp\left(i c_u \frac{a}{v} \right) \bar{u}_L u_R + m_d \exp\left(i c_d \frac{a}{v} \right) \bar{d}_L d_R + h.c. \right], \tag{3.19}$$

where the first term denotes the canonical kinetic term of the axion, m_u and m_d are the masses of the up- and down-quarks, respectively. Below the QCD scale, the quarks condensate, and the neutral pion field π^0 is produced,

$$\bar{u}_L u_R \to \frac{B_0 F_\pi^2}{2} \exp\left(i \frac{\pi^0}{f_\pi} \right), \ \bar{d}_L d_R \to \frac{B_0 F_\pi^2}{2} \exp\left(-i \frac{\pi^0}{f_\pi} \right) \tag{3.20}$$

Then, the low energy effective Lagrangian about the axion and the neutral pion is

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} (\partial a)^2 + \frac{1}{2} (\partial \pi^0)^2 + \left[-m_u \frac{B_0 F_\pi^2}{2} \exp\left(i c_u \frac{a}{v} + i \frac{\pi^0}{f_\pi} \right) \right. \\ &- \left. m_d \frac{B_0 F_\pi^2}{2} \exp\left(i c_d \frac{a}{v} - i \frac{\pi^0}{f_\pi} \right) + h.c. \right] && (3.21) \\ &= \frac{1}{2} (\partial a)^2 + \frac{1}{2} (\partial \pi^0)^2 && (3.22) \\ &- m_u B_0 F_\pi^2 \cos\left(c_u \frac{a}{v} + \frac{\pi^0}{f_\pi} \right) - m_d B_0 F_\pi^2 \cos\left(c_d \frac{a}{v} - \frac{\pi^0}{f_\pi} \right). && (3.23) \end{aligned}$$

Notice that the axion-pion couplings from the quark kinetic term can be always erased by appropriate choice of c_u and c_d keeping the condition $c_u + c_d = 3(v_u/v_d + v_d/v_u)$. The quadratic part of this Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial a)^2 + \frac{1}{2}(\partial \pi^0)^2 - \frac{1}{2}\begin{pmatrix} \pi^0 \\ a \end{pmatrix}^T M^2_{a,\pi^0} \begin{pmatrix} \pi^0 \\ a \end{pmatrix}, \tag{3.24}$$

where M^2_{a,π^0} is the 2×2 matrix of the axion and the pion mass squared,

$$M^2_{a,\pi^0} = \begin{pmatrix} (m_u + m_d)B_0 & (-m_u c_u + m_d c_d)B_0 F_\pi/v \\ (-m_u c_u + m_d c_d)B_0 F_\pi/v & (m_u c_u^2 + m_d c_d^2)B_0 F_\pi^2/v^2 \end{pmatrix}. \tag{3.25}$$

For $v \gg F_\pi$, the pion obtains the mass squared

$$m_\pi^2 \simeq (m_u + m_d)B_0, \tag{3.26}$$

and the other mass corresponds to the one for the axion,

$$m_a^2 \simeq (c_u + c_d)\frac{m_u m_d}{m_u + m_d} B_0 F_\pi^2 \frac{1}{v^2} \tag{3.27}$$

$$\simeq 6\frac{m_\pi^2 F_\pi^2}{v^2}\frac{m_u m_d}{(m_u + m_d)^2} \simeq \mathcal{O}(100\,\mathrm{keV})^2. \tag{3.28}$$

There, the axion field calms down to the vacua, which is equivalent to the $a = 0$ vacuum. In the other word, the axion cancels the θ-term dynamically.

The PQWW model is immediately ruled out by the experiment. In PQWW model, the PQ symmetry is spontaneously broken with the $SU(2)_L \times U(1)_Y$ breaking in SM. Therefore, the axion decay constant is at most $\mathcal{O}(100)\,\mathrm{GeV}$, and then the PQWW axion has a relatively large mixing with the neutral pion. For example, the branching ratio of $\mathcal{B}(K^\pm \to \pi^\pm + \text{axion})$ is estimated as [46],

$$\mathcal{B}(K^+ \to \pi^+ + \text{axion}) \simeq \left(\frac{F_\pi}{v}\right)^2 \mathcal{B}(K^\pm \to \pi^\pm + \pi^0) \tag{3.29}$$

$$\simeq 10^{-7}. \tag{3.30}$$

On the other hand, this branching ratio is upper-limited by the KEK [47],

$$\mathcal{B}(K^+ \to \pi^+ + \text{axion}) < 3.8 \times 10^{-8} \ (90\%\,\mathrm{C.L.}). \tag{3.31}$$

The PQWW axion is already excluded. This result leads to the invisible axion model, where the PQ symmetry is spontaneously broken by the scalar fields of the SM gauge singlet.

3.2 KSVZ model

One of the invisible axion models has been proposed by Kim [11] and Shifman, Vainshtein and Zakharov [12] (the so-called KSVZ model). They introduced a vector-like quark coupling with a scalar field which breaks the PQ symmetry spontaneously. Due to the large vacuum expectation value of that scalar field, a vector-like quark can obtain the mass around the PQ breaking scale.

We introduce a vector-like quark, Q and $\bar{Q}$, of $\bar{\mathbf{3}}$ and $\mathbf{3}$ representation in $SU(3)_c$, respectively. In the following, we call this vector-like quark as the KSVZ quark. The KSVZ quark couples with a complex scalar field ϕ,

$$\mathcal{L} = -\phi Q\bar{Q}. \tag{3.32}$$

The scalar potential of ϕ and SM Higgs boson H is

$$V(H,\phi) = -m_H^2|H|^2 - m_\phi^2|\phi|^2 + \lambda_H|H|^4 + \lambda_\phi|\phi|^4 + \lambda_{H\phi}|H|^2|\phi|^2. \tag{3.33}$$

In this Lagrangian, there exists a global PQ symmetry corresponding to the transformation,

$$\phi \to e^{2i\alpha}\phi,\ Q \to e^{-i\alpha}Q,\ \bar{Q} \to e^{-i\alpha}\bar{Q} \tag{3.34}$$

where α is a phase. Note that the SM Higgs boson has no charge of the PQ symmetry in KSVZ model. After the PQ symmetry is spontaneously broken by the VEV of the extra scalar field, $\langle\phi\rangle \equiv v_{\mathrm{PQ}}/\sqrt{2}$, the KSVZ quark generally obtain its mass around v_{PQ} from the term in Eq. (3.32). The axion component is the phase direction of the ϕ,

$$\phi = \frac{v_{\mathrm{PQ}}}{\sqrt{2}}\exp\left(i\frac{a}{v_{\mathrm{PQ}}}\right) \tag{3.35}$$

where the radial component is ignored. By the transformation of the Q

$$Q \to e^{-i\,a/v_{\mathrm{PQ}}}Q, \tag{3.36}$$

the axion-gluon coupling as in Eq. (3.16) is obtained,

$$\mathcal{L} = \frac{g^2}{32\pi^2}\frac{a}{v_{PQ}}G\tilde{G}. \tag{3.37}$$

The decay constant of the invisible axion v_{PQ} is constrained from astrophysical observation (See Sec. 3.6 for more details),

$$v_{PQ} \gtrsim 10^9\,\mathrm{GeV}. \tag{3.38}$$

The mass of the KSVZ axion is given by setting $v \to v_{PQ}$ and $c_u + c_d = 1$ in Eq. (3.27),

$$m_a \simeq \frac{\sqrt{m_u m_d}}{m_u + m_d} \frac{m_\pi F_\pi}{v_{PQ}} \lesssim 10^{-2} \left(\frac{10^9 \,\mathrm{GeV}}{v_{\mathrm{PQ}}} \right) \mathrm{eV}. \tag{3.39}$$

As the generic property of the QCD axion, the axion-photon coupling plays a important role for its detection and constraint,

$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} F\tilde{F}, \tag{3.40}$$

where F is the electromagnetic field strength tensor, $\tilde{F}$ is its dual *i.e.* $\tilde{F}_{\mu\nu} = 1/2\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, and

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi} \left(\frac{E}{N} - \frac{2}{3}\frac{4+z}{1+z} \right) \frac{1+z}{\sqrt{z}} \frac{m_a}{m_\pi F_\pi}, \tag{3.41}$$

where E and N are the electromagnetic and color anomaly of the axial current associated with the PQ symmetry, and z is the up- and down-quark mass ratio, $z = \frac{m_u}{m_d} \simeq 0.56$. In the KSVZ model, $E/N = 0$. Due to the mixing of the axion with the neutral pion and the eta meson, the axion has a non-zero coupling with photon even in the KSVZ model with only the $SU(3)_c$ charged KSVZ quark. See Appendix. E for the more detail derivation of the axion-photon coupling. Finally, let us also comment on the case of introducing N_K pair of Q, $\bar{Q}$. Such KSVZ quark can also obtain the mass with the coupling to the ϕ, and then the axion coupling with the gluon is

$$\mathcal{L} = \frac{g^2}{32\pi^2} N_K \frac{a}{v_{\mathrm{PQ}}} G\tilde{G}. \tag{3.42}$$

The mass formula of the axion is changed by,

$$m_a \simeq \frac{m_\pi F_\pi}{v_{PQ}} N_K. \tag{3.43}$$

Due to the blow-up of the SM gauge coupling at the GUT or the Planck scale, the number of N_K is generally upper-bounded by $N_K \lesssim 20$.

3.3 ZDFS model

Another invisible axion model has been proposed by Zhitnisky [48] and Dine, Fischler and Srednicki [49] (the so-called ZDFS model). There, an extra complex scalar field which breaks the PQ symmetry spontaneously is introduced to the PQWW model.

The Lagrangian in Yukawa sector is the same as the one in Eq. (3.1). The scalar potential of two doublet Higgs boson and an extra complex scalar field ϕ is

$$\begin{aligned} V(H_u, H_d, \phi) &= \lambda_u \left(|H_u|^2 - \frac{v_u}{2}\right)^2 + \lambda_d \left(|H_d|^2 - \frac{v_d^2}{2}\right)^2 && (3.44) \\ &+ \lambda_\phi \left(|\phi|^2 - \frac{v_{\rm PQ}^2}{2}\right)^2 && (3.45) \\ &+ (c_1|H_u|^2 + c_2|H_d|^2)|\phi|^2 + c_3(H_u H_d \phi^{2*} + h.c.) && (3.46) \\ &+ c_4|H_u H_d|^2 + c_5|H_u^\dagger H_d|^2. && (3.47) \end{aligned}$$

The PQ symmetry corresponds to the following transformation,

$$q_L \to e^{i\alpha} q_L, u_R \to e^{i\alpha} u_R,\ d_R \to e^{i\alpha} d_R, \tag{3.48}$$

$$H_u \to e^{-2i\alpha} H_u,\ H_d \to e^{-2i\alpha} H_d,\ \phi \to e^{-2i\alpha}\phi. \tag{3.49}$$

Due to the term $H_u H_d \phi^*$ in $V(H_u, H_d, \phi)$, the extra scalar field obtain the charge of the PQ symmetry. To find the axion component, let us decompose $H_u,\ H_d,\ \phi$ with their phase directions and the vacuum expectation value,

$$H_u = \frac{v_u}{\sqrt{2}}\exp\left(i\frac{x_u}{v_u}\right),\ H_d = \frac{v_d}{\sqrt{2}}\exp\left(i\frac{x_d}{v_d}\right),\ \phi = \frac{\tilde{v}_{\rm PQ}}{\sqrt{2}}\exp\left(i\frac{\tilde{a}}{\tilde{v}_{\rm PQ}}\right). \tag{3.50}$$

As discussed in the PQWW model, one of these components are absorbed into the Z boson,

$$h_Z = -\frac{v_u}{v}x_u + \frac{v_d}{v}x_d. \tag{3.51}$$

Furthermore, another component $H_{\rm heavy}$ obtains the mass from the term proportional to c_3 in Eq. (3.44),

$$V = c_3 H_u H_d \phi^* = \frac{c_3}{2} v_u v_d v_{\rm PQ}^2 \cos\left(\frac{x_u}{v_u} + \frac{x_d}{v_d} - 2\frac{\tilde{a}}{v_{\rm PQ}}\right). \tag{3.52}$$

Therefore, the axion component is the orthogonal direction to these two directions,

$$\begin{pmatrix} h_z \\ H_{\rm heavy} \\ a \end{pmatrix} = \begin{pmatrix} -v_u/v & v_d/v & 0 \\ -v_d/2\sqrt{N_1} & -v_u/2\sqrt{N_1} & v_u v_d/\tilde{v}_{\rm PQ}\sqrt{N_1} \\ 2v_u v_d^2/v' v^2 & 2v_u^2 v_d/v' v^2 & \tilde{v}_{\rm PQ}/v' \end{pmatrix} \begin{pmatrix} x_u \\ x_d \\ \tilde{a} \end{pmatrix}, \tag{3.53}$$

where,

$$\begin{aligned} N_1 &= \sqrt{v_u^2/4 + v_d^2/4 + v_u^2 v_d^2/v_{\rm PQ}^{\tilde{2}}}, && (3.54) \\ v' &= \sqrt{\tilde{v}_{\rm PQ}^2 + v_u^2\left(\frac{2x}{x + x^{-1}}\right)^2 + v_d^2\,(2x^{-1}x + x^{-1})^2}, && (3.55) \end{aligned}$$

and $x = v_d/v_u$. The axion component in $H_u^0,\ H_d^0,\ \phi$ is

$$H_u^0 = \frac{v_u}{\sqrt{2}}\exp\left(i\frac{2x}{x+x^{-1}}\frac{a}{v'}\right),\ H_d^0 = \frac{v_d}{\sqrt{2}}\exp\left(i\frac{2x^{-1}}{x+x^{-1}}\frac{a}{v'}\right), \tag{3.56}$$

$$\phi = \frac{\tilde{v_{\rm PQ}}}{\sqrt{2}}\exp\left(i\frac{a}{v'}\right). \tag{3.57}$$

By the transformation,

$$u_R \to \exp\left(-i\frac{2x}{x+x^{-1}}\frac{a}{v'}\right)u_R,\ d_R \to \exp\left(-i\frac{2x^{-1}}{x+x^{-1}}\frac{a}{v'}\right)d_R, \tag{3.58}$$

the axion-gluon coupling is obtained as

$$\mathcal{L}_{agg} = +\frac{g^2}{32\pi^2}6\frac{a}{v'}G\tilde{G}. \tag{3.59}$$

The mass of the DFSZ axion is

$$m_a \simeq \frac{m_\pi F_\pi}{v'/6} \lesssim 6\times 10^{-2}\left(\frac{10^9\,\text{GeV}}{v'/6}\right). \tag{3.60}$$

The axion-photon coupling in ZDFS model is

$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4}F\tilde{F}, \tag{3.61}$$

where $g_{a\gamma\gamma}$ with $E/N = 8/3$ in Eq. (3.41) (See Appendix E).

3.4 Composite axion model

The composite invisible axion model has been proposed by Kim [50], Kim and Choi [51]. There, the axion is the composite meson by an extra confining force.

Let us consider the fermions charged under a new $SU(N)$ confining gauge interaction in the representation,

$$(N,3)+(\bar{N},\bar{3})+(N,1)+(\bar{N},1), \tag{3.62}$$

where N ($\bar{N}$) denotes the (anti-)fundamental representation in $SU(N)$, and 3 ($\bar{3}$) is the (anti-)fundamental representation in $SU(3)_c$. The last two fermions are singlet in $SU(3)_c$. For these representations, we assign

$$Q^{A\alpha},\ \bar{Q}_{A\alpha},\ q^A,\ \bar{q}_A, \tag{3.63}$$

respectively. There, A is the $SU(N)$ index, and α is the $SU(3)_c$ index. Neglecting the mass terms of these fermions and the coupling constant of the $SU(3)_c$, the

Lagrangian possesses the $SU(4)_L \times SU(4)_R \times U(1)_V \times U(1)_A$ global symmetry,

$$\begin{pmatrix} Q^{A1} \\ Q^{A2} \\ Q^{A3} \\ q^A \end{pmatrix} \to \exp(i\alpha_L^a T^a_{SU(4)_L}) \begin{pmatrix} Q^{A1} \\ Q^{A2} \\ Q^{A3} \\ q^A \end{pmatrix}, \tag{3.64}$$

$$\begin{pmatrix} \bar{Q}_{A1} \\ \bar{Q}_{A2} \\ \bar{Q}_{A3} \\ \bar{q}_A \end{pmatrix} \to \exp(i\alpha_R^a T^a_{SU(4)_R}) \begin{pmatrix} \bar{Q}_{A1} \\ \bar{Q}_{A2} \\ \bar{Q}_{A3} \\ \bar{q}_A \end{pmatrix}, \tag{3.65}$$

$$\begin{pmatrix} Q^A \\ \bar{Q}^\dagger_A \\ q^A \\ \bar{q}^\dagger_A \end{pmatrix} \to \exp(i\alpha_V) \begin{pmatrix} Q^A \\ \bar{Q}^\dagger_A \\ q^A \\ \bar{q}^\dagger_A \end{pmatrix}, \tag{3.66}$$

$$\begin{pmatrix} Q^A \\ \bar{Q}_A \\ q^A \\ \bar{q}_A \end{pmatrix} \to \exp(i\alpha_A) \begin{pmatrix} Q^A \\ \bar{Q}_A \\ q^A \\ \bar{q}_A \end{pmatrix} \tag{3.67}$$

where $T^a_{SU(4)_L}$ ($T^a_{SU(4)_R}$) is the generator of $SU(4)_L$ ($SU(4)_R$), α_L^a, α_R^a, α_V, α_A are the phase of $SU(4)_L$, $SU(4)_R$, $U(1)_V$, $U(1)_A$, respectively. Note that the $U(1)_A$ symmetry is the anomalous symmetry. These global symmetries are broken down to $SU(4)_V \times U(1)_V$ below the confinement scale of $SU(N)$ by the fermion bilinear condensates,

$$\langle Q^{A\alpha}\bar{Q}_{A\alpha}\rangle = \langle q^A \bar{q}_A\rangle = \Lambda_N^3 \tag{3.68}$$

where only the index A is contracted, Λ_N is a parameter of mass dimension 1. The pseudo-NG boson by the representations in $SU(3)_c$ is given by

$$\mathbf{8} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1} + \mathbf{1}. \tag{3.69}$$

The $SU(3)_c$ charged pseudo-NG bosons obtain their masses from the knowledge of the mass difference of the charged pion and the neutral pion $m^2_{\pi^+} - m^2_{\pi^0} = (33.6\,\mathrm{MeV})^2$, which is order-estimated from the radiative correction with the photon propagation as

$$m^2_{\pi^+} - m^2_{\pi^0} \simeq (\sqrt{\alpha_{em}} F_\pi)^2 \simeq \mathcal{O}(10\,\mathrm{MeV})^2, \tag{3.70}$$

where $\alpha_{\rm em}$ denotes the fine-structure constant. Here, the radiative correction with the gluon produces the masses of the colored pseudo-NG bosons around Λ_N,

$$m_{\text{colored pNG boson}} \simeq (\sqrt{\alpha_c}\Lambda_N)^2 \simeq \mathcal{O}(\Lambda_N)^2. \tag{3.71}$$

There still remains two $SU(3)_c$ singlets. One of them should be heavy like the η' in QCD. Then, the last one corresponds to the axion, which gets the mass from the $SU(3)_c$ anomaly and the mixing with the neutral pion. The order of the axion mass is estimated as

$$m_a \simeq \mathcal{O}\left(\frac{m_\pi F_\pi}{\Lambda_N}\right). \tag{3.72}$$

3.5 Periodic axion potential and domain wall number

Before going to the discussion about the constraints on the axion model, let us discuss the periodicity of the axion potential. To be specific, let us consider the KSVZ model with the N_K flavor, where the interval of the axion under the decomposition in Eq. (3.35) can be defined as,

$$\frac{a}{v_{PQ}} : \ [0, 2\pi]. \tag{3.73}$$

After the decoupling of the KSVZ quarks, the axion couples with the gluon as in Eq. (3.42),

$$\mathcal{L} = N_K \frac{a}{v_{PQ}} G\tilde{G}. \tag{3.74}$$

Notice that, in this basis, the axion couples with the gluon only by the above term. By the up and the down quark transformation in Eq. (3.17) of $c_u + c_d = N_K$, the axion-gluon coupling is erased. Below the QCD scale, the axion-pion potential in Eq. (3.22) arises, and the potential can be re-written as [52],

$$V = -m_\pi^2 F_\pi^2 \sqrt{1 - \frac{m_u m_d}{(m_u + m_d)^2}[1 + \cos(N_K\theta)]} \cos(\pi^0/F_\pi - \phi_a), \tag{3.75}$$

$$\phi_a = \frac{m_u - m_d}{m_u + m_d} \frac{-\sin(N_K\theta)}{1 - \cos(N_K\theta)}, \tag{3.76}$$

where $\theta = a/v_{PQ}$. On the vacuum, the pion π^0 gets a VEV to cancel the ϕ_a, and thus the axion potential is,

$$V = -m_\pi^2 F_\pi^2 \sqrt{1 - \frac{m_u m_d}{(m_u + m_d)^2}[1 + \cos(N_K\theta)]}. \tag{3.77}$$

This shows that the axion potential has N_K degenerate vacua under the interval $0 \leq \theta < 2\pi$. As we discussed later, such N_K becomes important for the axion domain wall. We define F_a as the axion decay constant accounting the correct axion interval (Here, $F_a = v_{PQ}$). The number of the degenerate vacua is called the domain wall number (here, N_K is the domain wall number). This number is generally determined by the color anomaly. For the domain wall number of different models, see Sec. 4.1.

3.6 Astrophysical constraints on axion model

3.6.1 Horizontal branch stars

The so-called horizontal branch (HB) stars are powered by helium fusion to carbon and oxygen with a core-averaged energy release of about $80\,\mathrm{erg\,g^{-1}s^{-1}}$. In existence of the axion, the transition of the photon into the axion provides another energy loss process. The energy loss rate per unit mass $\mathcal{E}$ is given by [53]

$$\mathcal{E} \simeq \frac{g_{a\gamma\gamma}^2 T^7}{4\pi\rho_G} \simeq 30\left(\frac{g_{a\gamma\gamma}}{10^{-10}\,\mathrm{GeV}^{-1}}\right)^2 \mathrm{erg\,g^{-1}\,s^{-1}}, \tag{3.78}$$

where T is a typical temperature $10^8\,\mathrm{K}$, and ρ_G is a typical density $10^4\,\mathrm{g\,cm^{-3}}$. Therefore, the lifetime of the HB stars is reduced by a factor $80/(80+30) \simeq 0.3$ for $g_{a\gamma\gamma} = 10^{-10}\,\mathrm{GeV}^{-1}$. On the other hand, the measurement of fifteen clusters agrees with the expected helium-burning lifetime within about $10\,\%$ [54]. Thus, as a reasonable upper-limit, we obtain

$$|g_{a\gamma\gamma}| \lesssim 10^{-10}\,\mathrm{GeV}^{-1}. \tag{3.79}$$

which corresponds to the v_{PQ} constraint,

$$v_{\mathrm{PQ}} \gtrsim 10^7\,\mathrm{GeV}. \tag{3.80}$$

3.6.2 Supernova 1987A

About twenty neutrinos from Supernova (SN) 1987A were observed at Kamiokande [55], IMB [56] and Baksan [57]. There, several properties such as the energy of the neutrinos, the number of neutrinos, and the distribution during several seconds match well with the theoretical expectation. If the axion exists, such properties will be modified. Because the axion has a coupling with the nucleon (n), the axion is emitted by nucleon bremsstrahlung $n+n \to n+n+a$. This process reduces the duration of the neutrino signal, and thus constrains the large nucleon-axion coupling. The axion emission rate per unit mass is roughly given by [53]

$$\mathcal{E}_a \simeq 10^{37} \left(\frac{v_{\rm PQ}}{\rm GeV}\right)^{-2} \left(\frac{T}{30\,{\rm MeV}}\right)^4 {\rm erg\,g^{-1}\,s^{-1}}, \tag{3.81}$$

where T is the temperature of the core of the SN. From the neutrino observation from SN1987, the energy loss rate [58] is upper-bounded as

$$\mathcal{E}_a \lesssim 10^{19}\,{\rm erg\,g^{-1}\,s^{-1}}. \tag{3.82}$$

Therefore, we obtain the bound,

$$v_{\rm PQ} \gtrsim 10^9\,{\rm GeV}. \tag{3.83}$$

Precisely, the constraint from the burst duration disappears at $v_{\rm PQ} \lesssim 10^6\,{\rm GeV}$ because the large interaction with nucleons reduces the axion emission outside the SN. Even smaller $10^2\,{\rm GeV} \lesssim v_{PQ} \lesssim 10^5\,{\rm GeV}$ is, however, constrained due to the increased detection rate of the axion emitted from SN1987A at Kamiokande [59].

Therefore, using the constraints from both the HB stars and the SN1987A, the axion decay constant is lower-bounded as,

$$v_{PQ} \gtrsim 10^9\,{\rm GeV}. \tag{3.84}$$

3.7 Axion detection

In this section, we briefly review direct detections of the axion by haloscopes and helioscopes. See *e.g.* Refs. [60, 61] for recent reviews about the axion detection.

3.7.1 Haloscope

In the haloscope [62, 63], the dark matter axion is detected by using the coherently oscillating axion conversion to the photon in a magnetic field. In a micro-wave cavity, the coherently oscillating axion produces the monochromatic spectrum of the photon, and the photon signal is maximized when a cavity resonant frequency matches with m_a. In the ADMX experiment [64], the frequency range is $460-890$ Hz which corresponds to the axion mass range $1.9-3.65\,\mu$eV. Recently, the experiment constrains the QCD axion parameter region for the axion mass $2.66-2.81\,\mu$eV [65].

3.7.2 Helioscope

In the helioscope [62], the axion from the sun is detected using the axion conversion to the photon in a static uniform magnetic field. The probability of the axion conversion to the photon under the transverse magnetic field over the length L is proportional to a factor F_c which parametrizes the coherence of the conversion [62],

$$F_c = \frac{2(1 - \cos qL)}{(qL)^2}, \tag{3.85}$$

where q is the momentum transfer from the axion to the photon, and $L \simeq 10\,\mathrm{m}$ in the CAST experiment [66, 67]. For $qL \ll 1$, the factor F_c is almost unity, which implies the probability is almost constant up to $m_a \simeq 10^{-2}\mathrm{eV}$ and decreases for the higher mass range. The CAST constrains $g_{a\gamma\gamma} < 8.8 \times 10^{-11}\,\mathrm{GeV}^{-1}$ (95% C.L.) for $m_a < 0.02\,\mathrm{eV}$ [66, 67].

Chapter 4

Axion Cosmology

In this chapter, we discuss the cosmological evolution of the axion field in the following two cases.

- The PQ symmetry is restored after inflation and then broken due to the temperature decreasing (PQ symmetry breaking after inflation)
- The PQ symmetry is broken during inflation and not restored after inflation (PQ symmetry breaking during inflation)

4.1 PQ symmetry breaking after inflation

When the global $U(1)_{PQ}$ symmetry is broken after the end of the inflation, the topological defects called the global cosmic strings[1] are formed around the temperature $T \simeq v_{PQ}$. In the numerical calculation [68, 69, 69–71], it is shown that the cosmic strings follow the scaling law in $O(1)$ Hubble time by emitting the axion,

$$\rho_s \simeq \frac{\mu}{t^2}, \tag{4.1}$$

where μ is the string tension,

$$\mu = \pi F_a^2 \ln\left(\frac{t}{\delta_W}\right). \tag{4.2}$$

Here, δ_W is the width of the global string. In accordance with the scaling law, the cosmic string density in the radiation and matter dominated era is scaled as

$$\rho_s \propto 1/a^4,\ \rho_s \propto 1/a^3, \tag{4.3}$$

respectively. This scaling is the same as the dominant one. Thanks to this property, the $\mathcal{O}(1)$ number of the cosmic string exits in the universe for a long time. When

[1] See Appendix F.2 for the cosmic string solution.

the temperature decreases further, the axion obtains the periodic potential below the QCD phase transition temperature.

To find out the cosmology below the QCD confinement scale, let us concretely consider the cases in the KSVZ model with N_K flavors of KSVZ quarks. As discussed in Sec. 3.5, the axion potential has N_K degenerate vacua for $0 \leq a/F_a < 2\pi$ (here $F_a = v_{PQ}$ in Eq. (3.77)). Around the center of the cosmic string, the phase changes as

$$a/F_a : 0 \to 2\pi. \tag{4.4}$$

Thus, the phase, $N_K a/F_a$, changes from 0 to $2\pi N_K$. This means the N_K number of the domain walls[2] are formed around the center of a cosmic string. On the other word, the global $U(1)_{PQ}$ symmetry is explicitly broken into the discrete global symmetry Z_{N_K} by the quantum anomaly, and thus the domain walls are formed with the Z_{N_K} breaking below the QCD transition temperature.

In the case of the $N_K = 1$, there is no degenerate vacuum, and thus the string-domain wall system disappears in $\mathcal{O}(1)$ Hubble time by emitting the axions. Such emitted axions contribute to the present dark matter density. By the classical lattice simulation [72], the total present density of the cold axion in $N_K = 1$ scenario is estimated as

$$\Omega h^2 \simeq 8 \left(\frac{F_a}{10^{12}\,\mathrm{GeV}} \right)^{1.19}. \tag{4.5}$$

Thus, the decay constant of the axion is upper-bounded by

$$F_a \lesssim 3 \times 10^{10}\,\mathrm{GeV}. \tag{4.6}$$

for $\Omega h^2 \lesssim 0.12$.

On the other hand, the string-domain wall system in $N_K > 1$ is stable due to the spontaneous breaking of the exact discrete symmetry. To clarify this string-domain wall system, let us compare the energy of the domain wall, E_W, and the string, E_S,

$$\frac{E_W}{E_S} \simeq \frac{\sigma(1/H)^2}{\mu(1/H)}, \tag{4.7}$$

$$\simeq \frac{m_a}{N_K^2 H}, \tag{4.8}$$

where $\sigma \simeq m_a F_a^2/N_K^2$ is the domain wall tension. From this ratio, we find that the dynamics of the domain wall dominates soon after its formation. The density of the

[2] See Appendix F.1 for the domain wall solution.

domain wall is given by

$$\rho_W \simeq \frac{m_a F_a^2 (1/H)^2}{N_K^2 (1/H)^3}, \tag{4.9}$$

$$= m_a F_a^2 H / N_K^2. \tag{4.10}$$

For the radiation and the matter domination era, the domain wall energy density is scaled as

$$\rho_W \propto 1/a^2, \ \rho_W \propto 1/a^{3/2}. \tag{4.11}$$

Thus, the domain wall dominates the universe when $\rho_W \simeq M_{\rm pl}^2 H^2$, *i.e.*,

$$T \simeq \frac{10^{-4}}{N_K} \sqrt{\left(\frac{F_a}{10^{12}\,\text{GeV}}\right)}\,\text{GeV}, \tag{4.12}$$

which conflict with the standard cosmology.

To solve the domain wall problem in $N_K > 1$, one can introduce the term which explicitly and completely breaks the Z_{N_K} symmetry,

$$\mathcal{L} = \mathcal{B} e^{-i\delta_B} \phi + h.c., \tag{4.13}$$

where $\mathcal{B}$ is the real parameter and δ_B is the phase. Because this term resolve the degeneracy of the vacuum, the energy between domain walls is given by

$$E_B \simeq \frac{\mathcal{B} F_a}{N_K (1/H)^3}. \tag{4.14}$$

Comparing with the domain wall energy,

$$\frac{E_B}{E_W} \simeq \frac{\mathcal{B} N_K}{m_a F_a H}. \tag{4.15}$$

Thus, the vacuum energy will dominate the dynamics at

$$H \lesssim \frac{\mathcal{B} N_K}{m_a F_a}. \tag{4.16}$$

To evade the domain wall problem, this time must be earlier than the domain wall domination time $H \simeq m_a F_a^2/(M_{\rm pl}^2 N_K^2)$. This condition puts the lower-bound for the parameter $\mathcal{B}$ as

$$\mathcal{B} \gtrsim \frac{m_a^2 F_a^3}{M_{\rm pl}^2 N_K^3}. \tag{4.17}$$

Using the astrophysical constraint $F_a \gtrsim 10^9$ GeV. The bias term changes the axion potential as

$$V \simeq m_a^2 F_a^2 / N_K^2 (1 - \cos(N_K a/F_a)) - \mathcal{B} F_a \cos(a/F_a + \delta_B). \tag{4.18}$$

For the small $a/F_a \ll 1$, the shift of the potential minimum of the axion is given by

$$\frac{a}{F_a} \simeq \frac{\mathcal{B} F_a \delta_B \sin(\delta_B)}{m_a^2 + \mathcal{B} F_a \cos(\delta_B)}. \tag{4.19}$$

To be consistent with the nEDM experiment, we require

$$\frac{\mathcal{B} F_a \delta_B \sin(\delta_B)}{m_a^2 + \mathcal{B} F_a \cos(\delta_B)} \lesssim 10^{-10}. \tag{4.20}$$

Furthermore, to be consistent with the observed cold dark matter abundance, the small $\mathcal{B}$ and F_a are required. Therefore, these three conditions generally give the non-trivial constraints for the three parameters F_a, $\mathcal{B}$, δ_B. From the numerical calculation of the axion abundance in this scenario [73], it is founded that δ_B must be tuned as $\delta_B \lesssim 10^{-2}$ for $N_K = 6$.

Finally, let us comment on the cosmology of the ZDFS model. In the ZDFS model, the domain wall number, N_K, is larger than 1. Thus, one needs to solve the domain wall problem *e.g.* by introducing the bias term. It should be noted that the domain wall number in the ZDFS model is 3 or 6 for the coupling between the PQ breaking scalar field and two Higgs doublets, $\mathcal{L} = \phi^{*2} H_u H_d$ or $\mathcal{L} = \phi^* H_u H_d$, respectively.

4.2 PQ symmetry breaking during inflation

When the PQ symmetry is broken during the inflation, the inflation exponentially expands a tiny domain, and then the topological remnants and the axions from them are diluted away. There, the axion field with some value, a_0, has spread in and beyond our observable Universe. At the time of the QCD phase transition, the axion obtains its mass and begins to roll down to its potential minimum because the a_0 generally does not sit in the CP-conserving point. This oscillation energy density contributes to the abundance of the cold dark matter. The axion dark matter production mechanism by this oscillation is called as the misalignment mechanism. In the following, let us estimate the current axion abundance from this mechanism.

The axion periodic potential is

$$V(a) = m_a^2(T) \left(\frac{F_a}{N_{\rm dom}}\right)^2 \left(1 - \cos\left[a/(F_a/N_{\rm dom})\right]\right), \tag{4.21}$$

where F_a and $N_{\rm dom}$ are the axion decay constant and the domain wall number defined in Sec. 3.5, $m_a(T)$ is the axion mass depending on the temperature T. Here, we use the power-law approximation obtained by the numerical calculation [74–76],

$$m_a(T) = 4 \times 10^{-4} \frac{\Lambda_{\rm QCD}^2}{F_a/N_{\rm dom}} \left(\frac{T}{\Lambda_{\rm QCD}}\right)^{-3.34} \quad \text{for } T > 0.26\,\Lambda_{\rm QCD} \tag{4.22}$$

$$= 3.8 \times 10^{-2} \frac{\Lambda_{\rm QCD}^2}{F_a/N_{\rm dom}} \quad \text{for } T \leq 0.26\Lambda_{\rm QCD}. \tag{4.23}$$

where $\Lambda_{\rm QCD} \simeq 400\,\text{MeV}$. The equation of motion for the axion is

$$\ddot{a} + 3H(T)\dot{a} + m_a^2(T)a = 0 \tag{4.24}$$

where the higher terms of a are ignored assuming the initial amplitude is not so large ($a \ll F_a/N$), $\dot{}$ denotes the physical time derivative, $H(T)$ is the Hubble parameter,

$$H(T) = \frac{\dot{R}}{R} = \sqrt{\frac{g^*}{90\pi^2}} \frac{T^2}{M_{\rm pl}} \tag{4.25}$$

where R is the scale factor, $M_{\rm pl} \simeq 2.4 \times 10^{18}\,\text{GeV}$ is the reduced Planck mass, g^* is the effective degree of freedom. The axion field oscillation starts at the temperature T_O determined by the condition,

$$m_a(T_O) = 3H(T_O), \tag{4.26}$$

Using Eq. (4.22),

$$T_O \simeq 1\,\text{GeV} \left(\frac{F_a/N_{\rm dom}}{10^{12}\,GeV}\right)^{-0.19} \left(\frac{\Lambda_{\rm QCD}}{400\,MeV}\right), \tag{4.27}$$

using $g^* \simeq 62$ at $1\,\text{GeV}$. The axion density ρ_a is

$$\rho_a(T) = \frac{\ddot{a}}{2} + m_a(T)^2 \frac{a^2}{2}. \tag{4.28}$$

Under the assumption that $m_a \gg H$ and thus the time average of the axion energy density satisfies $\rho_a(T) \simeq \langle m_a(T)^2 a^2 \rangle = \langle \ddot{a}^2 \rangle$, the variable $\rho_a R^3/m_a$ is time invariant. Thus, the current axion abundance is given by [77]

$$\Omega h^2 = \frac{\rho_a^0}{\rho_0} = \frac{s_0}{\rho_0}\frac{\rho_a^0}{s_0} = \frac{s_0}{\rho_0} m_a \frac{\rho_a^0/m_a}{s_0} \tag{4.29}$$

$$\simeq \frac{s_0}{\rho_0} m_a C_1 \frac{\rho_a(T_O)/m_a(T_O)}{s(T_O)} \tag{4.30}$$

$$\simeq 0.18\,\theta_m^2 \left(\frac{F_a/N_{\rm dom}}{10^{12}\,\text{GeV}}\right)^{1.19} \left(\frac{\Lambda_{\rm QCD}}{400\,\text{MeV}}\right) \tag{4.31}$$

where h is the present Hubble parameter in units of $100\,\text{km/s/Mpc}$, ρ_a^0 is the present axion density, $\rho_0 \simeq 1.05 \times 10^{-5} h^2\,\text{GeVcm}^{-3}$ is the critical density of the Universe,

$s_0 \simeq 2890\,\mathrm{cm}^{-3}$ is the current entropy density, m_a is the axion mass below $T \leq 0.26\Lambda_{\rm QCD}$ in Eq. (4.23), $C_1 = 1.85$ is the factor correcting the contribution during the era $m_a \gtrsim H$ calculated by [78], the coefficient θ_m is the free parameter relating to the initial misalignment angle.

4.2.1 Constraint from isocurvature density perturbation

During inflation, the massless axion has a quantum fluctuation. When the PQ symmetry is not restored after the end of the inflation, this fluctuation affects the axion energy density below the QCD transition temperature as

$$\Omega h^2 = 0.18\,\theta_{\rm ini}^2 \left(\frac{F_a/N_{\rm dom}}{10^{12}\,\mathrm{GeV}}\right)^{1.19} \left(\frac{\Lambda_{\rm QCD}}{400\,\mathrm{MeV}}\right), \tag{4.32}$$

$$\theta_m^2 \equiv \theta_{\rm ini}^2 + \delta\theta^2 = \theta_{\rm ini}^2 + \left(\frac{H_I}{2\pi F_a/N_{\rm dom}}\right)^2. \tag{4.33}$$

where H_I is the Hubble parameter during inflation, $H_I/(2\pi F_a/N_{\rm dom})$ or $\delta\theta$ is the quantum fluctuation [79–83] in the initial misalignment angle $\theta_{\rm ini}$. Such axion fluctuations are independent of the inflaton's one in the flat time slice, called the isocurvature density perturbations. Using the energy density fluctuation $\delta\Omega h^2$ due to the isocurvature density perturbations, *i.e.* $\delta\Omega h^2 = 2(\delta\theta/\theta_m)\Omega h^2$, the power spectrum of the isocurvature density perturbation is given by

$$\mathcal{P}_{S_c} = \left(\frac{\delta\Omega}{\Omega_c}\right)^2 = \frac{4}{\theta_m^2}\left(\frac{H_{\rm ini}}{2\pi F_a/N_{\rm dom}}\right)^2 \left(\frac{\Omega}{\Omega_c}\right)^2, \tag{4.34}$$

where Ω_c is the density parameter of the cold dark matter, $\Omega_c h^2 \simeq 0.12$. The isocurvature density perturbation is stringently constrained by the observation of the cosmic microwave background (CMB) as [84]

$$\mathcal{P}_{S_c}/(\mathcal{P}_\zeta + \mathcal{P}_{S_c}) < 0.038, \tag{4.35}$$

where $\mathcal{P}_{S_c} \simeq 2.20 \times 10^{-9}$ is the power spectrum of the curvature perturbations measured by CMB observations [84]. Therefore, we obtain the following condition

$$\frac{1}{\theta_m^2}\left(\frac{H_I}{2\pi F_a/N_{\rm dom}}\right)^2 \left(\frac{\Omega}{\Omega_c}\right)^2 \lesssim 2 \times 10^{-11}, \tag{4.36}$$

equivalently,

$$\left[\theta_{\rm ini}^2 + \left(\frac{H_I^2}{2\pi F_a/N_{\rm dom}}\right)^2\right]\left(\frac{H_I}{2\pi F_a/N_{\rm dom}}\right)^2 \left(\frac{F_a/N_{\rm dom}}{10^{12}\,\mathrm{GeV}}\right)^{2.38} \lesssim 10^{-11}. \tag{4.37}$$

For *e.g.* $F_a/N_{\rm dom} = 10^{12}$ GeV, $\theta_{\rm ini} = 2/3$, the constraint on H_I is obtained as

$$H_I \lesssim 3 \times 10^7 \, \text{GeV}. \tag{4.38}$$

Therefore, the Hubble parameter during inflation must be much smaller than the current observation bound $H_I \lesssim 10^{13}$ GeV if the PQ symmetry is broken during or before the inflation and not restored after the end of the inflation.

Chapter 5

Global symmetry and quantum gravity

In this chapter, let us discuss the conjecture that all global symmetries are accidental ones. In particular, we show that the $U(1)$ global symmetry seems to be broken by the wormhole effect.[1]

5.1 Global symmetries broken by wormhole

5.1.1 Wormhole solution

Historically, in 1987, Giddings and Strominger [85] found a wormhole solution as an instanton from the Euclidean action of a massless axion coupled to gravity, where they introduced a three form field as the dual version of the axion theory. The wormhole solution in the pseudo-scalar representation was obtained by Lee [86] in 1988, where it was shown that the wormhole solution appears if the charge conservation condition is added as the Lagrange multiplier. In these papers, only the axion component is accounted, on the other word, the radial component of an original complex scalar field is assumed to be frozen. As was first pointed out by Abbott and Wise [87], the radial component is not fixed in reality. In Refs. [87, 88], the effect is investigated without the spontaneous symmetry breaking. Thus, in 1995, Linde *e.t.al* studied and found the wormhole solution with a dynamical complex scalar field and spontaneous symmetry breaking [30].

Here, let us derive the wormhole solution from the Euclidean action[2] as in [30],

$$\hat{S}_E = \int d^4x \sqrt{g_g} \left(-\frac{M_{\rm pl}}{2}\mathcal{R} + \frac{1}{2}(\partial_\mu f)^2 + \frac{1}{2} f^2 (\partial_\mu \theta)^2 + V(f) \right) \tag{5.1}$$

[1]The meaning of the wormhole will be explained soon.

[2]Here, we omit the Gibbons-Hawking surface term for simplicity. It should be noted that its effect is not so large in our following analysis (See Ref. [30] for more details).

where $f,\ \theta$ is defined from a complex scalar field $\Phi = \frac{f}{\sqrt{2}}e^{i\theta}$, g_g is the determinant of the metric tensor (we will explain the metric soon), $\mathcal{R}$ is the scalar curvature, $V(f)$ is the potential which has a minimum at $f = f_0$ and

$$\frac{d}{df}V(f)|_{f=f_0} = V(f)|_{f=f_0} = 0. \tag{5.2}$$

Here, we assume that the solution is $O(4)$ symmetric, and thus the metric becomes

$$\begin{aligned} ds^2 &= d\rho^2 + R(\rho)^2 d\Omega_3^2, &(5.3)\\ d\Omega_3 &= d\chi^2 + \sin(\chi)^2(d\theta^2 + \sin(\theta)^2 d\phi^2), &(5.4)\end{aligned}$$

where $d\Omega_3$ is the line element of a unit three-sphere, $R(\rho)$ is a function of ρ. Following Ref. [86], the following charge conservation condition,

$$\partial_\mu(j^\mu \equiv \sqrt{g_g}g_g^{\mu\nu}f^2\partial_\nu\theta) = 0, \tag{5.5}$$

is introduced as a Lagrange multiplier. This charge conservation originates from the global $U(1)$ symmetry induced by the phase rotation of the complex scalar Φ, where the current is given by

$$j^\mu \equiv \sqrt{g_g}g_g^{\mu\nu}i(\partial_\nu\Phi^*\Phi - \Phi^*\partial_\nu\Phi) = \sqrt{g_g}g_g^{\mu\nu}f^2\partial_\nu\theta. \tag{5.6}$$

Therefore, the action S_E is

$$S_E = \hat{S}_E + \int d^4x\partial\lambda(x)\partial_\mu j^\mu, \tag{5.7}$$

where $\lambda(x)$ is the lagrange multiplier. From the variational principle $\delta S_E = 0$, three equations for $f,\ \theta,\ \lambda$ is obtained,

$$\begin{aligned} -(\sqrt{g_g}f') + f(\theta')^2 + dV/df - 2f\theta'\lambda' &= 0, &(5.8)\\ -(\sqrt{g_g}f^2\theta')' + \partial_\mu(\sqrt{g_g}f^2\partial^\mu\lambda) &= 0, &(5.9)\\ \partial_\mu j^\mu &= 0, &(5.10)\\ R_{\mu\nu} - \frac{1}{2}g_{g\mu\nu}R &= \frac{1}{M_{\rm pl}}T_{\mu\nu}, &(5.11)\end{aligned}$$

where the subscript $'$ shows $d/d\rho$, the assumption of the $O(4)$ symmetric solution is applied, $R_{\mu\nu}$ is the Ricci curvature tensor, $T_{\mu\nu}$ is the energy-momentum tensor which is given by

$$T_{\mu\nu} = \partial_\mu f\partial_\nu f - f^2\partial_\mu\theta\partial_\nu\theta - g_{g\mu\nu}\left[\frac{1}{2}\partial_k f\partial^k f - \frac{1}{2}\partial_k\theta\partial^k\theta + V\right]. \tag{5.12}$$

The equation in Eq. (5.9) denotes $\lambda = \theta + \text{constant}$. The current conservation equation in Eq. (5.10) has a solution,

$$2\pi^3 R^3 f^2 \theta' = Q_W, \tag{5.13}$$

where Q_W denotes the charge. The non-trivial equations[3] are

$$f'' + \frac{3R'}{R} f' + \frac{Q_W^2}{4\pi^4 f^3 R^6} - dV/df = 0, \tag{5.14}$$

$$(R')^2 - 1 + \frac{8\pi}{3M_p^2} R^2 \left[\frac{Q_W^2}{8\pi^4 f^2 R^6} + V - \frac{f'^2}{2} \right] = 0. \tag{5.15}$$

To obtain the finite action, and avoid singularities in $\rho = 0$, let us take the following boundary condition,

$$R'(0) = 0, \ \ f'(0) = 0, \ \ f(\rho)|_{\rho \to +\infty} \to f_0. \tag{5.16}$$

In Ref. [30], the numerical solution assuming the potential,

$$V(f) = \frac{\lambda}{4}(f^2 - f^0)^2, \tag{5.17}$$

gives the value of the effective action[4], $S_E \simeq \mathcal{O}(10)$ for $\lambda \simeq \mathcal{O}(1)$, $f_0 \simeq \mathcal{O}(10^{12}\,\text{GeV})$.

Let us interpret the wormhole solution physically [85]. First, for $\rho \to \pm\infty$, the equation in Eq. (5.15) is

$$(R')^2 = 1, \tag{5.18}$$

which leads to $R(\rho)^2|_{\rho \to \pm\infty} \to \rho^2$. Thus, the metric for $\rho \to \pm\infty$ becomes,

$$ds^2 = d\rho^2 + \rho^2 d\Omega_3, \tag{5.19}$$

which shows that there are asymptotically Euclidean regions. On the other hand, in particular, for $|\rho| < \infty$, the universe is topologically S^3 or the closed universe for fixed ρ. The schematic picture by this interpretation is in Fig. 5.1. The wormhole seems to represent the tunneling between two asymptotically Euclidean regions. In the figure, each circle (denoted as S^3) around the "throat" or the "tube" shows a three-sphere S^3 for fixed w. From this interpretation, some types of wormholes are considered in large four-dimensional volume as in Fig. 5.2. In Fig. 5.2 on the upper left, the tunneling between the same Euclidean region (a parent universe)

[3] The equations are invariant under the transformation, $\rho \leftrightarrow -\rho$.

[4] In Ref. [30], the effect from the Gibbons-Hawking surface term is included in the analysis, and then it decreases the action by about 10 %.

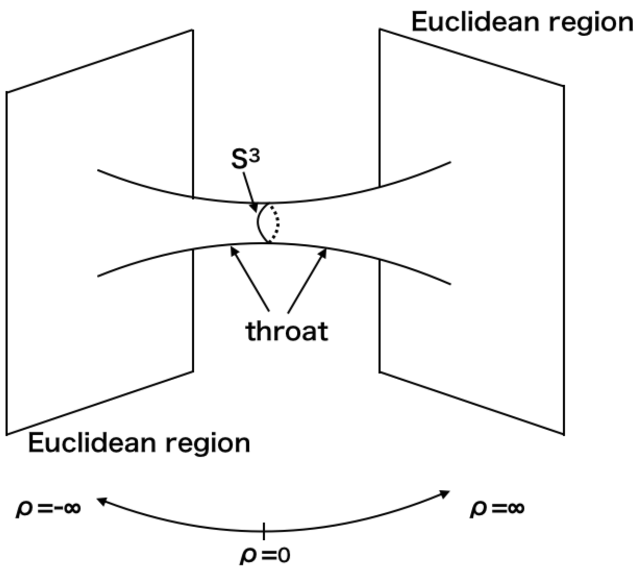

Figure 5.1: *The schematic picture of a wormhole.*

is represented. On the other word, it describes the creation (the extinction) of a closed universe or a baby universe of topologically S^3 from (into) an asymptotically Euclidean region. A type of wormhole as in Fig. 5.2 on the upper right is called the semi-wormhole, where the baby universe is created although both ends of the throat must be connected into asymptotically Euclidean regions far away (outside the picture). As in Fig. 5.2 on the bottom, two different Euclidean regions are connected by the wormhole.

5.1.2 Effects of wormhole

In 1998, Coleman [6, 7], Giddings and Strominger [5] show that the effect of the wormholes is to add local interactions to the Lagrangian. Here, let us see this in a simple set-up.

To concretely see the wormhole effect in the theory, let us consider the transition amplitude from an initial state to a final state with wormholes under the dilute-gas approximation. For simplicity, we assume that there is only one kind of the wormhole (the baby universe). The configuration of wormholes is given in Fig. 5.3. There, the black curve lines are the baby universes, and one large square denotes the parent universe (the Euclidean region), and the black dots are the wormhole locations on the parent universe. The initial state has one parent universe and n_i baby universes. The final state has one parent universe and n_f baby universe. The

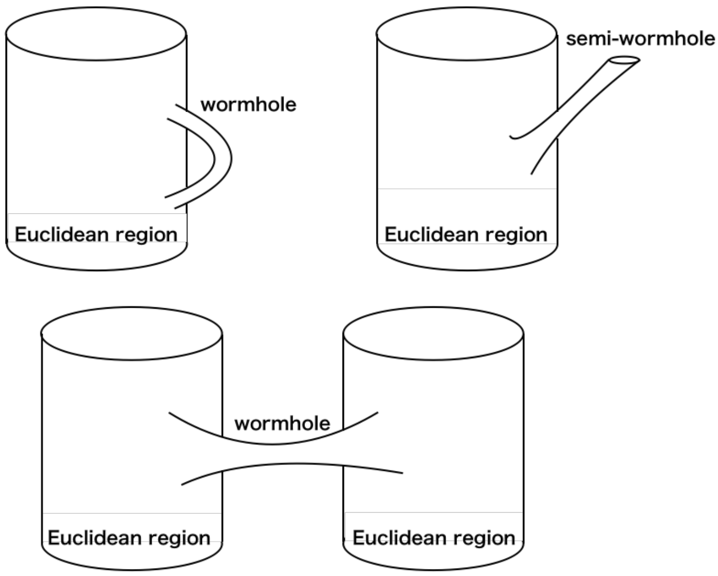

Figure 5.2: *The different types of wormholes.*

m baby universes in the initial state directly evolve into the m baby universes in the final state. There are n wormholes that begin and end in the parent universe. Here, we denote the action of a semi-wormhole by S_0 ($S_0 > 0$). The transition amplitude of one wormhole is proportional to $\exp(-S_0)$. Thus, the amplitude of one geometry *e.g.* of the wormhole configuration in Fig. 5.3 is weighted by the factor $\exp[-S_0(2n+n_i+n_f-2m)]$. To obtain the total transition amplitude, we must sum over all such geometries including the integration over the locations of the wormhole on the parent universe. It should be noted that each inequivalent geometry should be counted once and only once. For the fixed n_i, n_f, m, n, the number of inequivalent geometries is,

$$\frac{(KV_4)^{2n+n_i+n_f-2m}}{(n_i-m)!(n_f-m)!n!2^n}, \tag{5.20}$$

where $V_4 = \int d^4x\sqrt{g}$ and K is assumed to be some dimensionful constant to keep things simple [6].[5] By the factor $(V_4)^{2n+n_i+n_f-2m}$, the wormhole location on the parent universe is integrated over. But, the geometry is redundantly summed over only by this factor. By the factor $(n_i - m)!(n_f - m)!$ in the denominator, the redundancy of the initial and the final semi-wormholes are canceled. The factor $n!2^n$ is needed to correctly count the inequivalent geometry of the n wormholes, where 2^n is required because we cannot distinguish the start and the final point of

[5] Actually, the amplitude will include terms constructed from the fields on the manifold.

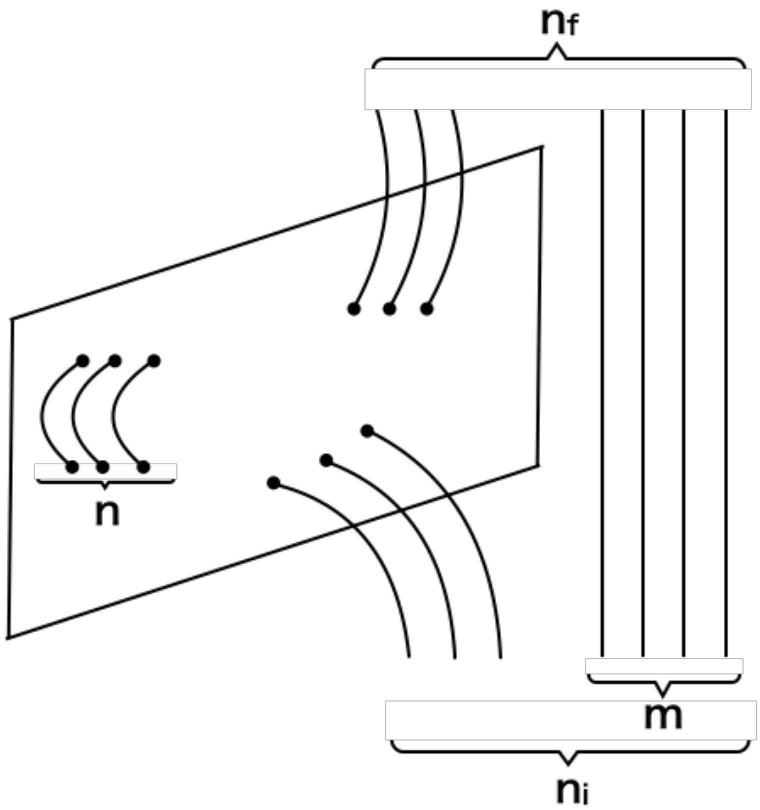

Figure 5.3: *The schematic picture of the wormhole configuration. The sheet shows the parent universe. The black dot denotes the wormhole connection points with the parent universe.*

each wormhole. Combining these factors, we obtain the amplitude,

$$(A_{n_i,n_f})_{n,m} = \frac{(e^{-S_0}KV_4)^{2n+n_i+n_f-2m}}{2^n(n_i-m)!(n_f-m)!n!}, \tag{5.21}$$

for fixed $n_i,\ n_f,\ m,\ n$. However, the simple sum over $(A_{n_in_f})_{n,m}$ is not quite equal to the transition amplitude $\langle n_f|e^{-S_{eff}}|n_i\rangle$, where $|n_i\rangle,\ \langle n_f|$ are the normalized initial and final states with n_i and n_f baby universes, respectively. S_{eff} is the Euclidean action. Actually, for $\langle n'|n''\rangle = \delta_{n'n''}$, we obtain the composition rule, *e.g.*

$$\langle 0|e^{-2S_{eff}}|0\rangle = \sum_n \langle 0|e^{-S_{eff}}|n\rangle\langle n|e^{-S_{eff}}|0\rangle. \tag{5.22}$$

On the other hand, the corresponding transition amplitude by $(A_{n_in_f})_{n,m}$ is

$$\sum_n (A_{0,0})_{n,0} \quad = \quad \exp\left(2(e^{-S_0}KV_4)^2\right). \tag{5.23}$$

If the composition rule is established, $A_{0,0} = \sum_{n'} A_{0,n'}A_{n',0}$[6] would be shown, but

$$\sum_{n'} A_{0,n'}A_{n',0}n'! = \sum_{n'}(\sum_{n''}(A_{0,n'})_{n'',0})(\sum_{n'''}(A_{n',0})_{n''',0})n'! \tag{5.24}$$

$$= \sum_{n'}\left(\frac{\exp\left((e^{-S_0}KV_4)^2/2\right)k^{n'}}{n'!}\right)^2 n'! \tag{5.25}$$

$$= \exp\left(2(e^{-S_0}KV_4)^2\right) = A_{0,0}. \tag{5.26}$$

Thus, we need the extra factor $n'!$. This factor arises from the $n'!$ ways of identifying the final state of $A_{0,n'}$ with the initial state of $A_{n',0}$, and thus

$$\langle n|e^{-S_{eff}}|0\rangle = \sqrt{n!}A_{n,0}. \tag{5.27}$$

Then, let us consider the general case of A_{n_i,n_f} using the schematic picture in Fig. 5.4. It is enough to consider the final state of one geometry labeled as M_1 and the initial state of another geometry labeled by M_2. To see the composition rule, the number of baby universes (n') is the same between the final state of M_1 and the initial state of M_2. In M_1 (M_2) geometry, there are m_1 (m_2) baby universes which do not connect with the parent universe in M_1 (M_2). In this situation, the inequivalent identifying way is $n'!/m_1!m_2!$. Therefore, one obtains the amplitude for fixed n_i, n_f, m, n denoted as $\langle n_f|e^{-S_{eff}}|n_i\rangle_{n,m}$,

$$\langle n_f|e^{-S_{eff}}|n_i\rangle_{n,m} = \frac{\sqrt{n_i n_f}}{m!}(A_{n_i,n_f})_{n,m} = \frac{\sqrt{n_i n_f}}{m!}\frac{(e^{-S_0}KV_4)^{2n+n_i+n_f-2m}}{2^n(n_i-m)!(n_f-m)!n!}. \tag{5.28}$$

By summing up for n, m, the total amplitude is

$$\langle n_f|e^{-S_{eff}}|n_i\rangle = e^{1/2e^{-2S_0}(KV_4)^2}\sum_{m=0}^{\min(n_i,n_f)}\frac{\sqrt{n_i!n_f!}}{m!(n_i-m)!(n_f-m)!}(e^{-S_0}KV_4)^{n_i+n_f-2m} \tag{5.29}$$

Now let us calculate the following quantity,

$$\langle n_2|e^{\beta(\hat{a}+\hat{a}^\dagger)}|n_1\rangle, \tag{5.30}$$

where β is some definite value, $\hat{a}^\dagger$, $\hat{a}$ are creation and annihilation operators satisfying the bose commutation relation, $[\hat{a},\hat{a}^\dagger]$ and $|n\rangle = \sqrt{1/n!}(\hat{a}^\dagger)^n|0\rangle$. Using the wick's theorem, the result is,

$$\langle n_2|e^{\beta(\hat{a}+\hat{a}^\dagger)}|n_1\rangle = e^{\beta^2/2}\sum_{r=0}^{\min(n_1,n_2)}\frac{\sqrt{n_1!}\sqrt{n_2!}\beta^{n_1+n_2-2r}}{(n_1-r)!(n_2-r)!r!}, \tag{5.31}$$

[6] Here, we omit some summation for simplicity, and thus $A_{a,b}$ denotes $\sum_{n,m}(A_{a,b})_{n,m}$ with the appropriate summation of n, m.

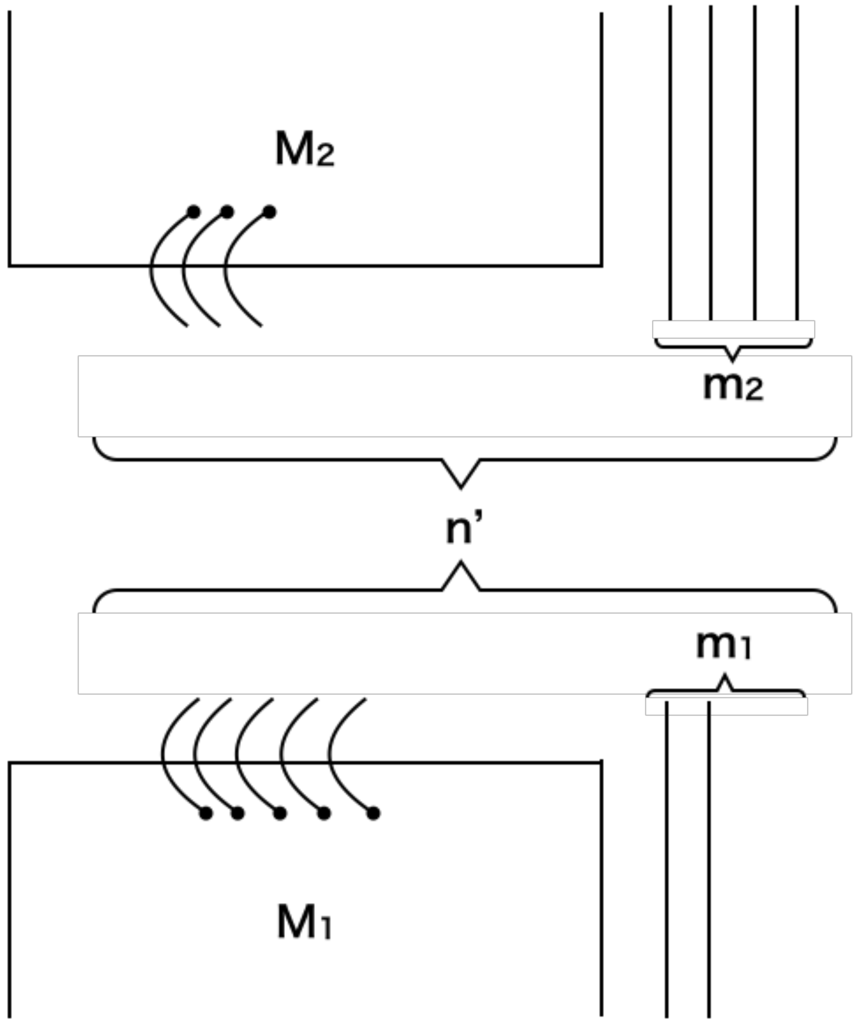

Figure 5.4: *The schematic picture of the wormhole decomposition rule.*

where we used Baker-Campbell-Hausdorff in the form,

$$\exp(\beta(\hat{a}+\hat{a}^\dagger)) = \exp(\beta\,\hat{a})\exp(\beta\,\hat{a}^\dagger)\exp(\beta^2/2). \tag{5.32}$$

Comparing Eq. (5.31) with Eq. (5.29), one obtains $\beta = e^{-S_0}KV_4$. Thus, we find the wormhole effect in the $-S_{eff}$,

$$e^{-S_0}KV_4(\hat{a}+\hat{a}^\dagger). \tag{5.33}$$

Here, the operator $A_{wm} \equiv (\hat{a}+\hat{a}^\dagger)$ is Hermitian, and thus let us take its eigenvalue by α_{wm}. If we write the transition amplitude in term of the eigenstate of $\hat{a}+\hat{a}^\dagger$, we find the following term in the action S_{eff},

$$-S_{eff} \ni \alpha_{wm} e^{-S_0} K V_4 = \int d^4x \sqrt{g_g} \alpha_{wm} K e^{-S_0}. \tag{5.34}$$

No local operator connects any A_{wm}-eigenstate to the other A_{wm}-eigenstate [6,89]. This implies that, for an observer who cannot detect baby universes like the human being, the α is just an (arbitrary) parameter like θ-angle in QCD. Therefore, the wormhole effect arises in the local Lagrangian as $\alpha_{wm} K e^{-S_0}$.

Although we assume the factor K is some constant, the K is actually constructed from the field, and thus we expect that the K can be described by an (infinite) series of local operators in the effective field theory which is valid as distance greater than some cutoff scale (*e.g.* the wormhole size $\simeq 2\pi R(0)$). There are some hints to obtain such local operators. The wormhole carries the charge, Q_W, defined in Eq. (5.13). This effect can be seen as the loss (production) of the charge from the parent universe. Thus, for the wormhole with the charge q, the local operator will reflect this symmetry breaking in the effective action,

$$\int d^4x \sqrt{g_g} \left[g_{wm} L^{4-q} \Phi^q + h.c. \right] = \int d^4x \sqrt{g_g} \left[g_{wm} L^{4-q} \left(\frac{1}{\sqrt{2}} f e^{i\theta} \right)^q + h.c. \right], \tag{5.35}$$

where g_{wm} is some dimensionless coupling, and L is some dimensionful parameter. Furthermore, if we limit the Plank scale to infinitely large, the wormhole effect must completely decouple from the effective field theory. Therefore, the dominant wormhole effect may be written by the local operator in the effective Lagrangian,

$$\frac{1}{M_{\rm pl}} |\Phi|^4 \Phi, \tag{5.36}$$

where the charge is assumed to be quantized.[7] In the following of this thesis, we expect the global symmetry breaking by such as the above operator.

To protect the global symmetry, there are (at least) two ways. One is to use the gauge symmetry as we discuss later. Another one is to use the topological suppression of wormhole effects (See Appendix for details).

5.2 Some comments about global and gauge symmetries

In this section, we discuss some comments about symmetries.

[7]In the following, we use the reduced Planck mass for the non-renormalized Planck mass suppressed terms. It should be noted that $M_{\rm PL} = \sqrt{8\pi} M_{\rm pl}$ is used in *e.g.* Ref. [87], where the explicit PQ breaking terms become more suppressed by inverse powers of $\sqrt{8\pi}$.

5.2.1 Non-compact gauge groups

Here, let us consider the relation between global symmetries and non-compact gauge symmetries [9]. To obtain the model with a non-compact gauge group, let us introduce fields with two relatively irrational charges, *e.g.* 1 and $\sqrt{2}$ under an abelian gauge symmetry. In any gauge invariant Lagrangian, these two fields never couple with each other, thus there is a global abelian symmetry under which the field of charge $\sqrt{2}$ rotates. If we accept that there are no global symmetries (no conserved global charges) in quantum gravity, such a model cannot couple with gravity. On the other word, in quantum gravity, all continuous gauge symmetries are compact.

5.2.2 Weak gravity conjecture

If we take the limit that gauge couplings equal to zero, we obtain the global symmetries, and thus the gauge couplings must be lower-bounded in quantum gravity. For a concrete example, let us consider the electrically charged black hole is given by classical Einstein-Maxwell theory, *i.e* $U(1)$ gauge theory coupling with gravity. There, the metric is given as the Reissner-Nordstrom solution,

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega_2^2 \tag{5.37}$$

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2 g_1^2}{r^2} \tag{5.38}$$

where $G = 1/(8\pi M_{\rm pl}^2)$, M is the mass of the black hole, Q is the integer charge, g_1 is the coupling constant of $U(1)$ gauge. The event horizon of the black hole is given by the condition $f(r) = 0$,

$$r_\pm = GM \pm \sqrt{(GM)^2 - GQ^2 g_1^2}. \tag{5.39}$$

An imaginary square root is considered to be unphysical, and thus

$$(GM)^2 \geq GQ^2 g_1^2 \to M \geq \frac{|Q| g_1 M_{\rm pl}}{\sqrt{8\pi}}. \tag{5.40}$$

We can make a black hole of mass $M \sim M_{\rm pl}$, and a gauge coupling is arbitrarily small. Then, a black hole can obtain any charge as long as the condition in Eq. (5.40) is satisfied. If there are no light particles with $U(1)$ charge, such black holes eventually evaporate into Planck scale remnants. Such remnants are believed to cause inconsistencies (See *e.g.* Ref. [90]). This problem is avoided if there are particles with $m|Q| \leq M$. Combining the condition in Eq. (5.40), we obtain the following bound,

$$g_1 \geq \frac{\sqrt{8\pi} m}{M_{\rm pl}}, \tag{5.41}$$

equivalently,

$$g_1^2 \geq m^2 G. \tag{5.42}$$

This bound implies that the gauge coupling constants are lower-bounded [91].

Chapter 6

Gauged Peccei-Quinn symmetry and its applications

In this chapter, we show our three papers [13, 39, 40] about the origin of the PQ symmetry.

Before going to our works, let us discuss the so-called quality problem of the PQ symmetry. As we discussed in Ch. 5, the PQ symmetry may be badly and explicitly broken by the quantum gravity effect. Actually, the following non-renormalizable term may be produced at the lowest order,[1]

$$\mathcal{L} = \lambda \frac{|\phi|^4 \phi}{M_{\rm pl}} + h.c., \tag{6.1}$$

where ϕ is the PQ breaking complex scalar field with some PQ charge, and the coefficient λ is a complex coupling. Apparently, this potential breaks the PQ symmetry explicitly, and thus the axion potential is modified by

$$\mathcal{L} \simeq |\lambda| \frac{v^5}{M_{\rm pl}} \cos(a/v + \delta) \tag{6.2}$$

where $|\lambda|$ denotes the absolute value of λ, δ is the phase of λ, v is the VEV of ϕ, and the a is the axion field, *i.e.* $\phi = \frac{1}{\sqrt{2}} v\, e^{ia/v}$. Because it is expected $\delta = \mathcal{O}(1)$, this term drastically changes the axion potential, and generally leads the $\mathcal{O}(1)$ shift from the CP-conserving potential minimum denoted as $\Delta\theta = \mathcal{O}(1)$ for a typical scale of $v \simeq 10^{9-12}\,\text{GeV}$. To be consistent with the measurement of the nEDM, the shift from the CP-conserving vacuum, $\Delta\theta$ must be smaller than the measured

[1] The other terms like $\frac{\phi^5}{M_{\rm pl}}$ or more higher non-renormalizable terms may arise in the Lagrangian. The generalization of the following discussion including such other terms is straightforward, and thus we only consider the term in (6.1) for simplicity.

upper-bound of the θ-parameter,

$$\Delta\theta \simeq |\lambda| \frac{v^5}{\Lambda_{\rm QCD}^4 M_{\rm pl}} \lesssim 10^{-10}. \tag{6.3}$$

For the typical value of the v and the QCD confinement scale $\Lambda_{\rm QCD}$, the coupling λ is required to be extremely tiny,

$$|\lambda| \lesssim 10^{-51} \left(\frac{10^{11}\,{\rm GeV}}{v^5}\right)\left(\frac{\Lambda_{\rm QCD}^4}{0.1\,{\rm GeV}}\right). \tag{6.4}$$

Therefore, the Strong CP problem still remains unsolved. This is so-called the quality problem of the PQ symmetry.

6.1 Gauged Peccei-Quinn symmetry

In this section[2], we show our mechanism to solve the quality problem, where the gauge symmetry, namely gauged PQ symmetry, can protect the PQ symmetry from the quantum gravity effect.

6.1.1 General prescription

Let us recall invisible axion models such as the KSVZ model [11, 12] or the DSFZ model [48, 49]. There, the postulated anomalous global PQ symmetry is spontaneously broken with which the axion field associates. The non-perturbative effects of QCD generate the axion potential through the axial anomalies.

Now let us bring two sectors of the invisible axion models. The two PQ symmetries in each sector, $U(1)_{PQ}$ and $U(1)_{PQ'}$, are explicitly broken by the QCD anomalies, and the corresponding Noether currents j^{μ}_{PQ} and $j^{\mu}_{PQ'}$ satisfy the anomalous ward identities,

$$\partial j_{PQ} = \frac{g_s^2}{32\pi^2} N_1 G\tilde{G}\ , \quad \partial j_{PQ'} = \frac{g_s^2}{32\pi^2} N_2 G\tilde{G}\ . \tag{6.5}$$

Here, G the gauge field strength of QCD, g_s the QCD coupling constant. The Lorentz indices and the color indices are suppressed. The coefficients N_1 and N_2 depend on each invisible axion model.

In the two anomalous symmetries, there is a linear combination which is free from the QCD anomaly. Hereafter, we consider that the anomaly free combination

[2]Most part of this section is based on Ref. [13] published in the physics letters B,

is a gauge symmetry, which we name the $U(1)_{gPQ}$ symmetry. Here, we assume that the $U(1)_{gPQ}$ is free from all anomalies [3].

In each sector, breaking operators of the global PQ symmetries are forbidden by the $U(1)_{gPQ}$ symmetry. Therefore, the $U(1)_{gPQ}$ symmetry provides protection of the PQ symmetries in each sector.

Let us further assume that there are no interactions between the two sectors except for the gauge interactions. In this limit, the PQ symmetries in each sector are broken only by the anomalies. It should be noted that the radiative corrections generate interactions between the two sectors. Those corrections, however, do not break the PQ symmetries in each sector since they are broken only by the $U(1)_{gPQ}$ and the QCD anomalies. Therefore, in this limit, the theory possesses an accidental $U(1)$ symmetry in addition to the $U(1)_{gPQ}$ gauge symmetry. In the following, we call this anomalous accidental symmetry, $U(1)_{aPQ}$. As it has been noted, the $U(1)_{aPQ}$ symmetry plays the role of the PQ symmetry for the PQ mechanism.

In reality, there are interaction terms between the two sectors. In particular, there are terms which are invariant under the $U(1)_{gPQ}$ gauge symmetry but break the $U(1)_{aPQ}$ symmetry. For example, let us consider operators $\mathcal{O}_1$ and $\mathcal{O}_2$ which consist of fields in each sector, respectively. When these two operators have non-vanishing and opposite $U(1)_{gPQ}$ charges, the interaction terms

$$\mathcal{L}_{\not{aPQ}} = \frac{1}{M_{\rm pl}^{d_{\mathcal{O}_1}+d_{\mathcal{O}_2}-4}}\mathcal{O}_1\mathcal{O}_2 + h.c. \ , \tag{6.6}$$

explicitly break the $U(1)_{aPQ}$ symmetry. Here, $d_{\mathcal{O}_{1,2}}$ denote the mass dimensions of the corresponding operators, and $M_{\rm pl}$ denotes the reduced Planck scale. Given the general discussion that all global symmetries are broken by quantum gravity effects, there is no principle to suppress these terms since it is consistent with gauge symmetries.

From the current experimental upper limit on the θ angle, $\theta \lesssim 10^{-10}$ [1], such explicit breaking terms of the $U(1)_{aPQ}$ symmetry are, however, acceptable as long as $d_{\mathcal{O}_1} + d_{\mathcal{O}_2} > 10$ when the PQ symmetries are spontaneously broken at 10^{10-12} GeV [10, 20, 21].

6.1.2 Decomposition of $U(1)_{gPQ}$ and $U(1)_{aPQ}$

Before moving to explicit examples, let us discuss how to decompose the $U(1)_{gPQ}$ and the $U(1)_{aPQ}$ symmetries. For that purpose, let us consider a simple example where

[3] The $[U(1)_{gPQ}]^3$ anomaly and the gravitational anomaly of $U(1)_{gPQ}$ can be cancelled by adding fermions which are singlet under the Standard Model gauge groups.

the invisible axion candidates in the two sectors correspond to the axial components of complex SM gauge singlet scalar fields ϕ and ϕ',

$$\phi = \frac{1}{\sqrt{2}} f_a \, e^{i\tilde{a}/f_a} \ , \quad \phi' = \frac{1}{\sqrt{2}} f_b \, e^{i\tilde{b}/f_b} \ . \tag{6.7}$$

Here, $f_{a,b}$ are the decay constants of each sector and we keep only the axial components, $\tilde{a}$ and $\tilde{b}$. The domains of them are given

$$\tilde{a}/f_a = [0, 2\pi) \ , \quad \tilde{b}/f_b = [0, 2\pi) \ , \tag{6.8}$$

respectively.

Let us assume that the $U(1)_{gPQ}$ gauge charges of the complex scalars are q and q', respectively. In this case, the axial components are shifted by,

$$\tilde{a}/f_a \to \tilde{a}/f_a + q\alpha \ , \quad \tilde{b}/f_b \to \tilde{b}/f_b + q'\alpha \ , \tag{6.9}$$

under the $U(1)_{gPQ}$ symmetry. Hereafter, we take the normalization of α such that q and q' are relatively prime integers without loosing generality.

From the covariant kinetic terms of ϕ and ϕ', we obtain

$$\begin{aligned} \mathcal{L} &= |D_\mu \phi|^2 + |D_\mu \phi'|^2 \\ &= \frac{1}{2}(\partial\tilde{a})^2 + \frac{1}{2}(\partial\tilde{b})^2 - gA_\mu(qf_a\partial^\mu\tilde{a} + q'f_b\partial^\mu\tilde{b}) \\ &\quad + \frac{g^2}{2}(q^2f_a^2 + q'^2f_b^2)A_\mu A^\mu \\ &= \frac{1}{2}(\partial a)^2 + \frac{1}{2}m_A^2\left(A_\mu - \frac{1}{m_A}\partial_\mu b\right)^2 \ . \end{aligned} \tag{6.10}$$

where, g is the gauge coupling constant of $U(1)_{gPQ}$. The mass of the $U(1)_{gPQ}$ gauge boson, A_μ, is given by,

$$m_A^2 = g^2(q^2 f_a^2 + q'^2 f_b^2) \ . \tag{6.11}$$

In the final expression, we redefine the axial fields by

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{q^2f_a^2 + q'^2f_b^2}} \begin{pmatrix} q'f_b & -qf_a \\ qf_a & q'f_b \end{pmatrix} \begin{pmatrix} \tilde{a} \\ \tilde{b} \end{pmatrix} \ . \tag{6.12}$$

The field b is the would-be Nambu-Goldstone boson, while the gauge invariant field a corresponds to the PQ axion.

To extract an gauge invariant $U(1)_{aPQ}$ global symmetry, let us remember that a gauge orbit of $U(1)_{gPQ}$ winds the domain of $(\tilde{a}, \tilde{b})$ more than once for $q \neq q'$ (see Fig. 6.1). Then, the domain of a is given by the interval of the gauge orbit in the

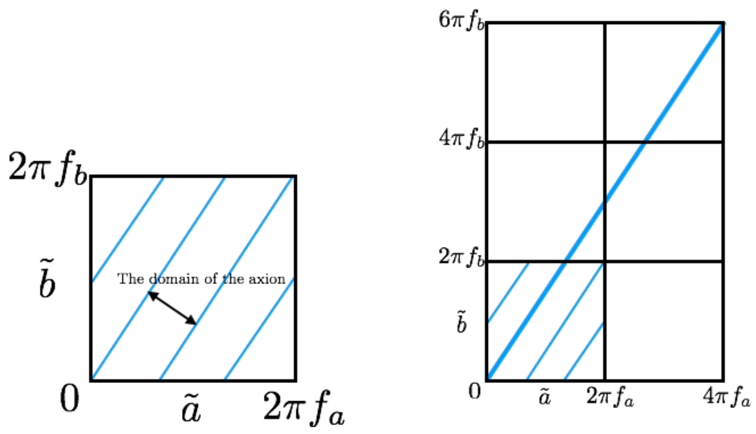

Figure 6.1: *(Left) A gauge orbit in the domain of $(\tilde{a}, \tilde{b})$ for $q = 2$, $q' = 3$. The domain of a is given by the interval between the orbits. (Right) The unwind gauge orbits.*

domain since the field points connected by a gauge orbit is physically equivalent. When we take that q and q' are relatively prime integers, we find the axion interval in the figure is given by,

$$a = \left[0, \frac{2\pi f_a f_b}{\sqrt{q^2 f_a^2 + q'^2 f_b^2}}\right) \ . \tag{6.13}$$

Thus, with a decay constant,

$$F_a = \frac{f_a f_b}{\sqrt{q^2 f_a^2 + q'^2 f_b^2}} \ , \tag{6.14}$$

the $U(1)_{aPQ}$ symmetry is realized by the shift of the axion,

$$\frac{a}{F_a} \to \frac{a}{F_a} + \delta_{PQ} \ , \tag{6.15}$$

with δ_{PQ} ranging from 0 to 2π [4].

The anomalous coupling of the axial components depends on models of the invisible axion models. In order for the $U(1)_{gPQ}$ symmetry is free from the anomaly, the anomalous coupling should appear in the form of

$$\mathcal{L}_{\rm QCD} = \frac{g_s^2}{32\pi^2} N \left(\frac{q'\tilde{a}}{f_a} - \frac{q\tilde{b}}{f_b}\right) G\tilde{G} \ , \tag{6.16}$$

$$= \frac{g_s^2}{32\pi^2} N \frac{a}{F_a} G\tilde{G} \ . \tag{6.17}$$

Here, N is a model dependent integer.

[4] We may extend our analysis where there is a kinetic mixing between $\tilde{a}$ and $\tilde{b}$, although the kinetic mixing does not change our discussion.

6.1.3 Examples

Barr-Seckel Model

As the simplest example, let us discuss a model based on two KSVZ axion models [11, 12]. This example corresponds to the model discussed in [10].

In each KSVZ sector, the PQ symmetry is spontaneously broken by the VEVs of complex scalars ϕ and ϕ' whose PQ charges are unity. In each sector, the scalars couple to extra vector-like quarks via

$$\mathcal{L} = y\phi Q\bar{Q} + h.c. \ , \tag{6.18}$$

and

$$\mathcal{L} = y'\phi' Q'\bar{Q}' + h.c. \ . \tag{6.19}$$

The PQ charges of the extra quarks are taken to be $Q(0)$ and $\bar{Q}(-1)$ in the first KSVZ sector and $Q'(0)$ and $\bar{Q}'(-1)$ in the second sector. We assume that there are N_f and N_f' flavors of the extra quarks in each sector.

Due to the QCD anomaly, the axion candidates in each sector have anomalous coupling,

$$\mathcal{L} = \frac{g_s^2}{32\pi^2}\left(\frac{N_f\,\tilde{a}}{f_a} + \frac{N_f'\,\tilde{b}}{f_b}\right) G\tilde{G} \ . \tag{6.20}$$

Here, we define the axial components of the KSVZ scalars as in Eq. (6.7). From this expression, we find that a linear combination of the two PQ symmetries with the charge assignments $\phi(q)$ and $\phi(q')$ is free from the anomaly for

$$q'/q = -N_f/N_f' \ . \tag{6.21}$$

As discussed in the previous section, we regard the anomaly free PQ symmetry as the $U(1)_{gPQ}$ gauge symmetry, where q and q' are normalized so that they are relatively prime integers.

Under the $U(1)_{gPQ}$ symmetry, no explicit PQ breaking operators appear in each sector. The interaction terms between the two KSVZ sectors, on the other hand, generically break $U(1)_{aPQ}$. In fact, the lowest dimensional operator which breaks the $U(1)_{aPQ}$ symmetry is given by,

$$\mathcal{L}_{\not{a}\not{P}\not{Q}} = \frac{1}{M_{\rm pl}^{|q|+|q'|-4}}\phi^{|q'|}\phi'^{|q|} + h.c. \tag{6.22}$$

As we have seen in the previous section, the explicit breaking of the PQ symmetry is acceptable when $|q| + |q'| > 10$. Once this condition is satisfied, the anomalous $U(1)_{aPQ}$ of an acceptable quality appears as a result of the $U(1)_{gPQ}$ gauge symmetry.

Let us comment here that q and q' in our normalization are given by,

$$q = N'_f / n_{gcd} \ , \quad q' = -N_f / n_{gcd} \ , \tag{6.23}$$

when N_f and N'_f has common divisors, $n_{gcd} > 1$. In this case, the anomalous coupling of the axion is given by,

$$\mathcal{L}_{\rm QCD} = \frac{g_s^2}{32\pi^2} n_{gcd} \frac{a}{F_a} G\tilde{G} \ , \tag{6.24}$$

which means $N = n_{gcd}$ in Eq. (13).

Composite Axion Model

As a second example, let us apply our prescription to the so-called composite axion model [50, 51] [5]. There, we consider an $SU(N_c)$ gauge theory with vector-like fermions of $SU(N_c)\times$QCD quantum numbers,

$$Q(N_c, 3), \quad \bar{Q}(\bar{N}_c, \bar{3}), \quad q(N_c, 1), \quad \bar{q}(\bar{N}_c, 1) \ . \tag{6.25}$$

This model possesses an axial $U(1)$ symmetry with the charge assignments,

$$Q(1), \quad \bar{Q}(1), \quad q(-3), \quad \bar{q}(-3) \ . \tag{6.26}$$

This symmetry is free from the anomaly of $SU(N_c)$ but broken by the QCD anomaly. We identify this symmetry with the anomalous PQ symmetry in the first sector. The anomalous PQ symmetry is spontaneously broken at the dynamical scale of $SU(N_c)$, where the axion appears as an composite field [6].

According to the general prescription, we further introduce another sector of the composite composite axion where N_c is replaced by N'_c. The PQ symmetry in this sector is also broken spontaneously at the dynamical scale of $SU(N'_c)$.

In this model, the anomalous couplings of the axion candidates are given by

$$\mathcal{L} = \frac{g_s^2}{32\pi^2} \left(\frac{N_c \, \tilde{a}}{f_a} + \frac{N'_c \, \tilde{b}}{f_b} \right) G^a \tilde{G}^a \ . \tag{6.27}$$

Here, the decay constants are taken so that the domains of $\tilde{a}/F_a = [0, 2\pi)$ and $\tilde{b}/f_b = [0, 2\pi)$ coincide with the domains of the axial components of the quark

[5] For other attempts to obtain a high-quality PQ symmetry in the composite axion model, see e.g. [27, 92].

[6] There are 15 light pseudo-goldstone modes associated with the chiral symmetry breaking at Λ_N, which are color charged except for the axion candidate. The colored pseudo-Nambu-Goldstone bosons obtain masses of $\mathcal{O}(\alpha_s \Lambda_N)$ where $\alpha_s = g_s^2/4\pi$. See e.g. [33].

bilinears, $Q\bar{Q}$ and $Q'\bar{Q}'$, respectively. From Eq. (6.27), we find an anomaly free combination is given by taking

$$q'/q = -N_c/N_c', \tag{6.28}$$

with which we identify the $U(1)_{gPQ}$ gauge symmetry in our general prescription. The anomalous $U(1)_{aPQ}$ symmetry is, on the other hand, given by Eq. (6.15). The axion domain wall number corresponds to the greatest common devisor of N_c and N_c'.

Under the $U(1)_{gPQ}$ symmetry, there are explicit breaking terms of the $U(1)_{aPQ}$ symmetry,

$$\mathcal{L} \sim \frac{1}{M_{\rm pl}^{3|q|+3|q'|-4}} (Q\bar{Q})^{|q'|} (Q'\bar{Q}')^{|q|} \ . \tag{6.29}$$

These operators does not spoil the PQ solution for $3(|q| + |q'|) > 10$. Thus, for example, a model with $N_c = 2$ and $N_c' = 5$ provides the origin of the anomalous PQ symmetry for the successful PQ mechanism.

For $q = 3k$ or $q' = 3k' (k, k' \in \mathbb{Z}\backslash\{0\})$, there are additional lower dimensional operators which break the $U(1)_{aPQ}$ symmetry,

$$\mathcal{L} \sim \frac{1}{M_{\rm pl}^{|q|+3|q'|-4}} (Q\bar{Q})^{|q'|} (q'\bar{q}')^{|q|/3} \ , \tag{6.30}$$

or

$$\mathcal{L} \sim \frac{1}{M_{\rm pl}^{3|q|+|q'|-4}} (q\bar{q})^{|q'|/3} (Q'\bar{Q}')^{|q|} \ , \tag{6.31}$$

Those operators are harmless for $|q| + 3|q'| > 10$ or $3|q| + |q'| > 10$, which can be satisfied for $N_c = 3$ and $N_c' = 4$ for example.

6.2 Domain wall problem in gauged PQ mechanism

Before going to the applications of the mechanism, let us discuss more details about the domain wall problem in our mechanism. For convenience, we use $N_{\rm dom}$ to denote the domain wall number in this section.

As discussed in Sec. 4.1, the model with $N_{\rm dom} > 1$ suffers from the domain wall problem if the global PQ symmetry is broken after inflation since the average of the axion field value in each Hubble volume is randomly distributed. To avoid the domain wall problem, spontaneous breaking of the global PQ symmetry is required to take place before inflation, which in turn requires a rather small inflation scale to avoid the axion isocurvature problem (see, e.g. Refs. [93, 94]).

In our mechanism, if N_f and N'_f are relatively prime integers in Eq. (6.23), the domain wall number is given as $N_{\rm dom} = 1$, and thus it seems to be no discrete symmetry which is broken by the VEV of the axion field and no domain wall problem as discussed in Sec. 4.1. Still, however, there can be domain wall problems when the global PQ breaking takes place after inflation. To see this problem, let us remember that there can be various types of cosmic string configurations formed at spontaneous symmetry breaking of the PQ symmetries. For example, in the Barr-Seckel model, when both the gauged and the global PQ symmetries are broken spontaneously after inflation, there can be cosmic string configurations in which either the phase of ϕ or ϕ' takes $0 - 2\pi$ around configurations. It should be noted that those configurations are the global strings and not the local string. Thus, the string tensions diverge in the limit of infinite volume which is cut off by the Hubble volume. The local string, on the other hand, corresponds to the configurations in which the phases of ϕ and ϕ' wind N'_f times and N_f times simultaneously. With the $U(1)_{gPQ}$ gauge field winding simultaneously, the tension of the local string is finite even in the limit of infinite volume for the local string.

A striking difference between the global strings and the local strings is how the axion field winds around the strings. Around the local strings, only the would-be-Goldstone field winds, while the axion winds around the global strings. Thus, when the axion potential is generated at around the QCD scale, the axion domain walls are formed only around the global strings, while they are not formed around the local strings. Once the domain walls are formed around the global strings, they immediately dominate over the energy density of the universe, which causes the domain wall problem. Therefore, for the domain wall problems not to occur, the local strings should be formed preferentially at the phase transition.

The string tensions of the global strings and the local strings, however, depend on model parameters. Thus, there is no guarantee that only the local strings preferentially survive in the course of the cosmic evolution. As an example, let us consider a case with $\langle\phi\rangle \gg \langle\phi'\rangle$. In this case, the cosmic strings are formed at the first phase transition, i.e. $\langle\phi\rangle \neq 0$ with $\langle\phi'\rangle = 0$. At this stage, strings around which the phase of ϕ winds just once are expected to be dominantly formed. They are *local* because we can take an appropriate charge normalization for the $U(1)_{gPQ}$. As the temperature of the universe decreases, the string networks follow the scaling solution where the number of the cosmic strings in each Hubble volume becomes constant (see, e.g., Ref. [95]).

Once the temperature becomes lower than the scale of the second phase transition, i.e., $\langle\phi'\rangle \neq 0$, the *local* strings formed at the first phase transition become no more the local strings.[7] Besides, formations of the global strings of ϕ' are also expected at the second phase transition in which the phase of ϕ' winds just once. To form a genuine local string, it is required to bundle N'_f ex-local strings (formed by ϕ) and N_f global strings (formed by ϕ') into a single string. However, the confluence of global strings into a local string is quite unlikely as there is no correlation between the nature of the cosmic strings in the adjacent Hubble volumes. Therefore, when $\langle\phi\rangle \gg \langle\phi'\rangle$, the domain wall problem is expected to be not avoidable even if $N_{\rm dom} = 1$.[8]

In summary, let us list up possibilities to avoid the domain wall problem. The first possibility is a trivial one where both the gauged and the global PQ symmetries are broken before inflation. This solution does not require $N_{\rm dom} = 1$. In this possibility, there is a constraint on the Hubble scale during inflation from the axion isocurvature problem.

The next possibility is only applicable for $N_{\rm dom} = 1$ with $N'_f = 1$ and $N_f > 1$. Here, it is assumed that the first phase transition (i.e. $\langle\phi\rangle \neq 0$) takes place before inflation while the second phase transition (i.e. $\langle\phi'\rangle \neq 0$) occurs after inflation. In this second possibility, the *local* strings formed at the first phase transition are inflated away. The global strings formed at the second phase transition, on the other hand, do not cause the domain wall problem as each of the global string is attached to only one domain wall [72, 73]. Notice that the global PQ symmetry is broken after inflation end in this scenario solving the domain wall problem and the quality problem of the global PQ symmetry.

In addition to these two possibilities, there can be another possibility which is applicable for $N_{\rm dom} = 1$ with $\langle\phi\rangle \sim \langle\phi'\rangle$. In this case, there can be a possibility where the local strings are preferentially formed at the phase transition. Besides, the axion domain wall attached to the global strings may have very short lifetime for $N_{\rm dom} = 1$ even if they are formed. To confirm this possibility, detailed numerical simulations are required, which goes beyond the scope of this paper.

It should be noted that the second possibility (and the third possibility if nu-

[7] The configuration of the gauge field formed at the first phase transition does not coincide with the one required for the local string with $\langle\phi\rangle' \neq 0$.

[8] As there is no corresponding discrete symmetry, the domain wall is not stable completely. For $\langle\phi\rangle \gg \langle\phi'\rangle$, however, the decay rate (i.e., the puncture rate and/or the rate of the breaking off) is highly suppressed.

merically confirmed) is one of the advantages of the gauged PQ mechanism over the models in which the global PQ symmetry results from an exact discrete symmetry, such as $\mathbb{Z}_N$. In such models, the axion potential also respects the $\mathbb{Z}_N$ symmetry, and hence, the domain wall problem is not avoidable when the global PQ symmetry is spontaneously broken after inflation. In the gauged PQ models, on the other hand, it is possible that the global PQ symmetry is broken after inflation without causing the domain wall problem nor the axion isocurvature problem.

6.3 Application to SUSY model

In this section[9], let us discuss a further application of the gauged PQ mechanism to the SUSY breaking model, where the PQ symmetry is simultaneously broken with the dynamical SUSY breaking.

6.3.1 Gauged PQ mechanism in SUSY

First, let us briefly summarize a supersymmetric version of the gauged PQ mechanism [13].

Would-be goldstone and axion superfields

As a simple example, let us consider two global PQ symmetries $U(1)_{PQ_1}$ and $U(1)_{PQ_2}$, which are broken by the VEVs of Φ_1, $\bar{\Phi}_1$ and Φ_2, $\bar{\Phi}_2$, respectively. For instance, such vacuum is achieved by the superpotential,

$$W = \lambda_1 X_1(2\Phi_1\bar{\Phi}_1 - \Lambda_1^2) + \lambda_2 X_2(2\Phi_2\bar{\Phi}_2 - \Lambda_2^2) \ . \tag{6.32}$$

Here, Φ_i and $\bar{\Phi}_i$ $(i = 1, 2)$ have charges ± 1 under $U(1)_{PQ_i}$ and have vanishing charges under $U(1)_{PQ_j}$ $(j \neq i)$, respectively. The superfields, $X_{1,2}$, have vanishing charges under both the PQ symmetries. The parameters $\lambda_{1,2}$ are coupling constants, and $\Lambda_{1,2}$ are dimensionful parameters. After the spontaneous breaking of the PQ symmetries, Φ's lead to the Goldstone superfields $A_{1,2}$,[10]

$$\Phi_1 = \frac{1}{\sqrt{2}}\Lambda_1 e^{A_1/\Lambda_1} \ , \quad \bar{\Phi}_1 = \frac{1}{\sqrt{2}}\Lambda_1 e^{-A_1/\Lambda_1} \ , \tag{6.33}$$

$$\Phi_2 = \frac{1}{\sqrt{2}}\Lambda_2 e^{A_2/\Lambda_2} \ , \quad \bar{\Phi}_2 = \frac{1}{\sqrt{2}}\Lambda_2 e^{-A_2/\Lambda_2} \ . \tag{6.34}$$

[9] Most part of this section is based on Ref. [39]

[10] Here, we set the origins of A_i at which $\Phi_i = \bar{\Phi}_i$, while $\langle \Phi_i \rangle \neq \langle \bar{\Phi}_i \rangle$ for $\langle A_i \rangle \neq 0$.

By using the Goldstone superfields, the PQ symmetries are realized by,

$$A_1/\Lambda_1 \rightarrow A_1/\Lambda_1 + i\alpha_1 , \quad (\alpha_1 = 0 - 2\pi) , \tag{6.35}$$

$$A_2/\Lambda_2 \rightarrow A_2/\Lambda_2 + i\alpha_2 , \quad (\alpha_2 = 0 - 2\pi) . \tag{6.36}$$

The PQ symmetries are communicated to the supersymmetric Standard Model (SSM) sector by introducing extra quark multiplets as in the KSVZ axion model [11, 12]. Throughout this paper, we assume that the extra multiplets form $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations of the $SU(5)$ gauge group of the Grand Unified Theory (GUT). Let us suppose that $\Phi_{1,2}$ couple to N_1 and N_2 flavors of the KSVZ extra multiplets $\mathbf{5}_i$, $\bar{\mathbf{5}}_i$ $(i = 1, 2)$, respectively,

$$W = \Phi_1 \mathbf{5}_1 \bar{\mathbf{5}}_1 + \bar{\Phi}_2 \mathbf{5}_2 \bar{\mathbf{5}}_2 . \tag{6.37}$$

Through the above coupling, both the global PQ symmetries are broken by the Standard Model anomaly. The anomalous breaking of the global PQ symmetries lead to the anomalous coupling of the Goldstone superfields,

$$W_{\rm anom} = \frac{1}{8\pi^2} \left(N_1 \frac{A_1}{\Lambda_1} - N_2 \frac{A_2}{\Lambda_2} \right) \sum_l W_l^\alpha W_{l\alpha} , \tag{6.38}$$

where, W_l^α $(l = 1, 2, 3)$ denote the field strength superfields of the Standard Model gauge interactions.[11] We normalize the gauge field strength so that the gauge kinetic functions are given by

$$\mathcal{L} = \frac{1}{2i} \left[\sum \tau_l W_l^\alpha W_{l\alpha} \right]_F + h.c. , \tag{6.39}$$

with

$$\tau_l = \frac{i}{g_l^2} + \frac{\theta_l}{8\pi^2} , \tag{6.40}$$

where g_l and θ_l are the gauge coupling constants and the vacuum angles of the corresponding gauge interactions.

An important observation here is that there is a linear combination of the PQ symmetries for which the Standard Model anomalies are absent. In fact, a $U(1)$ symmetry under which $\Phi_{1,2}$ have charges q_1 and q_2 is free from the Standard Model anomaly for

$$q_1 N_1 - q_2 N_2 = 0 . \tag{6.41}$$

[11] Here, the gauge indices of $SU(3)_c$ and $SU(2)_L$ are suppressed, and the GUT normalization is used for $U(1)_Y$.

In the gauged PQ mechanism, we identify the anomaly-free combination to be a gauge symmetry $U(1)_{gPQ}$. The gravitational anomaly and the self-anomaly of the $U(1)_{gPQ}$ are canceled by adding $U(1)_{gPQ}$ charged singlet fields. Hereafter, we take q_1 and q_2 are both positive and relatively prime numbers without loss of generality.

In the gauged PQ mechanism, one of the linear combinations of $A_{1,2}$ is the would-be Goldstone supermultiplet, and the other combination corresponds to the physical axion superfield. To see how the physical axion is extracted, let us consider the Kähler potential of Φ's,

$$K = \Phi_1^\dagger e^{-2q_1 gV}\Phi_1 + \bar{\Phi}_1^\dagger e^{2q_1 gV}\bar{\Phi}_1 + \Phi_2^\dagger e^{-2q_2 gV}\Phi_2 + \bar{\Phi}_2^\dagger e^{2q_2 gV}\bar{\Phi}_2 \ , \tag{6.42}$$

where V and g are the $U(1)_{gPQ}$ gauge supermultiplet and the gauge coupling constant, respectively. Under the $U(1)_{gPQ}$ gauge transformation, the gauge field is shifted by,

$$2gV \to 2gV' = 2gV - i\Theta + i\Theta^\dagger \ , \tag{6.43}$$

with Θ being the gauge parameter superfield. By substituting Eqs.(6.33) and (6.34), the Kähler potential is reduced to

$$K = \Lambda_1^2 \cosh\left(2q_1 gV - \frac{A_1^\dagger + A_1}{\Lambda_1}\right) + \Lambda_2^2 \cosh\left(2q_2 gV - \frac{A_2^\dagger + A_2}{\Lambda_2}\right) . \tag{6.44}$$

The physical axion and the would-be Goldstone superfields A and G are obtained by rearranging $A_{1,2}$ by

$$\begin{pmatrix} A^{(\dagger)} \\ G^{(\dagger)} \end{pmatrix} = \frac{1}{\sqrt{q_1^2\Lambda_1^2 + q_2^2\Lambda_2^2}} \begin{pmatrix} q_2\Lambda_2 & -q_1\Lambda_1 \\ q_1\Lambda_1 & q_2\Lambda_2 \end{pmatrix} \begin{pmatrix} A_1^{(\dagger)} \\ A_2^{(\dagger)} \end{pmatrix} . \tag{6.45}$$

By using A and G, the Kähler potential is rewritten by,

$$K = \Lambda_1^2 \cosh\left(2q_1 \tilde{V} - \frac{2q_2}{m_V}\frac{\Lambda_2}{\Lambda_1}(A^\dagger + A)\right) + \Lambda_2^2 \cosh\left(2q_2 \tilde{V} + \frac{2q_1}{m_V}\frac{\Lambda_1}{\Lambda_2}(A^\dagger + A)\right), \tag{6.46}$$

where

$$\tilde{V} = V - \frac{g}{m_V}(G^\dagger + G) \ , \tag{6.47}$$

$$m_V = 2g\sqrt{q_1^2\Lambda_1^2 + q_2\Lambda_2^2} \ . \tag{6.48}$$

The final expression of Eq. (6.46) shows there is no bi-linear term which mixes A and $\tilde{V}$. Therefore, we find that A corresponds to the physical axion superfield, while G is the would-be Goldstone superfield which is absorbed by V in the unitarity gauge.

It should be noted that the physical axion A is invariant under the gauge $U(1)_{gPQ}$ transformation.

For a later purpose, let us discuss the domain and the effective decay constant of the axion. The domains of the imaginary parts of $A_{1,2}$ (corresponding to the phases of $\Phi_{1,2}$) are given by

$$\frac{\mathrm{Im}[A_i]}{\Lambda_i} = \frac{a_i}{f_i} = [0, 2\pi) \ , \quad (i = 1, 2) \ , \tag{6.49}$$

where $a_i = \sqrt{2}\,\mathrm{Im}[A_i]$ and $f_i = \sqrt{2}\Lambda_i$. When q_1 and q_2 are relatively prime integers, the gauge invariant axion interval is given by [13],

$$a = \sqrt{2}\,\mathrm{Im}[A] = \left[0, \ \frac{2\pi f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}}\right) . \tag{6.50}$$

Accordingly, the global $U(1)_{PQ}$ symmetry is realized by

$$\frac{a}{F_a} \to \frac{a'}{F_a} = \frac{a}{F_a} + \delta_{PQ} \ , \quad (\delta_{PQ} = 0 - 2\pi) \ , \tag{6.51}$$

where F_a is defined as an effective decay constant,

$$F_a = \frac{f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} = \frac{\sqrt{2}\Lambda_1\Lambda_2}{\sqrt{q_1^2\Lambda_1^2 + q_2^2\Lambda_2^2}} \ . \tag{6.52}$$

Accidental global PQ symmetry

In the present model, the lowest dimensional $U(1)_{gPQ}$ invariant operators which break the global PQ symmetries are given by,

$$W \sim \frac{1}{M_{\mathrm{pl}}^{q_1+q_2-3}} \left(\Phi_1^{q_2}\bar{\Phi}_2^{q_1} + \bar{\Phi}_1^{q_2}\Phi_2^{q_1}\right) \ , \tag{6.53}$$

where $M_{PL} = 2.4 \times 10^{18}$ GeV denotes the reduced Planck scale. When supersymmetry is spontaneously broken in a separate sector, the above superpotential contributes to the axion potential through the supergravity effects,[12]

$$V \sim \frac{1}{2} m_a^2 F_a^2 \left(\frac{a}{F_a}\right)^2 + \frac{m_{3/2}}{M_{\mathrm{pl}}^{q_1+q_2-3}} \Lambda_1^{q_2}\Lambda_2^{q_1} \frac{a}{F_a} + h.c. + \cdots \ , \tag{6.55}$$

[12] In supergravity, a superpotential term W_i directly appears in the scalar potential as

$$V = (n_i - 3) m_{3/2} \times W_i + h.c. \ , \tag{6.54}$$

with n_i being the mass dimension of W_i.

where $m_{3/2}$ denotes the gravitino mass. In the final expression, we use $\Phi_1^{q_2}\bar{\Phi}_2^{q_1} = \Lambda^{q_2+q+1}/2^{(q_2+q_1)/2}e^{ia/F_a}$, and the intrinsic θ angle of QCD is absorbed by the definition of the axion field. The first term represents the axion mass term due to the QCD effects [44],

$$m_a^2 \simeq \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{F_a^2}\ , \tag{6.56}$$

where $m_{u,d}$ are the u- and d-quark masses, m_π the pion mass, and $f_\pi \simeq 93\,\mathrm{MeV}$ the pion decay constant.

As a result, the effective θ angle at the vacuum of the axion is given by,

$$\begin{aligned}\theta_{\mathrm{eff}} &\simeq \frac{1}{m_a^2F_a^2}\frac{m_{3/2}}{M_{\mathrm{pl}}^{q_1+q_2-3}}\Lambda_1^{q_2}\Lambda_2^{q_1} \\ &\sim 10^{66-6.4(q_1+q_2)} \qquad (6.57)\\ &\times \left(\frac{m_{3/2}}{10^6\,\mathrm{GeV}}\right)^2\left(\frac{0.08\,\mathrm{GeV}}{\sqrt{m_aF_a}}\right)^4\left(\frac{\Lambda_1}{10^{12}\,\mathrm{GeV}}\right)^{q_2}\left(\frac{\Lambda_2}{10^{12}\,\mathrm{GeV}}\right)^{q_1}. \qquad (6.58)\end{aligned}$$

Thus, for $q_1+q_2 \gtrsim 12$, $m_{3/2} = \mathcal{O}(10^6)\,\mathrm{GeV}$, and $\Lambda_{1,2} = \mathcal{O}(10^{12})\,\mathrm{GeV}$, the explicit breaking terms of the global PQ symmetries are small enough to be consistent with the measurement of the neutron EDM, i.e. $\theta_{\mathrm{eff}} < 10^{-11}$ [1]. In this way, a *high quality* global PQ symmetry appears as an accidental symmetry in the gauged PQ mechanism.

6.3.2 Dynamical supersymmetry/PQ symmetry breaking

In this section, we discuss a model of a simultaneous breaking of supersymmetry and the *global* PQ symmetry. As we are interested in solutions to the strong CP-problem without severe fine-tuning, it is natural to seek models in which the PQ breaking scale is generate by dynamical transmutation. Thus, in the following, we construct a model of a simultaneous supersymmetry/PQ symmetry breaking sector based on a strong dynamics. For now, we do not consider the gauged PQ mechanism which will be implemented in the next section.

Simultaneous breaking of supersymmetry and global PQ symmetry

As the simplest example of the dynamical supersymmetry breaking models, we consider a model of supersymmetry breaking based on $SU(2)$ gauge dynamics (the IYIT model) [96, 97]. The advantage of this model is that the nature of dynamical supersymmetry breaking is calculable by using effective composite states.

The model consists of four $SU(2)$ doublets, Q_i ($i = 1-4$), and six singlets, $Z_{ij} = -Z_{ji}$ ($i, j = 1-4$). Those superfields couple via the superpotential

$$W_{IYIT} = \sum \lambda_{ij}^{kl} Z^{ij} Q_k Q_l \tag{6.59}$$

where λ_{ij}^{kl} denote coupling constants with $\lambda_{ij}^{kl} = -\lambda_{ji}^{kl} = -\lambda_{ij}^{lk}$. The maximal non-abelian global symmetry of the IYIT model is $SU(4)$ flavor symmetry, $SU(4)_f$, which is broken by λ_{ij}^{kl}.

The superpotential Eq. (6.59) respects a global $U(1)_A$ symmetry with charges, Z's$(+2)$, Q's(-1), and a continuous R-symmetry, $U(1)_R$, with Z's$(+2)$, Q's(0) (Tab. 6.1). The former is broken down to the discrete subgroup, $\mathbb{Z}_4$, by the $SU(2)$ anomaly, while the latter is free from the $SU(2)$ anomaly. As we seek a solution to the strong CP problem not relying on global symmetries, we consider that the $\mathbb{Z}_4$ and $U(1)_R$ symmetries are accidental symmetries and are broken by Planck suppressed operators.

It should be noted, however, that a discrete subgroup of $U(1)_R$, $\mathbb{Z}_{NR}$ ($N > 2$), plays crucial roles in constructing the SSM. Without $\mathbb{Z}_{NR}$ ($N > 2$) symmetry, the VEV of the superpotential is expected to be of the order of the Planck scale. Such a large VEV of the superpotential, in turn, does not allow a supersymmetry breaking scale lower than the Planck scale due to the condition for the flat present universe. In addition, it is also known that R-symmetry (or at least an approximate R-symmetry) is relevant for supersymmetry breaking vacua to be stable [98, 99]. Given its importance, we assume that the $\mathbb{Z}_{NR}$ ($N > 2$) symmetry is an exact discrete gauge symmetry [100–106].[13] In this paper, we take the simplest possibility, $\mathbb{Z}_{4R}$, assuming a presence of an extra multiplet of the $\mathbf{5}$, $\bar{\mathbf{5}}$ representations of the $SU(5)$ GUT. The $\mathbb{Z}_{4R}$ symmetry is free from the Standard Model anomaly when the R-charges of the bilinear term of the Higgs doublets and that of the extra multiplets are vanishing [107–110].[14]

In this model, we identify the global PQ symmetry with a $U(1)$ subgroup of $SU(4)_f$ (Tab. 6.1). As it is a subgroup of $SU(4)_f$, the PQ symmetry is free from the $SU(2)$ anomaly. Under the global $U(1)_{PQ}$ symmetry, the superpotential is reduced to

$$W_{IYIT} = \lambda_{12}^{12} Z^{12} Q_1 Q_2 + \lambda_{34}^{34} Z^{34} Q_3 Q_4 + \sum \tilde{\lambda}_{ij}^{kl} Z^{ij} Q_k Q_l \tag{6.60}$$

[13] In Ref. [25], it is proposed to achieve the global PQ symmetry as an accidental symmetry protected by the exact discrete R-symmetry without relying on the gauged PQ mechanism.

[14] For GUT models which are consistent with the $\mathbb{Z}_{4R}$ symmetry, see, e.g., [111, 112].

Table 6.1: *Charge assignment of the simultaneous symmetry breaking model. The chiral superfields, Q's, and Z's, are the $SU(2)$ doublets and singlets of the IYIT model, respectively. The $U(1)_{PQ}$ symmetry is a subgroup of the maximum flavor symmetry of the IYIT model. The KSVZ extra multiplets consist of the* $\mathbf{5}$ *and* $\bar{\mathbf{5}}$ *representations of the $SU(5)$ GUT group. The U(1)$_R$ and U(1)$_A$ symmetries are accidental symmetries of the IYIT model. A discrete subgroup of U(1)$_R$, i.e. $\mathbb{Z}_{4R}$ is assumed to be an exact symmetry. The R-charges of the KSVZ extra multiplets are taken to be $r_5 + r_{\bar{5}} = 2$.*

	$Q_{1,2}$	$Q_{3,4}$	$Z_{12}(Z_-)$	$Z_{34}(Z_+)$	$Z_{13,14,23,24}(Z_0^a\ (a=1-4))$	$\mathbf{5}$	$\bar{\mathbf{5}}$
$SU(2)$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_{PQ}$	$+1$	-1	-2	$+2$	0	-2	0
$U(1)_R$	0	0	$+2$	$+2$	$+2$	r_5	$r_{\bar{5}}$
$U(1)_A$	-1	-1	$+2$	$+2$	$+2$	$+2$	$+0$

where λ's are dimensionless coupling constants with $\tilde{\lambda}_{ij}^{kl} = 0$ for $ij = 12, 34$ or $kl = 12, 34$. Hereafter, we take $\lambda_{12}^{12} = \lambda_{34}^{34} = \lambda$ for simplicity,although it is straightforward to extend the following analysis for $\lambda_{12}^{12} \neq \lambda_{34}^{34}$. As we will see shortly, the PQ symmetry is spontaneously broken by the VEV of Q_1Q_2 and Q_3Q_4.

By assuming the KSVZ axion model, the PQ symmetry is communicated to the SSM sector through couplings to the KSVZ extra multiplets in $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations of the $SU(5)$ GUT,[15]

$$W = \frac{1}{M_{\rm pl}} Q_1 Q_2 \mathbf{5}\,\bar{\mathbf{5}}\ . \tag{6.61}$$

The PQ charges of the KSVZ extra multiplets are given in Tab. 6.1. Hereafter, we assume that there are N_f flavors of the KSVZ extra multiplets. Once $\langle Q_3Q_4\rangle$ spontaneously breaks the PQ symmetry, the axion couples to the SSM sector via Eq. (6.123) and the extra multiplets obtain masses of $\mathcal{O}(\langle Q_3Q_4\rangle/M_{PL})$.

Now, let us discuss how supersymmetry and the PQ symmetry are broken spontaneously. Below the dynamical scale of $SU(2)$ dynamics, Λ, the IYIT model is well described by using the composite fields, $M_{ij} \sim Q_iQ_j$, with an effective superpotential,

$$W_{\rm eff} \sim \lambda\Lambda Z_- M_+ + \lambda\Lambda Z_+ M_- + \tilde{\lambda}_{ab}\Lambda Z_0^a M_0^b + \mathcal{X}(2M_+M_- + M_0^a M_0^a - \Lambda^2)\ . \tag{6.62}$$

Here, $M_+ \sim Q_1Q_2/\Lambda$ and $M_- \sim Q_3Q_4/\Lambda$ denote the PQ charged mesons, while M_0^a ($a = 1-4$) are the PQ neutral mesons. The coupling constants $\tilde{\lambda}$ and the singlets

[15] The KSVZ extra multiplets should be distinguished the extra multiplets required to cancel the Standard Model anomaly of the $\mathbb{Z}_{4R}$ symmetry.

Z_0's are also rearranged accordingly. In the effective superpotential, the quantum modified constraint $2M_+M_- + M_0M_0 - \Lambda^2 = 0$ [113] is implemented by a Lagrange multiplier field $\mathcal{X}$.

By assuming that λ's are perturbative, and $\lambda_\pm(=\lambda)$ are smaller than $\tilde{\lambda}$'s, the VEVs of $M_\pm$ are given by

$$\langle M_+\rangle = \frac{1}{\sqrt{2}}\Lambda, \quad \langle M_-\rangle = \frac{1}{\sqrt{2}}\Lambda \ . \tag{6.63}$$

Other fields do not obtain VEVs of $\mathcal{O}(\Lambda)$.[16] At this vacuum, the PQ symmetry is spontaneously broken by $\langle M_\pm\rangle$ while supersymmetry is broken by the VEVs of the F-components of $Z_\pm$, i.e.,

$$F_{Z_\pm} \sim \frac{1}{\sqrt{2}}\lambda\Lambda^2 \ , \tag{6.64}$$

simultaneously.

Here, let us comment that the $\mathbb{Z}_{4R}$ is not enough to restrict the superpotential in the form of Eq. (6.59). In fact, there can be superpotential terms such as Z_0^3 or $Z_0Z_+Z_-$ without the $U(1)_A$ (or $\mathbb{Z}_4$) symmetry. As those terms make the supersymmetry breaking vacuum in Eqs. (6.63) and (6.64) metastable, the coefficients of those terms should be rather suppressed to make the vacuum long lived. Such suppression can be achieved, for example, by assuming that a subgroup of $\mathbb{Z}_4$ and $U(1)_{PQ}$ is an exact symmetry where Z_0's are charged but $Z_\pm$ are neutral.[17] It is also possible to suppress the unwanted terms by extending the SU(2) dynamics of the IYIT sector into a conformal window by adding extra doublets [114–116].

Axion supermultiplet

The degeneracy due to the PQ symmetry breaking is parametrized by the axion superfield A,

$$M_+ = \frac{1}{\sqrt{2}}\Lambda e^{A/\Lambda} \ , \quad M_- = \frac{1}{\sqrt{2}}\Lambda e^{-A/\Lambda} \ , \tag{6.65}$$

with which the PQ symmetry is realized by

$$A/\Lambda \to A/\Lambda + i\alpha \ , \quad (\alpha = 0 - 2\pi) \ . \tag{6.66}$$

Here, we reduce the domain of the $U(1)_{PQ}$ rotation parameter from $\alpha = 0 - 4\pi$ to $\alpha = 0 - 2\pi$, since all the $SU(2)$ gauge invariant fields have the PQ charge of ± 2 (see

[16] The scalar components of $Z_\pm$ and $\mathcal{X}$ obtain small VEVs of $\mathcal{O}(m_{3/2})$.

[17] As this symmetry is not broken spontaneously at the vacuum, and hence, Z_0's and M_0's are predicted to be stable. Thus, the simultaneous breaking of the IYIT sector should take place before inflation to avoid the production of those stable particles if we assume the above symmetry.

Tab. 6.1). In other words, the sign changes of Q's by a phase rotation with $\alpha = 2\pi$ can be absorbed by a part of $SU(2)$ transformation.

The effective Kähler potential and superpotential of $M_\pm$ and $Z_\pm$ are given by,

$$K_{\rm eff} \sim |Z_+|^2 + |Z_-|^2 + |M_+|^2 + |M_+|^2 + \cdots \ , \tag{6.67}$$

$$W_{\rm eff} \sim \lambda\Lambda M_+ Z_- + \lambda\Lambda M_- Z_+ \ , \tag{6.68}$$

where the ellipses denote the higher dimensional operators. By substituting the axion superfield, the effective theory is reduced to

$$K_{\rm eff} \sim |Z_+|^2 + |Z_-|^2 + \frac{1}{2}(A^\dagger + A)^2 + \cdots \ , \tag{6.69}$$

$$W_{\rm eff} \sim \frac{1}{\sqrt{2}}\lambda\Lambda^2(Z_- e^{A/\Lambda} + Z_+ e^{-A/\Lambda}) \ , \tag{6.70}$$

with some irrelevant holomorphic terms omitted in the Kähler potential. The scalar potential is accordingly given by,[18]

$$V \sim \lambda^2\Lambda^4 \cosh\left(\frac{A^\dagger + A}{\Lambda}\right) + \frac{1}{2}\lambda^2\Lambda^4 \left|Z_+ e^{-A/\Lambda} - Z_- e^{A/\Lambda}\right|^2 \tag{6.71}$$

$$\sim \lambda^2\Lambda^4 \cosh\left(\frac{A^\dagger + A}{\Lambda}\right) + \lambda^2\Lambda^4 |T|^2 \ . \tag{6.72}$$

In the final expression, we rearranged the scalar fields by introducing complex scalar fields S and T,

$$Z_+ = \frac{1}{\sqrt{2}}(S+T)e^{-A/\Lambda} \ , \tag{6.73}$$

$$Z_- = \frac{1}{\sqrt{2}}(S-T)e^{A/\Lambda} \ , \tag{6.74}$$

so that the PQ symmetry is manifest in the scalar potential.

The above scalar potential shows that the complex scalar T and the real component of A (the saxion) obtain masses of $\lambda\Lambda$, around their origins. The complex scalar filed S (the pseudo-flat direction) and the imaginary part of A (the axion a), on the other hand, remain massless. The pseudo-flat direction eventually obtains a mass from the higher order terms in the Kähler potential. For perturbative λ and $\tilde{\lambda}$, the mass is dominated by the one-loop contributions [117–119],

$$m_S^2 \simeq \frac{1}{32\pi^2}\left(\lambda^2(2\log 2 - 1) + \frac{4\lambda^4}{3\tilde{\lambda}^2}\right)\frac{F_S^2}{\Lambda^2} \ , \quad (F_S = \lambda\Lambda^2) \ , \tag{6.75}$$

[18] Throughout the paper, we use the same symbols to describe the superfields and their scalar components.

with which the pseudo-flat direction is stabilized at its origin.[19]

The superpotential in Eq. (6.70) also shows that the fermion partners of A (the axino) and T obtain a Dirac mass of $\lambda\Lambda$, with each other. The fermion partner of S corresponds to the goldstino which is absorbed into the gravitino by the super-Higgs mechanism.

Putting together, the model achieves dynamical breaking of supersymmetry and the PQ breaking simultaneously. The axion supermultiplet splits into a massless axion and massive saxion/axino with masses of the supersymmetry/PQ breaking scale. The axion couples to the SSM sector via the coupling in Eq. (6.123), i.e.,

$$W \sim \frac{\Lambda^2}{\sqrt{2}M_{\rm pl}} e^{A/\Lambda}\,\bar{\mathbf{5}}\,\mathbf{5} \sim \frac{\Lambda^2}{\sqrt{2}M_{\rm pl}} e^{ia/F_a}\,\bar{\mathbf{5}}\,\mathbf{5}\ , \tag{6.76}$$

where $a = \sqrt{2}\,{\rm Im}[A]$ denotes the axion field and $F_a = \sqrt{2}\Lambda$. After integrating out the extra KSVZ multiplets, the axion couples to the SM gauge fields through

$$W_{\rm anom} = \frac{N_f}{8\pi^2}\frac{A}{\Lambda}\sum_l W_l^\alpha W_{l\alpha}\ , \tag{6.77}$$

with which the strong CP problem is solved

Explicit breaking of the PQ symmetry

Now, let us discuss explicit breaking of the global PQ symmetry expected in quantum gravity. In this model, the most relevant terms which break the global PQ symmetry are given by, [20]

$$W \sim \frac{\kappa}{M_{\rm pl}^2} Z_+ (Q_1 Q_2)^2 + \frac{\kappa}{M_{\rm pl}^2} Z_- (Q_3 Q_4)^2 \sim \frac{\kappa\Lambda^2}{M_{\rm pl}^2} Z_\pm M_\pm^2\ . \tag{6.78}$$

with κ being a dimensionless coupling constant.[21] The corresponding symmetry breaking terms in the scalar potentials are given by,

$$V \sim \frac{1}{2} m_a^2 F_a^2 \left(\frac{a}{F_a}\right)^2 + \lambda\kappa \left(\frac{\Lambda}{M_{\rm pl}}\right)^2 \Lambda^4 e^{i\frac{a}{F_a}} + h.c.\ . \tag{6.79}$$

[19] Here, we neglect the one-loop contributions from the $U(1)_{gPQ}$ gauge interaction by assuming that the gauge coupling constant is small. The contributions from the gauge interaction, in fact, destabilize the origin of the pseudo flat direction [118–120].

[20] Here, we require that $U(1)_{PQ}$ is not broken by renormalizable interactions as a part of definition of the global symmetry.

[21] Lower dimensional operators which break the PQ symmetry, such as Z_+^4/M_{PL}, are forbidden by the $\mathbb{Z}_{4R}$ symmetry.

Here, we inserted the VEVs of $M_\pm$ and those of F-terms of $Z_\pm$. Due to the explicit breaking, the VEV of the axion, and hence, the effective θ angle is shifted to

$$\theta_{\rm eff} = \frac{\langle a \rangle}{F_a} \simeq \mathrm{Im}[\kappa\lambda] \left(\frac{\Lambda}{M_{\rm pl}}\right)^2 \left(\frac{\Lambda^4}{m_a^2 F_a^2}\right)\Bigg|_{\rm mod\ 2\pi} \tag{6.80}$$

$$\simeq 10^{40} \times \mathrm{Im}[\kappa\lambda] \left(\frac{0.08\,\mathrm{GeV}}{\sqrt{m_a F_a}}\right)^4 \left(\frac{\Lambda}{10^{12}\,\mathrm{GeV}}\right)^6\Bigg|_{\rm mod\ 2\pi} . \tag{6.81}$$

Thus, unless $\mathrm{Im}[\kappa\lambda]$ is finely tuned to be smaller than $\mathcal{O}(10^{-11})$, the effective θ angle is too large to be consistent with the measurement of the neutron electric dipole moment (EDM) [1].

6.3.3 Gauged PQ extension of simultaneous breaking model

Let us now implement the gauged PQ mechanism to the model of the simultaneous breaking of supersymmetry and the PQ symmetry in section 6.3.2. For that purpose, we introduce an additional sector based on $SU(3)$ dynamics which breaks a PQ symmetry spontaneously. In the following, we call this model the $SU(3)'$ model, and put primes on the superfields and the symmetry groups in this sector.

$SU(3)'$ PQ symmetry breaking model

The $SU(3)'$ model consists of three flavors of the (anti-)fundamental representation of $SU(3)$, Q', $\bar{Q}'$, and nine $SU(3)$ singlets, Z'. The charge assignment of the global symmetries is given in Tab. 6.2. Under these symmetries, they couple via the superpotential

$$W_{PQ} = \lambda'^{kl}_{ij} Z'^{ij} Q'_k \bar{Q}'_l\ , \tag{6.82}$$

where λ'^{kl}_{ij} denote coupling constants with $(i,j,k,l = 1-3)$. The baryon symmetry, $U(1)_B$, is identified with the global PQ symmetry, $U(1)'_{PQ}$, while the maximal flavor symmetry, $SU(3)_L \times SU(3)_R$, is completely broken by λ''s.

In addition to the global $U(1)'_{PQ}$ symmetry, the superpotential possesses a continuous R-symmetry and a $U(1)'_A$ symmetry (broken down to a $\mathbb{Z}_6$ symmetry by the $SU(3)'$ anomaly) in Tab. 6.2. As discussed previously, however, we consider that only $\mathbb{Z}_{4R}$ is an exact symmetry, and assume that $U(1)_R$ and $U(1)'_A$ are accidental symmetries broken by higher dimensional operators.[22]

[22]Without $U(1)'_A$ (or $\mathbb{Z}_6$), the superpotential terms such as Z'^3 are allowed even if we assume the $\mathbb{Z}_{4R}$ symmetry. Such terms, however, do not change the following discussion.

Table 6.2: *Charge assignment of the dynamical PQ symmetry breaking sector. The chiral superfields, Q''s, and Z''s, are the $SU(3)'$ triplets and singlets, respectively. The $U(1)'_{PQ}$ symmetry corresponds to $U(1)_B$ symmetry in the $SU(3)'$ sector. The KSVZ extra multiplets are denoted by $\mathbf{5}'$ and $\bar{\mathbf{5}}'$. The U(1)$_R$ and $U(1)'_A$ symmetries are accidental symmetries, with $\mathbb{Z}_{4R}$ being an exact symmetry. The R-charges of the KSVZ extra multiplets are taken to be $r'_5 + r'_{\bar{5}} = 2$.*

	Q'	$\bar{Q}'$	Z'	$\mathbf{5}'$	$\bar{\mathbf{5}}'$
$SU(3)'$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)'_{PQ}$	$+1$	-1	0	3	0
$U(1)_R$	0	0	$+2$	r'_5	$r'_{\bar{5}}$
$U(1)'_A$	$+1$	$+1$	-2	-3	0

Below the dynamical scale of $SU(3)'$, Λ', the $SU(3)'$ sector is well described by the composite mesons and baryons,

$$M' \sim Q'\bar{Q}'/\Lambda' \ , \quad B'_+ \sim Q'Q'Q'/\Lambda'^2 \ , \quad B'_- \sim \bar{Q}'\bar{Q}'\bar{Q}'/\Lambda'^2 \ , \tag{6.83}$$

with an effective superpotential,

$$W'_{\rm eff} \ = \ \lambda'\Lambda' Z' M' + \mathcal{X}'(B'_+ B'_- + \det(M')/\Lambda' - \Lambda'^2) \ . \tag{6.84}$$

Here, the second term implements the deformed moduli constraint by a Lagrange multiplier field $\mathcal{X}'$ [113]. The mesons are neutral under $U(1)'_{PQ}$ while the baryons have charges ± 3.

From the superpotential in Eq. (6.84), we find that the PQ symmetry is spontaneously broken by the VEVs of $B_\pm$. Accordingly, the vacuum is parametrized by the Goldstone superfield A',[23]

$$B'_+ \ = \ \frac{1}{\sqrt{2}}\Lambda_2 e^{A'/\Lambda_2} \ , \tag{6.85}$$

$$B'_- \ = \ \frac{1}{\sqrt{2}}\Lambda_2 e^{-A'/\Lambda_2} \ , \tag{6.86}$$

with $\Lambda_2 = \sqrt{2}\Lambda'$. By using A', the PQ symmetry is non-linearly realized by

$$\frac{A'}{\Lambda_2} \to \frac{A'}{\Lambda_2} + i\alpha' \ , \quad (\alpha' = 0 - 2\pi) \ . \tag{6.87}$$

As in the case of the IYIT sector, the domain of the PQ symmetry is reduced from $\alpha' = 0 - 6\pi$ to $\alpha' = 0 - 2\pi$ as the $SU(3)'$ invariant fields have the PQ charges of ± 3.

[23] The origin of A' is set at which $B'_+ = B'_-$, and $\langle B'_+ \rangle \neq \langle B'_- \rangle$ for $\langle A' \rangle \neq 0$, accordingly.

Table 6.3: The charge assignment of the gauged PQ symmetry and the $\mathbb{Z}_{4R}$ symmetry. The singlet fields Y's and Y''s are introduced to cancel the self-triangle and gravitational anomalies of $U(1)_{gPQ}$ (see subsection 6.3.3).

	$Z_\pm$	$M_\pm$	B'_+	B'_-	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{5}'$	$\bar{\mathbf{5}}'$	Y	Y'
$U(1)_{gPQ}$	$\pm q_1$	$\pm q_1$	q_2	$-q_2$	$-q_1$	0	q_2	0	q_1	$-q_2$
$\mathbb{Z}_{4R}$	2	0	0	0	r_5	$r_{\bar{5}}$	r'_5	$r'_{\bar{5}}$	1	1

The $U(1)'_{PQ}$ symmetry in this sector is also communicated to the SSM sector through the couplings to N'_f flavors of the KSVZ extra multiplets, $\mathbf{5}'$ and $\bar{\mathbf{5}}'$. With the charge assignment in Tab. 6.2, the baryons couple to the extra multiplets in the superpotential,

$$W = \frac{1}{M_{\rm pl}^2}\bar{Q}'\bar{Q}'\bar{Q}'\mathbf{5}'\bar{\mathbf{5}}' \sim \frac{\Lambda'^2}{M_{\rm pl}^2}B'_-\mathbf{5}'\bar{\mathbf{5}}' \ . \tag{6.88}$$

Once $U(1)'_{PQ}$ is broken, the axion obtains the anomalous coupling to the SSM gauge fields, while the extra multiplets obtain masses of $\mathcal{O}(\Lambda'^3/M_{\rm pl}^2)$.

Gauged PQ symmetry

Now, we are ready to find out a model of the gauged PQ symmetry by combining the simultaneous supersymmetry and the PQ symmetry breaking model in section 6.3.2 and the PQ symmetry breaking model in subsection 6.3.3. To apply the prescription in section 6.3.1, let us first identify Φ_1 with the meson operator M_+ in section 6.3.2 and Φ_2 with the baryon operator B'_+, i.e.,

$$\Phi_1 = M_+ \ , \tag{6.89}$$

$$\bar{\Phi}_2 = B'_- \ , \tag{6.90}$$

and assign $U(1)_{gPQ}$ charges of q_1 and $-q_2$ to them (Tab. 6.3).[24] Then, the anomaly-free condition of the $U(1)_{gPQ}$ symmetry in Eq. (6.41) is given by,

$$q_1 N_f - q_2 N'_f = 0 \ . \tag{6.91}$$

Once the two sectors are put together by the gauged PQ symmetry, spontaneous breaking of the PQ symmetries in the two sectors lead to the would-be Goldstone and the axion superfield. The would-be Goldstone is absorbed into the massive

[24]The $U(1)_{gPQ}$ charges of $Q_{1,2}$ and $\bar{Q}'$'s corresponds to $q_1/2$ and $q_2/3$, respectively.

$U(1)_{gPQ}$ gauge multiplet, and the saxion and the axino in the axion supermultiplet obtain masses of the order of the supersymmetry breaking scale. As a result, the simultaneous breaking model with the gauged PQ mechanism leaves only a light axion which couples to the Standard Model gauge fields via Eq. (**??**).

Accidental global PQ symmetry

As discussed in the previous section, the global PQ symmetry can be explicitly broken by the $U(1)_{gPQ}$ invariant operator consisting of the fields in the two sectors. Among the explicit breaking terms, the most relevant ones are given by,[25]

$$W \sim \frac{\kappa}{M_{\rm pl}^{3q_1+2q_2-4}} \left(Z_+(Q_1Q_2)^{q_2-1}(\bar{Q}'\bar{Q}'\bar{Q}')^{q_1}\right. \tag{6.92}$$

$$\left. +Z_+(Q_3Q_4)^{q_2-1}(Q'Q'Q')^{q_1}\right) , \tag{6.93}$$

$$\sim \frac{\kappa\Lambda^{q_2-1}\Lambda'^{2q_1}}{M_{\rm pl}^{3q_1+2q_2-4}} \left(Z_+M_+^{q_2-1}B_-'^{q_1} + Z_-M_-^{q_2-1}B_+'^{q_1}\right) , \tag{6.94}$$

with κ being a dimensionless coupling constant. It should be noted that these terms are consistent with the $\mathbb{Z}_{4R}$ symmetry, and hence, no factor of $m_{3/2}$ is required unlike the terms in Eq. (6.53). These operators roughly contribute to the axion potential,

$$V \sim \frac{1}{2}m_a^2F_a^2\left(\frac{a}{F_a}\right)^2 + \frac{{\rm Im}[\kappa\lambda]}{M_{\rm pl}^{3q_1+2q_2-4}}\Lambda^{2q_2}\Lambda'^{3q_1}\frac{a}{F_a} + h.c. + \cdots , \tag{6.95}$$

where the VEVs of $M_\pm$, $B'_\pm$, and those of the F-terms of $Z_\pm$ are inserted, Therefore, in the simultaneous breaking model with the gauged PQ mechanism, the effective θ angle at the vacuum is given by,

$$\theta_{\rm eff} \simeq \frac{1}{m_a^2F_a^2}\frac{\Lambda^{2q_2}\Lambda'^{3q_1}}{M_{\rm pl}^{3q_1+2q_2-4}} \tag{6.96}$$

$$\sim 10^{77.5-6.4(3q_1+2q_2)} \times \left(\frac{0.08\,{\rm GeV}}{\sqrt{m_aF_a}}\right)^4 \left(\frac{\Lambda}{10^{12}\,{\rm GeV}}\right)^{2q_2} \left(\frac{\Lambda'}{10^{12}\,{\rm GeV}}\right)^{3q_1} \tag{6.97}$$

Thus, for $3q_1 + 2q_2 \gtrsim 14$, the explicit breaking of the global PQ symmetries are small enough to be consistent with the measurement of the neutron EDM, i.e., $\theta_{\rm eff} < 10^{-11}$ [1].

[25] There are lower dimensional operators which break the global PQ symmetry with $M_\pm$ replaced by $M_{PL} \times Z_\pm$ in Eq. (6.92). The explicit breaking effects of those operators are comparable to the ones of Eq. (6.92) due to suppressed A-term VEVs of $Z_\pm = \mathcal{O}(m_{3/2})$.

Mass spectrum of the KSVZ multiplets

The KSVZ multiplets, $(\bar{\mathbf{5}}, \bar{\mathbf{5}})$ and $(\bar{\mathbf{5}}', \bar{\mathbf{5}}')$ were introduced to communicate the PQ symmetries to the SSM sector. After PQ symmetry breaking, those extra multiplets obtain supersymmetric masses of the order of

$$m_{KSVZ} \sim \frac{\Lambda^2}{M_{\rm pl}} , \qquad (6.98)$$

$$m'_{KSVZ} \sim \frac{\Lambda'^3}{M_{\rm pl}^2} , \qquad (6.99)$$

respectively (see Eqs. (6.123) and (6.125)). The scalar components of the KSVZ multiplets also obtain masses of $\mathcal{O}(m_{3/2})$ through supergravity effects. Thus, most of the KSVZ multiplets become heavy and beyond the reach of the LHC experiments except for the fermion components of $(\bar{\mathbf{5}}', \bar{\mathbf{5}}')$.[26]

The KSVZ extra multiplets are assumed to couple to the SSM particle via,

$$W \sim \frac{\epsilon}{M_{\rm pl}} Q_1 Q_2 \mathbf{5}\,\bar{\mathbf{5}}_{SM} + \frac{\epsilon'}{M_{\rm pl}^2} \bar{Q}'\bar{Q}'\bar{Q}'\mathbf{5}'\,\bar{\mathbf{5}}_{SM} , \qquad (6.100)$$

$$\sim \epsilon\, m_{KSVZ}\, \mathbf{5}\,\bar{\mathbf{5}}_{SM} + \epsilon'\, m'_{KSVZ}\, \mathbf{5}'\,\bar{\mathbf{5}}_{SM} , \qquad (6.101)$$

where $\bar{\mathbf{5}}_{SM}$ denotes the SSM matter multiplet, and $\epsilon^{(\prime)}$ are coefficients. Here, we take $r_5 = r'_5 = 1$ so that $\bar{\mathbf{5}}$ and $\bar{\mathbf{5}}'$ have the same R-charges with $\bar{\mathbf{5}}_{SM}$. Through the mixing terms, the KSVZ extra multiplets decay immediately into the SSM particles.

Finally, let us note that there can be mixing terms between $(\mathbf{5},\ \bar{\mathbf{5}})$ and $(\bar{\mathbf{5}}',\ \bar{\mathbf{5}}')$ through,

$$W \sim \frac{1}{M_{\rm pl}} Q_1 Q_2 \mathbf{5}\,\bar{\mathbf{5}}' + \frac{1}{M_{\rm pl}^2} \bar{Q}'\bar{Q}'\bar{Q}'\mathbf{5}'\,\bar{\mathbf{5}} . \qquad (6.102)$$

Although these operators consist of the fields in the two PQ symmetric sectors, they are invariant under not only the gauged PQ symmetry but also under the global PQ symmetries. Thus, these terms do not affect $\theta_{\rm eff}$. They do not affect the KSVZ mass spectrum significantly neither. From these reasons, we neglect these mixing terms throughout this paper.

PQ charges in the $SU(3)'$ model

For a given q_1 and q_2, there are upper limits on Λ and Λ' to achive a high-quality global PQ symmetry (see Eq. (6.96)). The dynamical scales are also constrained

[26] The extra multiplet to achieve the $\mathbb{Z}_{4R}$ symmetry also obtains the mass of $\mathcal{O}(m_{3/2})$ from the R-symmetry breaking effects [121].

from below for an appropriate supersymmetry breaking scale and for heavy enough KSVZ extra multiplets. As a lower limit on the supersymmetry breaking scale, i.e., Λ, we require

$$m_{3/2} \simeq \frac{\lambda\Lambda^2}{\sqrt{3}M_{\rm pl}} \gtrsim 10\,\text{TeV}\ , \tag{6.103}$$

so that the observed Higgs boson mass, $m_H \simeq 125\,\text{GeV}$, is achieved by the gravity mediated sfermion masses of $\mathcal{O}(m_{3/2})$. As a lower limit on the KSVZ extra multiplets, we put

$$m'_{KSVZ} \simeq \frac{\Lambda'^3}{M_{\rm pl}} \gtrsim 750\,\text{GeV}\ , \tag{6.104}$$

from the null results of the searches for a heavy b-type quark at the LHC experiments [122–125].

In Fig. 6.2, we show the charge choices for $SU(3)'$ model for $N_{\rm eff} = 1$ and $\lambda = 1$. The charges colored by blue are excluded, with which $\theta_{\rm eff}$ cannot be suppressed enough for $m_{3/2} \gtrsim \mathcal{O}(1)\,\text{TeV}$ and $m'_{\rm KSVZ} \gtrsim 750\,\text{GeV}$.[27] In the figure, we require $\theta_{\rm eff} \lesssim 10^{-10}$ given $\mathcal{O}(1)$ uncertainties of the coefficients of the explicit breaking terms. The figure shows that these constraints exclude relatively small charges as the suppression of the explicit breaking term relies on large PQ charges.

The perturbative coupling unification of the SSM gauge coupling constants also puts constraints on the charges. From the anomaly-free condition in Eq.(6.91), N_f and N'_f are given by

$$N_f = N_{\rm GCD} \times q_2\ , \quad N'_f = N_{\rm GCD} \times q_1\ . \tag{6.105}$$

As the extra multiplets contribute to the renormalization group evolutions of the SSM gauge coupling constants and make them asymptotically non-free, the perturbative unification puts upper limits on N_f and N'_f, and hence, on q_1 and q_2.

In Fig. 6.2, we color the charges by red, with which $\theta_{\rm eff} \lesssim 10^{-10}$ is not compatible with the perturbative unification. Here, we use the renormalization group equation at the one-loop level and require that $g_{1,2,3} < 4\pi$ below the GUT scale, i.e., $M_{\rm GUT} \simeq 10^{16}$ GeV. We also take the masses of the sfermions, the heavy charged/neutral Higgs boson, and the Higgsinos to be at the gravitino mass scale. The gaugino masses are assumed to be dominated by the anomaly mediation effects [126, 127] which are

[27] The charges colored by blue are not changed even if the lower limits on $m_{3/2}$ and m'_{KSVZ} are relaxed to $m_{3/2} \gtrsim 100\,\text{GeV}$ and $m'_{KSVZ} \gtrsim 100\,\text{GeV}$.

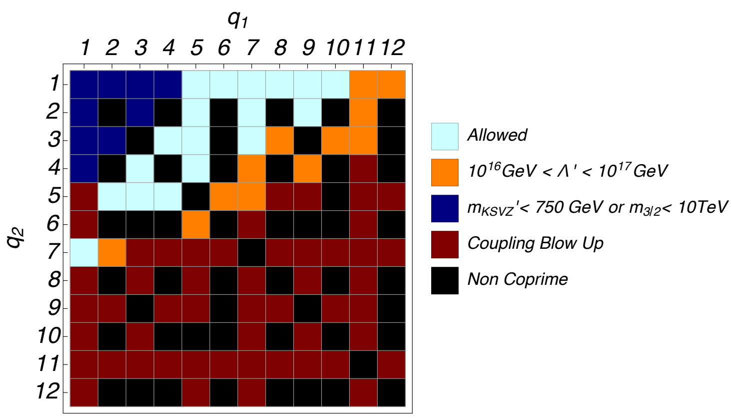

Figure 6.2: *Constraints on charges q_1 and q_2 in the $SU(3)'$ model for $N_{\rm GCD} = 1$. The allowed charges are colored by light blue and orange, although the orange colored charges are allowed only for $\Lambda' \gtrsim M_{\rm GUT}$. The charges colored by blue lead to too large $\theta_{\rm eff}$ or too light KSVZ extra multiplets. The gauge coupling constants of the SSM blow up below the GUT scale for the charges colored by red. The black colored charges are excluded as they are not relatively prime.*

roughly given by (see, e.g. [128]),

$$m_{\rm bino} \simeq 10^{-2} \times m_{3/2} \ , \qquad (6.106)$$

$$m_{\rm wino} \simeq 3 \times 10^{-3} \times m_{3/2} \ , \qquad (6.107)$$

$$m_{\rm gluino} \simeq 2.5 \times 10^{-2} \times m_{3/2} \ , \qquad (6.108)$$

although the constraints do not depend on them significantly as long as they are in the TeV range. The gravitino mass is take to be within $10\,{\rm TeV} \leq m_{3/2} \leq 10\,{\rm PeV}$. These choices are motivated by the pure gravity mediation model in Refs. [129–131] (see also Refs. [132–135] for closely related models).[28] In the renormalization group evolution, we also take into account an extra multiplet required for the anomaly free condition of the $\mathbb{Z}_{4R}$ symmetry, whose masses are also at the gravitino mass scale.

The figure shows that the requirement for perturbative unification excludes the charges with $q_2 > 7$ ($N_f > 7$). This is expected as N_f flavors of the KSVZ extra multiplets have masses of $10\,{\rm TeV} \lesssim m_{KSVZ} \lesssim 10\,{\rm PeV}$.[29] On the other hand, a large q_1 is allowed. This is because the explicit breaking terms are suppressed by $(\Lambda'/M_{\rm pl})^{3q_1}$, and hence, a high-quality global PQ is possible even for a large Λ' as long as q_1 is large. For a large Λ', m'_{KSVZ} also becomes large, with which the

[28] Here, the Higgsino mediation effects neglected for simplicity. Besides, the gaugino spectrum is deflected from the anomaly mediation in the presence of the KSVZ extra multiplets [136].

[29] If we restrict to $m_{3/2} < 1\,{\rm PeV}$, the constraint becomes tighter and the charges with $q_2 > 5$ are excluded.

perturbative unification is possible even if $N'_f = q_1$ is large. It should be noted, however, that the effective field theory approach is no more reliable when Λ' is too close to the Planck scale. In the figure, we color the charges by orange if they require a large Λ', i.e., $10^{16}\,\mathrm{GeV} \lesssim \Lambda' \lesssim 10^{17}\,\mathrm{GeV}$. For $N_{\mathrm{GCD}} \geq 2$, there are no appropriate charges with which $\theta_{\mathrm{eff}} < 10^{-10}$ and the perturbative unification are compatible.

Parameter regions in the $SU(3)'$ model

In Fig. 6.3, we show the parameter regions for a given q_1 and q_2. In each panel, we take $m_{3/2} < 10\,\mathrm{PeV}$ and $\lambda = 1,\ 10^{-1},\ 10^{-2}$, respectively. The gray shaded region is excluded, as $\theta_{\mathrm{eff}} < 10^{-10}$ is not satisfied (see Eq. (6.96)). The perturbative unification is not achieved in the blue shaded region. The red shaded region is excluded for too light KSVZ extra multiplets, i.e., $m'_{KSVZ} \lesssim 750\,\mathrm{GeV}$. The green dashed lines are contours of the effective decay constant in Eq. (6.52).

The figure shows that the dynamical scale Λ' is tightly constrained from above to achieve $\theta_{\mathrm{eff}} < 10^{-10}$ for the minimum charge choice, i.e., $q_1 = 5$ and $q_2 = 1$. This is understood as the explicit breaking terms are not effectively suppressed for rather small charges. As a result, the PQ breaking scales are required to be low to avoid large explicit breaking effects. The upper limit on Λ' becomes tighter for a larger Λ as is expected from Eq. (6.96). Furthermore, as the dynamical scale Λ becomes larger for a smaller λ, the upper limit becomes even tighter for a smaller λ for a given $m_{3/2}$. The constraints from the perturbative unification are, on the contrary, weaker since m_{KSVZ} becomes larger for a smaller λ for a given $m_{3/2}$.

An interesting property of the minimum choice is that the model predicts the KSVZ extra multiplets ($\mathbf{5}'$, $\bar{\mathbf{5}}'$) in the TeV range. This feature reflects the suppressed fermion masses of the KSVZ extra multiplet in Eq. (6.99) caused by the composite nature of the PQ breaking field, i.e., B'_-, with a tight upper limit on Λ'. Thus, the model with the minimum charge choice can be tested by searching for vector-like colored particles at the LHC experiments.

For $q_1 = 7$ and $q_2 = 1$, the upper limit on Λ' is weaker than for the minimum choice. This is because the suppression factor of the explicit breaking term, $(\Lambda'/M_{\mathrm{pl}})^{3q_1}$, can be very small even for a rather large Λ' due to a large exponent. The constraint form the perturbative unification is, on the contrary, tighter for a large q_1 as N'_f is proportional to q_1. For a large N'_f, the masses of the KSVZ extra multiplets, m'_{KSVZ}, is required to be high to avoid the blow-up of the gauge coupling constants below the GUT scale.

For $q_1 = 1$ and $q_2 = 7$, the upper limit on Λ' is also weaker than the minimum choice for $\lambda = 1$ due to a strong suppression of the explicit breaking terms by $(\Lambda/M_{\rm pl})^{2q_2}$. As the suppression factor is sensitive to Λ, the upper limit on Λ' becomes very tight for a smaller λ for a given gravitino mass.

In all cases, we find that the gravitino mass is required to be in the hundreds TeV or larger, and hence, the model can be consistent with the observed Higgs boson mass achieved by the gravity mediated sfermion masses. It is also notable that the dynamical scale Λ' is larger than Λ in the allowed parameter region. Therefore, both the accidental global PQ symmetry and supersymmetry are broken by the IYIT sector while the gauged PQ symmetry is mainly broken by the $SU(3)'$ sector. This feature is attractive as it explains the coincidence between the global PQ breaking scale and the supersymmetry breaking scale.

Before closing this subsection, let us comment on the axion dark matter abundance. The axion starts coherent oscillation when the Hubble expansion rate becomes comparable to the axion mass, which leads to the present axion dark matter density [77],

$$\Omega_{\rm axion} h^2 \simeq 0.2 \times \theta_i^2 \left(\frac{F_a}{10^{12}\,{\rm GeV}} \right)^{1.19} . \tag{6.109}$$

Here, θ_i is the initial misalignment angle of the axion field. Thus, the axion can be a dominant component for dark matter of $F_a = \mathcal{O}(10^{12})$ GeV, i.e., $\Omega_{\rm DM} \simeq 0.12$ [137]. As the figures show, $F_a = \mathcal{O}(10^{12})$ GeV is possible in a wide range of the parameter space. Therefore, the model based on $SU(3)'$ can be consistent with the axion dark matter scenario.[30]

Cancellation of self- and gravitational anomalies

As mentioned in section 6.3.1, the gravitational anomaly and the self-anomaly of $U(1)_{gPQ}$ are canceled by adding $U(1)_{gPQ}$ charged singlet fields. In this subsection, we show a concrete model of the anomaly cancelation.

In the IYIT sector and the $SU(3)'$ sector, the $U(1)_{gPQ}$ charged fields are paired with fields with opposite charges. Thus, the fields in these sectors do not contribute to the self-anomaly nor the gravitational anomaly. The charges of the KSVZ extra multiplets are, on the other hand, not paired, and hence, they contribute to the

[30] For $m_{3/2} \gg \mathcal{O}(1)$ PeV, the wino is expected to be heavier than $\mathcal{O}(1)$ TeV, whose relic abundance exceeds the observed dark matter density. In such parameter region, we need to assume either a dilution mechanism of dark matter or R-parity violation.

anomalies,

$$\mathcal{A}_{\rm self}^{\rm KSVZ} = -5N_f q_1^3 + 5N_f' q_2^3 \ , \tag{6.110}$$
$$\mathcal{A}_{\rm gravitational}^{\rm KSVZ} = -5N_f q_1 + 5N_f' q_2 \ , \tag{6.111}$$

respectively. The easiest way to cancel the anomaly is to introduce $5N_f$ singlet superfields Y with a charge q_1 and $5N_f'$ singlet superfields Y' with a charge $-q_2$. The charges of Y's and Y''s are given in Tab. 6.3.

As the singlet fields do not have mass partners with opposite charges, the supersymmetric masses of them are generated only after $U(1)_{gPQ}$ breaking. The mass terms of Y's are given by

$$W \sim \frac{1}{M_{\rm pl}^3}(Q_3Q_4)^2 YY \sim \frac{\Lambda^4}{M_{\rm pl}^3} YY \ . \tag{6.112}$$

Here, we take the $\mathbb{Z}_{4R}$ charge of Y's to be 1, so that their scalar and fermion components are odd and even under the R-parity, respectively. As a result, the fermionic components of Y's obtain

$$m_Y \sim 1\,{\rm MeV}\left(\frac{\Lambda}{10^{13}\,{\rm GeV}}\right)^4 \ , \tag{6.113}$$

while the masses of scalar components are dominated by the gravity mediated soft masses of $\mathcal{O}(m_{3/2})$.

The supersymmetric masses of Y''s are even smaller,

$$W \sim \frac{1}{M_{\rm pl}^5}(\bar{Q}'\bar{Q}'\bar{Q}')^2 Y'Y' \sim \Lambda'\left(\frac{\Lambda'}{M_{\rm pl}}\right)^5 Y'Y' \ . \tag{6.114}$$

Here, we take the $\mathbb{Z}_{4R}$ charge of Y''s to be 1. As a result, the fermionic components of Y''s obtain,

$$m_{Y'} \sim 5\times 10^{-2}\,{\rm eV}\left(\frac{\Lambda'}{10^{13.5}\,{\rm GeV}}\right)^5 \tag{6.115}$$

while the masses of the scalar components of Y''s are dominated by the gravity mediated soft masses as in the case of Y's.[31]

If those fermions are abundantly produced in the early universe, they contribute to the dark radiation or dark matter and result in an unacceptably large number of effective neutrino species, $N_{\rm eff}$, or the overabundance of dark matter. To evade these

[31] There are mass terms proportional to YY' which can be lager than Eqs. (6.112) and (6.114) depending on the parameters. Even in such cases, there remain light fermions with masses either m_Y or m_Y' as the numbers of Y's and Y''s are different.

problem, we assume that spontaneous breaking of $U(1)_{gPQ}$ takes place before the end of inflation. We also assume that the gauge superfields of $U(1)_{gPQ}$ are heavier than the reheating temperature after inflation. Furthermore, it is also assumed that the branching fraction of the inflaton into Y's and Y''s are suppressed. With these assumptions, we can achieve cosmologically consistent models where the self- and the gravitational anomalies are canceled by the $U(1)_{gPQ}$ charged singlets.

$SU(N)'$ dynamical PQ symmetry breaking model

So far, we have considered the dynamical PQ breaking sector based on the $SU(3)$ gauge theory. There, the deformed moduli constraint plays an important role to break the global PQ symmetry (i.e., the baryon symmetry) spontaneously. In this subsection, we discuss the models of dynamical PQ breaking based on $SU(N)$ gauge theory other than $N = 3$. We call such models, the $SU(N)'$ dynamical PQ breaking model.

First, let us consider the $SU(2)'$ model. With four fundamental representations of $SU(2)'$, Q', the model exhibits the deformed moduli constraint. In this model, there is no baryon symmetry, and the global PQ symmetry is identified with a subgroup of the maximal non-abelian group $SU(4)_f$ as in the case of the IYIT sector. Then, the global PQ symmetry breaking is achieved by introducing four PQ neutral singlet superfields, Z'.[32]

In this model, the KSVZ extra multiplets coupling to the $SU(2)'$ sector obtain masses via,

$$W \sim \frac{1}{M_{\rm pl}} Q'_1 Q'_2 \mathbf{5}'\, \bar{\mathbf{5}}' \ , \tag{6.116}$$

leading to

$$m_{KSVZ'} \sim \frac{\Lambda'^2}{M_{\rm pl}} \ . \tag{6.117}$$

Thus, the $SU(2)'$ model allows a rather small Λ' compared with the $SU(3)'$ model to achieve $m'_{KSVZ} \gtrsim 750\,\mathrm{GeV}$. It is even possible to be $\Lambda' \ll \Lambda$. The possibility of $\Lambda' \ll \Lambda$ is, however, not very attractive as the model does not explain a coincidence between the PQ breaking scale and the supersymmetry breaking scale.

Next, let us consider the $SU(N)'$ ($N > 3$) model. In this case, the global PQ symmetry is identified with the baryon symmetry which is broken by the deformed

[32] It is tempting to make the $SU(2)'$ sector also be the IYIT supersymmetry breaking sector by introducing six singlet fields, Z''s, instead. In this case, however, supersymmetry and the gauged PQ symmetry are broken by the dynamics, while the global PQ symmetry is broken separately.

moduli constraint as in the case of the $SU(3)'$ model. As the mass terms of the KSVZ extra multiplets, $\mathbf{5}'$ and $\bar{\mathbf{5}}'$, are given by,

$$W \sim \frac{1}{M_{\rm pl}^{N-2}}(\bar{Q}' \cdots \bar{Q}')\mathbf{5}'\,\bar{\mathbf{5}}' \ , \tag{6.118}$$

the dynamical scale Λ' should be much higher than Λ to satisfy $m'_{KSVZ} \gtrsim 750\,\text{GeV}$. Here, $\bar{Q} \cdots \bar{Q}$ denotes the baryon operators of the $SU(N)'$ sector. The $SU(N)'$ models are very similar to the $SU(3)'$ model except for the dynamical scale Λ', although we do not discuss details of the $SU(N)'$ model further.

6.4 B-L gauge symmetry as gauged PQ symmetry

As discussed in our mechanism application to the SUSY, the SM singlet fermions introduced to cancel the anomaly tend to have light mass due to its gauged PQ charge, and they are potentially produced from the decay of the inflaton and the gauge field of the gauged PQ symmetry, and thus they contribute to the dark radiation and result in an unacceptably large number of effective neutrino species. This problem seems to be a general cosmological problem with our mechanism.

In this section[33], we find the model with no extra singlet fermions as one simple solution to the above cosmological problem. To find out a realistic model, we have also sought the probable identification that the gauged PQ symmetry equals to the $B-L$ gauge symmetry which is the most authentic extension of the SM gauge symmetries. There, the $B-L$ charges for matters are assigned as motivated by the seesaw mechanism in the $SU(5)$ GUT.

6.4.1 $B-L$ as gauged PQ symmetry

Among the various extension of the Standard Model, $B-L$ is the most plausible addition. The anomalies of the $B-L$ gauge symmetry are canceled by simply introducing three SM singlet right-handed neutrinos $\bar{N}_R$. The $B-L$ extended Standard Model naturally implements the seesaw mechanism by the spontaneous breaking of $B-L$ at the intermediate scale.

Having the $SU(5)$ GUT in mind, it is more convenient to consider "fiveness", $5(B-L)-4Y$, instead of $B-L$, as it commutes with the $SU(5)$ gauge group. The fiveness charges of the matter fields are given by

$$\mathbf{10}_{\rm SM}(+1)\ , \quad \bar{\mathbf{5}}_{\rm SM}(-3)\ , \quad \bar{N}_R(+5)\ , \tag{6.119}$$

[33]Most part of this section is based on Ref. [40]

while the Higgs doublet, h, has a charge $+2$ (i.e. $B-L=0$).[34] Here, we use the $SU(5)$ GUT representations for the matter fields, i.e. $\mathbf{10}_{\rm SM} = (q_L, \bar{u}_R, \bar{e}_R)$ and $\bar{\mathbf{5}}_{\rm SM} = (\bar{d}_R, \ell_L)$, while $\bar{N}_R$ denotes the right-handed neutrinos.

The seesaw mechanism is implemented by assuming that the right-handed neutrinos obtain Majorana masses from spontaneous breaking of fiveness. In this paper, we assume that the Majorana masses are provided by the vacuum expectation value (VEV) of a gauge singlet scalar field with fiveness, -10, i.e.,

$$\phi(-10)\ , \tag{6.120}$$

which couples to the right handed neutrinos,

$$\mathcal{L} = -\frac{1}{2} y_N \phi \bar{N}_R \bar{N}_R + h.c.\ . \tag{6.121}$$

Here, y_N denotes a coupling constant, with which the Majorana mass is given by $M_N = y_N \langle\phi\rangle$. By integrating out the right-handed neutrinos, the tiny neutrino masses are obtained, via

$$\mathcal{L} = y_\ell \bar{\mathbf{5}}_{\rm SM} \bar{N}_R h^* + h.c., \tag{6.122}$$

where y_ℓ also denotes a coupling constant.

Now, let us identify the gauged PQ symmetry with $B-L$, i.e., fiveness. Following the general prescription of the gauged PQ mechanism in [13], let us introduce extra matter multiplets which obtain a mass from the VEV of ϕ;

$$\mathcal{L} = y_K \phi^*\, \mathbf{5}_K \bar{\mathbf{5}}_K + h.c.\ , \tag{6.123}$$

with y_K being a coupling constant.[35] Here, the extra multiplets ($\mathbf{5}_K$,$\bar{\mathbf{5}}_K$) are assumed to form the $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations of the $SU(5)$ gauge group, respectively. As in the KSVZ axion model [11, 12], the Ward identity of the fiveness current, j_5, obtains an anomalous contribution from the extra multiplets,

$$\partial j_5\big|_{\rm SM+N+K} = -\frac{g_a^2}{32\pi^2} 10 F^a \tilde{F}^a\ . \tag{6.124}$$

Here, F^a ($a = 1, 2, 3$) are the gauge field strengths of the Standard Model and g_a the corresponding SM gauge coupling constants. The Lorentz indices and the gauge group representation indices are suppressed. The factor -10 corresponds to the charge of the bi-linear, $\mathbf{5}_K \bar{\mathbf{5}}_K$ (see Eq. (6.123)).

[34] The colored Higgs is assumed to obtain a mass of the GUT scale.

[35] The reason why the extra multiplets couple not to ϕ but ϕ^* will become clear shortly.

In the gauged PQ mechanism, the $U(1)_{gPQ}$ gauge anomalies are canceled by a contribution from another set of the PQ charged sector. For that purpose, let us also introduce 10-flavors of extra matter multiplets $(\mathbf{5}'_K, \bar{\mathbf{5}}'_K)$. We assume that they obtain masses from a VEV of a complex scalar field ϕ' whose fiveness charge is $+1$;

$$\mathcal{L} = y'_K \phi'^* \, \mathbf{5}'_K \bar{\mathbf{5}}'_K + h.c. \ , \tag{6.125}$$

where the charge of the bi-linear, $\mathbf{5}'_K \bar{\mathbf{5}}'_K$, is set to be $+1$. With this choice, the anomalous contributions of the Ward identity in (6.124) are canceled by the one from $(\mathbf{5}'_K, \bar{\mathbf{5}}'_K)$, i.e.,

$$\partial j_5 \big|_{\mathrm{SM+N+K+K'}} = 0 \ . \tag{6.126}$$

The fiveness charges of the respective extra multiplets are chosen as follows. To avoid stable extra matter fields, we assume that $\bar{\mathbf{5}}_K$ and $\bar{\mathbf{5}}'_K$ can mix with $\bar{\mathbf{5}}_{\mathrm{SM}}$, so that

$$\mathbf{5}_K(-7) \ , \ \bar{\mathbf{5}}_K(-3) \ , \ \mathbf{5}'_K(+4) \ , \ \bar{\mathbf{5}}'_K(-3) \ , \tag{6.127}$$

respectively. As a notable feature of this charge assignment, it cancels the $[U(1)_{gPQ}]^3$ and the gravitational anomalies automatically without introducing additional SM singlet fields. In fact, the $[U(1)_{gPQ}]^3$ and the gravitational anomalies are proportional to

$$\begin{aligned} [U(1)_{gPQ}]^3 &\propto \left((-10-\bar{q}_K)^3 + (\bar{q}_K)^3\right) && (6.128)\\ &+ \ 10\left((1-\bar{q}'_K)^3 + (\bar{q}'_K)^3\right) \ , && (6.129)\\ [\text{gravitational}] &\propto \left((-10-\bar{q}_K) + (\bar{q}_K)\right) + 10\left((1-\bar{q}'_K) + (\bar{q}'_K)\right) \ , \end{aligned}$$

with $\bar{q}_K$ and $\bar{q}'_K$ are the charges of $\bar{\mathbf{5}}_K$ and $\bar{\mathbf{5}}'_K$, respectively. By substituting $\bar{q}_K = \bar{q}'_K = -3$, we find that both the anomalies are vanishing.

The anomaly cancellation without singlet fields other than the right-handed neutrinos is by far advantageous compared with the previous models [10,13,39]. The singlet fields required for the anomaly cancellation tend to be rather light and longlived, which make the thermal history of the universe complicated [39]. The anomaly cancellation of the present model is, therefore, a very important success as it is partly motivated by thermal leptogenesis which requires a high reheating temperature after inflation, i.e., $T_R \gtrsim 10^9$ GeV [138–140].

Under the fiveness symmetry, the interactions are restricted to

$$\mathcal{L} \ = \ \mathbf{10}_{\mathrm{SM}}\mathbf{10}_{\mathrm{SM}}h^* + \mathbf{10}_{\mathrm{SM}}\bar{\mathbf{5}}\,h + \bar{\mathbf{5}}\bar{N}_R h^* - \frac{1}{2}\phi\,\bar{N}_R\bar{N}_R + \phi^*\,\mathbf{5}_K\bar{\mathbf{5}} + \phi'^*\,\mathbf{5}'_K\bar{\mathbf{5}} + h.c.$$

$$-V(\phi,\phi',h)\ . \tag{6.130}$$

Here, $\bar{\mathbf{5}}$ collectively denotes ($\bar{\mathbf{5}}_{\rm SM}$,$\bar{\mathbf{5}}_K$,$\bar{\mathbf{5}}'_K$), and $V(\phi,\phi',h)$ is the scalar potential. The coupling coefficients are omitted for notational simplicity. At the renormalizable level, the above Lagrangian possesses a global $U(1)$ symmetry, which is identified with the global PQ symmetry. The global PQ symmetry corresponds to a phase rotation of a gauge invariant combination, $\phi\phi'^{10}$, while the other fields are rotated appropriately. The global PQ charges of the individual fields are generically given by

$$Q = -\frac{Q_\phi}{10}\times q_5\ ,\quad Q' = Q_{\phi'} - \frac{3}{10}Q_\phi\ , \tag{6.131}$$

for $\{{\rm SM}, \bar{N}_R, \mathbf{5}_K, \bar{\mathbf{5}}\}$ and $\{\mathbf{5}'_K\}$, respectively. Here, q_5 denotes the fivness charge of each field, and $Q_{\phi,\phi'}$ are the global PQ charges of ϕ and ϕ' with $Q_\phi/Q_{\phi'} \neq -10$, respectively.

The global PQ symmetry is broken by the QCD anomaly. In fact, under the global PQ rotation with a rotation angle α_{PQ},

$$\phi\phi'^{10} \to e^{i\alpha_{PQ}}\times\phi\phi'^{10}\ , \tag{6.132}$$

the Lagrangian shifts by,

$$\delta\mathcal{L}_{\not{PQ}} = \frac{\alpha_{PQ}g_a^2}{32\pi^2}F^a\tilde{F}^a\ . \tag{6.133}$$

It should be noted that the normalization factor of Eq. (6.133) is independent of the choice of the global PQ charge assignment for the individual fields.

Since the global PQ symmetry is just an accidental one, it is also broken by the Planck suppressed operators explicitly. However, due to the gauged fiveness symmetry, no PQ-symmetry breaking operators such as ϕ^n or ϕ'^n $(n>0)$ are allowed. As a result, the explicit breaking terms of the global PQ symmetry are highly suppressed, and the lowest dimensional ones are given by,

$$\mathcal{L}_{\not{PQ}} \sim \frac{1}{10!}\frac{\phi\phi'^{10}}{M_{\rm pl}^7} + h.c.\ , \tag{6.134}$$

where $M_{PL}\simeq 2.44\times 10^{18}$ is the reduced Planck scale. As we will see in the next section, the breaking terms are acceptably small not to spoil the PQ mechanism in a certain parameter space.

In the presence of the explicit breaking terms in Eq. (6.134), the QCD vacuum angle is slightly shifted by[36]

$$\mathit{\Delta}\theta \ \sim\ 2\frac{1}{10!}\frac{\langle\phi\rangle\langle\phi'\rangle^{10}}{M_{\rm pl}^7 m_a^2 F_a^2} \sim 3\times10^{-11}\left(\frac{\langle\phi\rangle}{10^{10}\,{\rm GeV}}\right)\left(\frac{\langle\phi'\rangle}{10^{11}\,{\rm GeV}}\right)^{10}\ . \tag{6.135}$$

[36]Hereafter, $\langle\phi\rangle$ and $\langle\phi'\rangle$ denote the absolute values of the VEVs of ϕ and ϕ'.

where m_a denotes the axion mass. Such a small shift should be consistent with the current experimental upper limit on the θ angle, $\theta \lesssim 10^{-10}$ [1].

In Fig. 6.4, we show the constraint on the VEVs of ϕ and ϕ' from the experimental upper limit on $\Delta\theta$. In the gray shaded region, the explicit breaking effect in Eq. (6.135) is too large to be consistent with $\Delta\theta \lesssim 10^{-10}$. The orange lines show the contours of the effective decay constant in Eq. (6.14), which is mainly determined by the smaller one between $\langle\phi\rangle$ and $\langle\phi'\rangle$. The figure shows that the model is consistent with the the experimental upper limit on $\Delta\theta$ for $\langle\phi'\rangle \lesssim 10^{11}$ GeV. As a result, we find that the gauged PQ mechanism based on the fiveness can solve the strong CP problem while satisfying the astrophysical constraint from the observation of supernova 1987A, $F_a \gtrsim 10^8$ GeV [141], and the condition for successful thermal leptogenesis, $M_N = y_N \langle\phi\rangle \gtrsim 10^9$ GeV [138–140].

Several comments are in order. Since ($\bar{\mathbf{5}}_{\rm SM}$,$\bar{\mathbf{5}}_K$,$\bar{\mathbf{5}}'_K$) have identical gauge charges, they are indistinguishable from each other. Once ϕ and ϕ' obtain VEVs in the intermediate scale, 11-flavors of them become mass partners of $\mathbf{5}$'s, and 3-flavors of them remain massless. The SM 3-flavors of $\bar{\mathbf{5}}$ are identified with those massless $\bar{\mathbf{5}}$'s.

It should also be noted that the "inter-sector" interactions via $\bar{\mathbf{5}}$ do not lead to explicit breaking of the global PQ symmetry. To see this, it is the most convenient to choose $Q_\phi = 0$ and $Q_{\phi'} = 1$ (see Eq. (6.131)), which leads to the global PQ charges,

$$\phi'(+1) \ , \quad \mathbf{5}'_K(+1) \ , \tag{6.136}$$

with the charges of $\{\mathrm{SM}, \bar{N}_R, \phi, \mathbf{5}_K, \bar{\mathbf{5}}\}$ vanishing. As the fiveness invariant interactions of $\bar{\mathbf{5}}$ in Eq. (6.130) are also invariant under the global PQ symmetry in Eq. (6.136), no explicit breaking terms are generated from the "inter-sector" interactions.[37]

In the low energy effective theory, the axion couplings to the SM fields are the same with those in the KSVZ axion model except for those to the neutrinos. As $B-L$ is an accidental symmetry of the SM except for the neutrino masses, the current couplings to the axion proportional to the fiveness can be absorbed by the $B-L$ rotation and $U(1)_Y$ rotation. The non-vanishing couplings to the neutrinos can also be understood from the fact that the axion in the present model also plays a role of the Majoron [142] which is obvious in the limit of $\langle\phi'\rangle \gg \langle\phi\rangle$. However, it seems very difficult to test the direct couplings between the axion and the neutrinos in laboratory experiments.

[37] Note that $\phi\phi'^{10}$ is the lowest dimensional operators among all the global PQ breaking operators. In this case, no larger explicit breaking terms are generated by radiative corrections other than the anomalous breaking terms given in Eq. (6.133).

6.4.2 Domain wall problem

Here, let us briefly discuss the domain wall problem and axion dark matter. As discussed in [39], the model suffers from the domain wall problem for $\langle\phi\rangle \gg \langle\phi'\rangle$ when global PQ symmetry breaking takes place after inflation. To avoid the domain wall problem, we assume either one of the following possibilities;

(i) Both phase transitions of $\langle\phi\rangle \neq 0$ and $\langle\phi'\rangle \neq 0$ take place before inflation.

(ii) The phase transition, $\langle\phi'\rangle \neq 0$, takes place before inflation while the transition, $\langle\phi\rangle \neq 0$, occurs after inflation.

The latter possibility is available as the fiveness charges of ϕ and ϕ' are relatively prime and $|q_a| : |q_b| = 10 : 1$.[38]

For the first possibility, the cosmic axion abundance is given by,

$$\Omega_a h^2 \simeq 0.18\,\theta_a^2 \left(\frac{F_a}{10^{12}\,\mathrm{GeV}}\right)^{1.19} , \tag{6.137}$$

for the initial misalignment angle $\theta_a = \mathcal{O}(1)$ [143]. Thus, in the allowed parameter region in Fig. 6.4, i.e., $F_a \lesssim 10^{10}\,\mathrm{GeV}$, relic axion abundance is a subdominant component of dark matter. It should be also noted that the Hubble constant during inflation is required to satisfy,

$$H_{\mathrm{inf}} \lesssim 10^8\,\mathrm{GeV} \times \theta_a^{-1} \left(\frac{F_a}{10^{10}\,\mathrm{GeV}}\right)^{-0.19} . \tag{6.138}$$

to avoid the axion isocurvature problem (see Refs. [93, 94]).[39]

For the second possibility, the cosmic axion abundance is dominated by the one from the decay of the string-domain wall networks [72] (see Refs. [144–146] for more recent up-to-date version),

$$\Omega_a h^2 \simeq 0.035 \pm 0.012 \left(\frac{F_a}{10^{10}\,\mathrm{GeV}}\right)^{1.19} . \tag{6.139}$$

Thus, the relic axion from the string-domain wall network can be the dominant component of dark matter at the corner of the parameter space in Fig. 6.4. To avoid symmetry restoration after inflation, we also require that the maximum temperature during reheating [147],

$$T_{\mathrm{MAX}} \simeq g_*^{-1/8} T_R^{1/2} H_{\mathrm{inf}}^{1/4} M_{PL}^{1/4} , \tag{6.140}$$

[38] The domain wall problem might also be solved for $\langle\phi\rangle \sim \langle\phi'\rangle$ even if both the phase transitions take place after inflation. To confirm this possibility, detailed numerical simulations are required.

[39] Here, we do not assume that the axion is the dominant component of dark matter but use the axion relic abundance in Eq. (6.137) to derive the constraint.

does not exceed $\langle \phi' \rangle$, which leads to

$$H_{\rm inf} \lesssim 5 \times 10^8\,{\rm GeV} \left(\frac{\langle \phi' \rangle}{10^{11}\,{\rm GeV}} \right)^4 \left(\frac{10^9\,{\rm GeV}}{T_R} \right)^2 . \tag{6.141}$$

Here, we use the effective massless degrees of freedom $g_* \simeq 200$, though the condition does not depend on g_* significantly.

6.4.3 Supersymmetric extension

The SUSY extension of the present model is straightforward. The SM matter fields, the right-handed neutrinos, and the extra multiplets are simply extended to corresponding supermultiplets with the same fiveness charges given in Eqs. (6.119) and (6.127). The Higgs doublets are extended to the two Higgs doublet supermultiplets H_u and H_d as in the minimal SUSY Standard Model (MSSM). The fiveness charges are assigned to be $H_u(-2)$ and $H_d(+2)$, respectively. The complex scalars ϕ and ϕ' are also extended to corresponding supermultiplets which are accompanied by supermultiplets with opposite fiveness charges, $\bar{\phi}$ and $\bar{\phi}'$ (see Tab. 6.4).

Under the fiveness symmetry, the superpotential is restricted to[40]

$$\begin{aligned} W &= \mathbf{10}_{\rm SM}\mathbf{10}_{\rm SM}H_u + \mathbf{10}_{\rm SM}\bar{\mathbf{5}}H_d + \bar{\mathbf{5}}\bar{N}_R H_u - \frac{1}{2}\phi\,\bar{N}_R\bar{N}_R + \bar{\phi}\,\mathbf{5}_K\bar{\mathbf{5}} + \bar{\phi}'\,\mathbf{5}'_K\bar{\mathbf{5}} \\ &\quad + X(2\phi\bar{\phi} - v^2) + Y(2\phi'\bar{\phi}' - v'^2) . \end{aligned} \tag{6.142}$$

Here, X and Y are introduced to make ϕ and ϕ' obtain non-vanishing VEVs, which are neutral under fiveness.[41] The coupling coefficients are again omitted for notational simplicity. The SUSY extension again possesses the global PQ symmetry as in the case of the non-SUSY model.

In addition to fiveness, we also assume that a discrete subgroup of $U(1)_R$, $\mathbb{Z}_{NR}$ $(N > 2)$, is an exact discrete gauge symmetry. This assumption is crucial to allow the VEV of the superpotential, and hence, the supersymmetry breaking scale much smaller than the Planck scale.[42] In the following, we take the simplest possibility, $\mathbb{Z}_{4R}$ with the charge assignment given in Tab. 6.4, which is free from $\mathbb{Z}_{4R}$–$SU(5)^2$ anomaly and the gravitational anomaly.[43] It should be noted that the

[40] More generally, the Higgs bi-linear, $H_u H_d$, also couples to X and Y. We assume that the soft masses of the Higgs doublets are positive and larger than those of ϕ's and ϕ''s, so that the Higgs doublets do not obtain VEVs from the couplings to X and Y. We may also restrict those couplings by some symmetry.

[41] See [39] for details of the SUSY extension of the gauged PQ mechanism.

[42] R-symmetry is also relevant for SUSY breaking vacua to be stable [98, 99].

[43] It should be noticed that there is no need to add extra $SU(5)$ singlet fields to cancel the anomalies.

Table 6.4: The charge assignment of the fiveness symmetry and the gauged $\mathbb{Z}_{4R}$ symmetry. Here, we fix the $\mathbb{Z}_{4R}$ charges of the Higgs doublets to 0 which is motivated by pure gravity mediation model [129–131]. An extra multiplet $(\mathbf{5}_E, \bar{\mathbf{5}}_E)$ is introduced to cancel the $\mathbb{Z}_{4R}$–$SU(5)^2$ anomaly [107].

	$\mathbf{10}_{\rm SM}$	$\bar{\mathbf{5}}$	$\bar{N}_R$	H_u	H_d
fiveness	+1	−3	+5	−2	+2
$\mathbb{Z}_{4R}$	+1	+1	+1	0	0

	$\mathbf{5}_K$	$\mathbf{5}'_K$	ϕ	$\bar{\phi}$	ϕ'	$\bar{\phi}'$	X	Y
fiveness	−7	+4	−10	+10	+1	−1	0	0
$\mathbb{Z}_{4R}$	+1	+1	0	0	0	0	+2	+2

	$\mathbf{5}_E$	$\bar{\mathbf{5}}_E$
fiveness	+3	−3
$\mathbb{Z}_{4R}$	−1	+1

mixed anomalies of $\mathbb{Z}_{4R}$ and fiveness do not put constraints on charges since they depend on the normalization of the heavy spectrum [100–106, 108–110].[44]

Under fiveness and the gauged $\mathbb{Z}_{4R}$ symmetry, the lowest dimensional operators which break the global PQ symmetry are given by,

$$W_{\not{PQ}} = \frac{m_{3/2}}{10!M_{\rm pl}}\frac{\phi\phi'^{10}}{M_{\rm pl}^8} + \frac{m_{3/2}}{10!M_{\rm pl}}\frac{\bar{\phi}\bar{\phi}'^{10}}{M_{\rm pl}^8}\ . \tag{6.143}$$

It should be noted that a lower dimensional PQ breaking term, $\bar{\phi}'^5\bar{N}_R$, is forbidden by the $\mathbb{Z}_{4R}$ symmetry. The above superpotential contributes to the shift of the QCD vacuum angle mainly through the scalar potential,

$$\mathcal{L}_{\not{PQ}} \sim \frac{8m_{3/2}^2}{10!M_{\rm pl}}\frac{\phi\phi'^{10}}{M_{\rm pl}^8} + \frac{8m_{3/2}^2}{10!M_{\rm pl}}\frac{\bar{\phi}\bar{\phi}'^{10}}{M_{\rm pl}^8} + h.c.\ , \tag{6.144}$$

where $m_{3/2}$ denotes the gravitino mass. Compared with Eq. (6.134), the explicit breaking is suppressed by a factor of $(m_{3/2}/M_{\rm pl})^2$. Accordingly, the shift of the QCD vacuum angle is given by,

$$\Delta\theta \sim 2\frac{1}{10!}\frac{8m_{3/2}^2\langle\phi\rangle\langle\phi'\rangle^{10}}{M_{\rm pl}^9 m_a^2 F_a^2} \tag{6.145}$$

$$\sim 10^{-25}\left(\frac{m_{3/2}}{100\,\text{TeV}}\right)^2\left(\frac{\langle\phi\rangle}{10^{11}\,\text{GeV}}\right)\left(\frac{\langle\phi'\rangle}{10^{12}\,\text{GeV}}\right)^{10}\ , \tag{6.146}$$

where we assume $\langle\phi\rangle = \langle\bar{\phi}\rangle$ and $\langle\phi'\rangle = \langle\bar{\phi}'\rangle$ for simplicity.[45]

[44] GUT models consistent with the $\mathbb{Z}_{4R}$ symmetry are discussed in, e.g., [111, 112].
[45] The following argument can be easily extended to the cases with $\langle\phi\rangle \neq \langle\bar{\phi}\rangle$ and $\langle\phi'\rangle \neq \langle\bar{\phi}'\rangle$.

In Fig. 6.5, we show the constraints on the VEVs of ϕ and ϕ' from the experimental upper limit on $\Delta\theta$. Here, we take the gravitino mass, $m_{3/2} \simeq 100\,\text{TeV}$, which is favored to avoid the cosmological gravitino problem for $T_R \gtrsim 10^9$ GeV [148–150]. For $m_{3/2} \simeq 100\,\text{TeV}$, the scalar partner and the fermionic partner of the axion also do not cause cosmological problems as they obtain the masses of the order of the gravitino mass and decay rather fast [151]. In the figure, the gray shaded region is excluded by the constraint on $\Delta\theta \lesssim 10^{-10}$. Due to the suppression of the breaking term in Eq. (6.144), the higher value of $\langle\phi'\rangle$ is allowed compared with the non-SUSY model. The higher $\langle\phi'\rangle$ is advantageous to avoid symmetry restoration after inflation (see Eq. (6.141)), with which the domain wall problem is avoided in the possibility (ii) (see section 6.4.2). Accordingly, the decay constant can also be as high as about 10^{11-12} GeV, which also allows the axion to be the dominant dark matter component (see Eq. (6.139)). Therefore, we find that the SUSY extension of the model is more successful.[46]

It should be noted that the 11-flavors of extra multiplets at the intermediate scale make the renormalization group running of the MSSM gauge coupling constants asymptotic non-free. Thus, the masses of them are bounded from below so that perturbative unification is achieved. In the figure, the gray shaded lower region shows the contour of the renormalization scale M_* at which at least one of $g_{1,2,3}$ becomes 4π. Here, we use the one-loop renormalization group equations assuming that the extra quarks obtain masses of $\langle\phi\rangle$ and $\langle\phi'\rangle$, respectively.[47] The result shows that the perturbative unification can be easily achieved for $\langle\phi'\rangle \gtrsim 10^{9-10}$ GeV even in the presence of 11-flavors of the extra multiplets.

[46] As in [39], we will discuss a possibility where SUSY and $B-L$ are broken simultaneously elsewhere.

[47] The masses of the sfermions, the heavy charged/neutral Higgs boson, the Higgsinos, and $(\mathbf{5}_E, \bar{\mathbf{5}}_E)$ are at the gravitino mass scale, $m_{3/2} \simeq 100$–$1000\,\text{TeV}$. The gaugino masses are, on the other hand, assumed to be in the TeV scale as expected by anomaly mediation [126, 127]. This is motivated by the pure gravity mediation model in [129–131] (see also Refs. [132–135] for similar models), where the Higgsino mass is generated from the R-symmetry breaking [121].

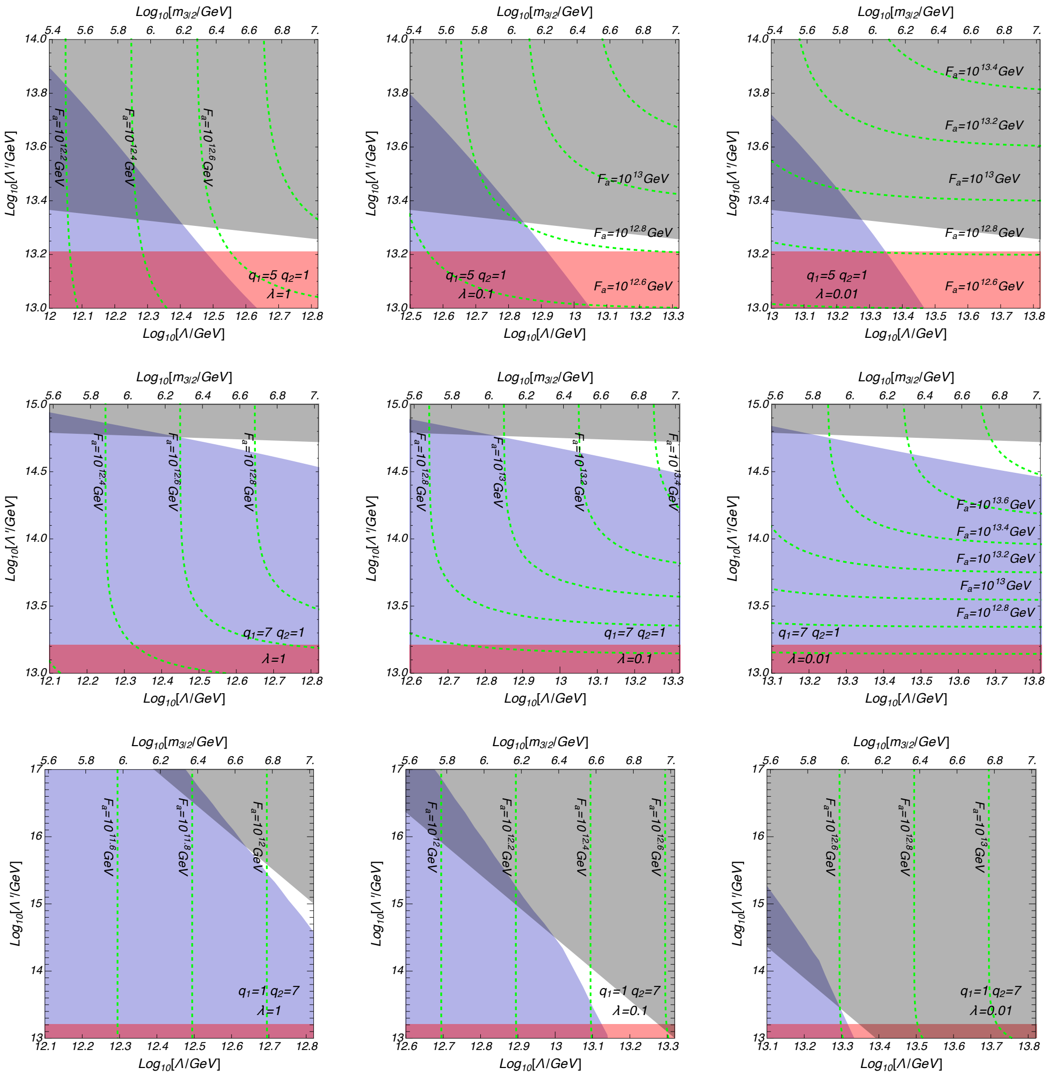

Figure 6.3: *The constraints on the parameter reions for given PQ charges, q_1 and q_2. The gray region is excluded as $\theta_{\rm eff} < 10^{-10}$ is not satisfied. The perturbative unification is not achieved in the blue region. The red regions are excluded by $m'_{KSVZ} \gtrsim 750\,\text{GeV}$. The green lines are the contours of the effective decay constant F_a.*

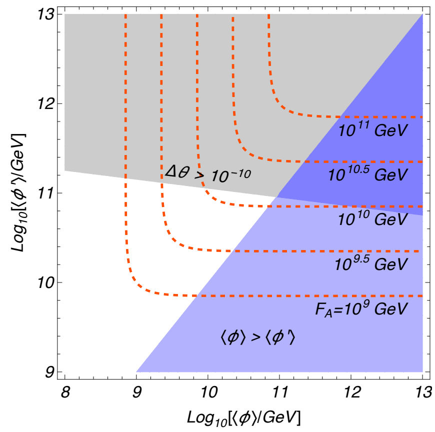

Figure 6.4: *The constraint on the VEVs of ϕ and ϕ'. The gray shaded region is excluded by $\Delta\theta < 10^{-10}$ for the non-SUSY model (see Eq. (6.135)). The orange lines are the contours of the effective decay constant F_a. In the blue shaded region, $\langle\phi\rangle > \langle\phi'\rangle$.*

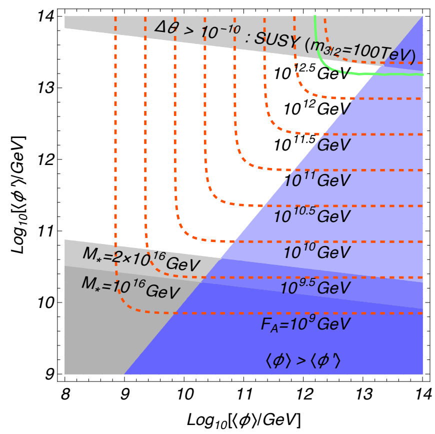

Figure 6.5: *The constraint on the VEVs of ϕ and ϕ' for the SUSY extension. The gray shaded upper region is excluded for the SUSY model with $m_{3/2} = 100\,\mathrm{TeV}$ (see Eq. (6.145)). The orange lines are the contours of the effective decay constant F_a. In the blue shaded region, $\langle\phi\rangle > \langle\phi'\rangle$. The gray shaded lower regions are excluded as the gauge coupling constants become non-perturbative below the GUT scale. The thin green region is excluded by the Axion Dark Matter eXperiment (ADMX) [65] where the dark matter density is assumed to be dominated by the relic axion.*

Chapter 7

Conclusion

As we have reviewed in Chap. 2, the Strong CP problem is one of the longstanding problems in particle physics. In the SM, the so-called θ-parameter associated with the violation of CP symmetry exists. The current measurement of the neutron electric dipole moment sets a limit on the parameter, $\theta \lesssim 10^{-10}$ [1]. Why is the θ so small? This is the Strong CP problem.

In Chap. 3, we have reviewed an elegant solution to the problem, the so-called Peccei-Quinn mechanism [2, 43–45]. There, one introduces the Peccei-Quinn symmetry which is almost exact but explicitly broken by the QCD anomaly.

In spite of the requirement of the exactness of the global Peccei-Quinn symmetry, it is widely discussed that any global symmetries are broken by quantum gravity effect as we have reviewed in Chap. 5. Unfortunately, if one takes such effect into account, the Peccei-Quinn mechanism does not work well to solve the Strong CP problem. One simple way to overcome this issue is by introducing the gauge symmetry. Actually, in the literatures, there are many attempts by using the (discrete) gauge symmetry [10, 14–28].

In this thesis, we have proposed a new general mechanism to provide an almost exact accidental Peccei-Quinn symmetry by using an abelian gauge symmetry and discussed its applications to models beyond the SM in Chap. 6.

In Sec. 6.1, we have shown our mechanism. Concretely, we have first introduced two sectors with apparent independent Peccei-Quinn symmetries. There, an anomaly free combination of two Peccei-Quinn symmetries always exists. We can gauge it, and the gauge symmetry can work well to protect the global Peccei-Quinn symmetry. We call the gauge symmetry as the gauged Peccei-Quinn symmetry and call the mechanism as the gauged Peccei-Quinn mechanism [13]. Our mechanism can obviously be applied to a wide range of axion models, *e.g.* the KSVZ model and the composite axion model.

For further application of the mechanism, we have first worked on the model, where the Peccei-Quinn symmetry is broken with the dynamical supersymmetry breaking simultaneously as we have discussed in Sec. 6.3. Following the gauged Peccei-Quinn mechanism, we have introduced a new sector where the apparent global Peccei-Quinn symmetry is broken dynamically and discussed the phenomenological and the theoretical constraints on the model [39]. In consideration of the cosmology in this model, we have noticed a general cosmological problem due to extra SM gauge singlet fermions added to cancel the self-triangle anomaly of the gauged PQ symmetry. Such singlets tend to become light and stable due to their charges of the gauged Peccei-Quinn symmetry, and thus result in *e.g.* an unacceptably large number of effective neutrino species if they are produced too much by *e.g.* the decay from the inflation or the gauge boson of the gauged Peccei-Quinn symmetry.

To resolve the above cosmological issue, we have constructed a model without un-wanted SM gauge singlets fermions in Sec. 6.4. Concretely, we have searched for such solutions by applying our mechanism, where the gauged Peccei-Quinn mechanism is identified as the $B-L$ gauge symmetry [40]. There, we have actually found a model without unwanted singlets. In addition, in this model, most of the issues such as the dark matter, the finiteness of neutrino masses, the baryon asymmetry, and the hierarchy problem (for the supersymmetric version) are also solved simultaneously. Furthermore, the domain wall problem can be avoided even if the Peccei-Quinn symmetry is broken after inflation end (See also Sec. 6.2). Because of these advantages and the simplicity in the extension of the SM, this model may serve as a prime candidate to be embedded in a further unified model with some GUT gauge group.

Acknowledgement

The author is especially grateful to his supervisor Masahiro Ibe for various instructive discussions, stimulating suggestions and collaborations. He also thanks Tsutomu Yanagida for fruitful discussion and collaborations, especially his insightful comments were invaluable. He also thanks Hajime Fukuda for various instructive discussions and collaborations. He also thanks Izumi Tsutsui, Koichi Hamaguchi, Masahiro Kawasaki, Masatake Ohashi, Kimihiro Okumura for judging his doctoral thesis examination. Finally, he also would like to express his gratitude to Masahiro Kawasaki and all the members of theory group at Institute for Cosmic Ray Research University of Tokyo and to his family for their hospitality and nontrivial supports.

This work is supported in part by a Research Fellowship for Young Scientists from the Japan Society for the Promotion of Science (JSPS).

Appendix A

More details about $U(1)$ problem

In this Appendix, let us discuss the predicted light meson mass comparable to the neutral pion mass.

Consider the QCD theory, where the mass term of the $u-, d-, s-$quarks are neglected, and thus the Lagrangian possesses $SU(3)_L \times SU(3)_R$ chiral symmetry. This chiral symmetry is spontaneously broken to $SU(3)$ by the condensation of the quarks, and then the eight Nambu-Goldstone (NG) boson will emerge in the low energy theory. Here, let us start from the chiral perturbation theory with the matrix U including the eight NG bosons,

$$U = \exp\left(i\frac{\mathcal{M}}{F_\pi}\right), \tag{A.1}$$

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & K^0 \\ \bar{K}^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta^0 \end{pmatrix}, \tag{A.2}$$

where $\mathcal{M}$ is a Hermitian matrix, $\pi^0,\ \pi^\pm,\ K^0,\ K^\pm,\ \eta$ are the scalar fields of the neutral pion, the charged pion, the neutral kaon, the charged kaon and eta meson, respectively. The F_π is a free parameter determined as $F_\pi \simeq 93\,\mathrm{MeV}$ by calculating the pion decay *e.g.* $\pi^+ \to \mu^+\nu_\mu$ in chiral perturbation theory. The matrix U is transformed under the $SU(3)_L \times SU(3)_R$ chiral symmetry by,

$$U \to RUL^\dagger, \tag{A.3}$$

where $R,\ L$ denotes the chiral rotation by the $SU(3)_R,\ SU(3)_L$, respectively. The chirally invariant canonical kinetic term is

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial U^\dagger\right]. \tag{A.4}$$

Actually, the $SU(3)_L \times SU(3)_R$ symmetry is explicitly broken by the quark mass term, and thus it leads to the masses of the eight Goldstone bosons. Concretely, the quark mass term is written by

$$\mathcal{L} = -\bar{q}_R M_q q_L + h.c.,\ M_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \tag{A.5}$$

where $m_u,\ m_d,\ m_s$ are the mass parameters of the up, down, strange quarks, respectively.[1] Because this Lagrangian would be invariant if M_q is transformed by

$$M_q \to R M_q L^\dagger, \tag{A.6}$$

the mass for the NG bosons below the condensation scale comes from

$$\mathcal{L} = \frac{F_\pi^2 B_0}{2} \mathrm{Tr}(M_q U^\dagger + U M_q^\dagger), \tag{A.7}$$

where B_0 is a parameter determined soon. Then, the meson mass terms are given by

$$\begin{aligned} -\frac{B_0}{2}\mathrm{Tr}\left[M_q \mathcal{M}^2\right] &= -\frac{B_0}{2}\left((m_u+m_d)\pi^0\pi^0 + 2(m_u+m_d)\pi^+\pi^-\right. && \text{(A.8)} \\ &+ 2(m_d+m_s)K^0\bar{K}^0 + 2(m_u+m_s)K^+K^- && \text{(A.9)} \\ &+ \left.\frac{2}{\sqrt{3}}(m_u-m_d)\pi^0\eta + \frac{m_u+m_d+4m_s}{3}\eta^2\right). && \text{(A.10)} \end{aligned}$$

Ignoring the mass mixing of the $\pi^0 - \eta$, we obtain the lowest order mass formula,

$$\begin{aligned} m_\pi^2 \equiv m_{\pi^0}^2 = m_{\pi^+}^2 &= B_0(m_u+m_d), && \text{(A.11)} \\ m_{K^0}^2 &= B_0(m_d+m_s), && \text{(A.12)} \\ m_{K^+}^2 &= B_0(m_u+m_s), && \text{(A.13)} \\ m_{\eta^0}^2 &= B_0\left(\frac{m_u+m_d+4m_s}{3}\right). && \text{(A.14)} \end{aligned}$$

From these mass squared, we obtain some formula about the quark mass ratio *e.g.*

$$\begin{aligned} \frac{m_d-m_u}{m_d+m_u} &= \frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_{\pi^0}^2} && \text{(A.15)} \\ &\simeq 0.3. && \text{(A.16)} \end{aligned}$$

In the second line, we subsitute $m_{K^0} = 498\,\mathrm{MeV}$, $m_{K^+} = 494\,\mathrm{MeV}$, $m_{\pi^0} = 135\,\mathrm{MeV}$ and $m_{\pi^+} \simeq 139\,\mathrm{MeV}$.

[1] We take these mass parameters as real and positive ones for simplicity.

By the way, in the three quarks massless limit, the $U(1)_A$ symmetry in Eq. (2.1) seems to be also the symmetry. Therefore, $\mathcal{M}$ includes another NG boson η' in the effective Lagrangian,

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & K^0 \\ \bar{K}^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta^0 \end{pmatrix} + \frac{F_\pi}{\sqrt{3}F_{\eta'}} \begin{pmatrix} \eta' & 0 & 0 \\ 0 & \eta' & 0 \\ 0 & 0 & \eta' \end{pmatrix} \tag{A.17}$$

where $F_{\eta'}$ is the unknown coefficient, but naively expected $F_{\eta'} \simeq F_\pi$ in QCD. The mass term of the $\pi^0,\ \eta^0,\ \eta'$ is obtained from the formula in Eq. (A.8) as

$$\mathcal{L} = -B_0 \left[m_u \left(\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 + \frac{F_\pi}{\sqrt{3}F_{\eta'}}\eta' \right)^2 \right. \tag{A.18}$$

$$+ \quad m_d \left(-\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 + \frac{F_\pi}{\sqrt{3}F_{\eta'}}\eta' \right)^2 \tag{A.19}$$

$$\left. + \quad m_s \left(-\sqrt{\frac{2}{3}}\eta^0 + \frac{F_\pi}{\sqrt{3}F_{\eta'}}\eta' \right)^2 \right]. \tag{A.20}$$

Then, the mass matrix of these mesons is

$$M^2_{\text{neutral}} = 2B_0 \begin{pmatrix} \frac{m_u+m_d}{2} & \frac{m_u-m_d}{2\sqrt{3}} & \frac{(m_u-m_d)F_\pi}{2\sqrt{3}F_{\eta'}} \\ \frac{m_u-m_d}{2\sqrt{3}} & \frac{m_u+m_d+4m_s}{6} & \frac{(m_u+m_d-2m_s)F_\pi}{3\sqrt{2}F_{\eta'}} \\ \frac{(m_u-m_d)F_\pi}{\sqrt{6}F_{\eta'}} & \frac{(m_u+m_d-2m_s)F_\pi}{3\sqrt{2}F_{\eta'}} & \frac{(m_u+m_d+m_s)F_\pi^2}{\sqrt{3}F_{\eta'}^2} \end{pmatrix}. \tag{A.21}$$

In the limit of $m_u,\ m_d \to 0$, there are two massless modes apparently. More concretely, the three eigenvalues in that limit are

$$\left(0, 0, \frac{2B_0(2F_{\eta'}^2 + F_\pi^2)m_s}{3F_{\eta'}^2} \right). \tag{A.22}$$

$$\tag{A.23}$$

Then, its eigenvectors are given as

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\ v_2 = \frac{1}{\sqrt{F_\pi^2 + 2F_{\eta'}^2}} \begin{pmatrix} 0 \\ F_\pi \\ \sqrt{2}F_{\eta'} \end{pmatrix},\ v_3 = \frac{1}{\sqrt{2F_\pi^2 + F_{\eta'}^2}} \begin{pmatrix} 0 \\ -\sqrt{2}F_{\eta'} \\ F_\pi \end{pmatrix} \tag{A.24}$$

respectively. In this basis, the meson mass squared matrix to the first order in $m_u,\ m_d$ corresponding to the massless mode is

$$\begin{pmatrix} v_1^T M_{\text{neutral}} v_1 & v_1^T M_{\text{neutral}} v_2 \\ v_2^T M_{\text{neutral}} v_1 & v_2^T M_{\text{neutral}} v_2 \end{pmatrix} = B_0 \begin{pmatrix} (m_u + m_d) & \frac{\sqrt{3}(m_u-m_d)F_\pi}{\sqrt{2F_{\eta'}^2+F_\pi^2}} \\ \frac{\sqrt{3}(m_u-m_d)F_\pi}{\sqrt{2F_{\eta'}^2+F_\pi^2}} & \frac{3(m_u+m_d)F_\pi^2}{2F_{\eta'}^2+F_\pi^2} \end{pmatrix}. \tag{A.25}$$

There, the off-diagonal term can be negligible due to the cancellation of u, d-quark masses. Thus, the component of $(1,\ 1)$ roughly corresponds to the mass squared of the neutral pion which is also obtained in Eq. (A.8),

$$m_\pi^2 \simeq B_0(m_u + m_d). \tag{A.26}$$

The mass of the extra meson is upper-bounded by

$$m_{\text{extra}} \simeq \frac{\sqrt{3} m_\pi F_\pi}{\sqrt{F_\pi^2 + 2F_{\eta'}^2}} \leq \sqrt{3} m_\pi. \tag{A.27}$$

Therefore, the spontaneously broken down $U(1)_A$ demands a pseudo-NG boson with mass less than $\sqrt{3}m_\pi$ although there is no observation of such meson. This is the $U(1)$ problem.

Appendix B

Instanton solution

In this section, let us discuss the instanton solution of the pure $SU(2)$ gauge theory in Euclidean space. The action is,

$$S_E = \int d^4x \frac{1}{2}\mathrm{tr}(F^{\mu\nu}F_{\mu\nu}). \tag{B.1}$$

Here, we use the $SU(2)$ gauge field and its field strength as

$$A_\mu = \frac{\tau^a}{2}A^a_\mu,\ F_{\mu\nu} = \frac{\tau^a}{2}F^a_{\mu\nu}, \tag{B.2}$$

with the Pauli-matrices τ^a $(a = 1-3)$. To make the action finite, we assume the boundary condition,

$$F_{\mu\nu} \to 0 \text{ for } |x| \to \infty, \tag{B.3}$$

where $|x|$ denotes the norm in the Euclidean space. This condition shows that the gauge field becomes a pure gauge at $|x| \to \infty$ and the gauge field,

$$A_\mu \to U(\hat{x})^{-1}\partial_\mu U(\hat{x}), \tag{B.4}$$

where $\hat{x}$ is the unit vector of x. U is some function of $\hat{x}$, which can be determined soon. U^{-1} is the inverse of the U. Note that the $U(\hat{x})$ shows the mapping from the S^3 to the $SU(2)$ space. This implies that there is some topological winding number by the mapping from S^3 to S^3 as $SU(2)$ space.

To obtain the concrete solution of U, let us find the condition that the action is minimized. For this purpose, the following relation is useful,

$$\int d^4x\mathrm{tr}(F_{\mu\nu} \pm \tilde{F}_{\mu\nu})^2 \geq 0, \tag{B.5}$$

where $\tilde{F}_{\mu\nu}$ is the dual of the field strength. Using,

$$(F_{\mu\nu} \pm \tilde{F}_{\mu\nu})^2 = 2(F_{\mu\nu}F_{\mu\nu} \pm F_{\mu\nu}\tilde{F}_{\mu\nu}), \tag{B.6}$$

the above condition in Eq. (B.5) leads to the inequality,

$$\int d^4x \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \geq \left| \int d^4x \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \right|. \tag{B.7}$$

The term on the right-hand side can be re-written by using,

$$K^\mu = 4\epsilon^{\mu\nu\lambda\rho}\,\mathrm{tr}\left[A_\nu \partial_\lambda A_\rho + \frac{2}{3}A_\nu A_\lambda A_\rho\right], \tag{B.8}$$

as,

$$\int d^4x\,\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) = \frac{1}{2}\int d^4x\,\partial_\mu K^\mu = \frac{1}{2}\int d\sigma^\mu K_\mu. \tag{B.9}$$

The last integral denotes the surface integral. The K_μ is so-called the Chern-Simons current, and thus the total derivative term can be written by the topological winding number, n_W, of the mapping from S^3 to S^3 as,

$$\frac{1}{16\pi^2}\int d^4x\,\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) = n_W, \tag{B.10}$$

where n is an integer. Then the inequality in Eq. (B.7) is

$$\int d^4x \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \geq \left| \int d^4x \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \right| = 16\pi^2|n_W| \tag{B.11}$$

This implies that the action is minimized when

$$F_{\mu\nu} = \pm\tilde{F}_{\mu\nu}. \tag{B.12}$$

The solution for satisfying the condition in Eq. (B.12) has been found by Belavin *et.al.* in 1975 [152] as

$$U = \frac{x_0 + i\sum_{a=1}^{3} \mathrm{x}^a \tau^a}{x_0^2 + \mathbf{x}^2}, \tag{B.13}$$

where x^0 is one component in the Euclidean coordinate, and the other three component is denoted as x^a $(a = 1, 2, 3)$. This is the instanton solution, which gives the $n = 1$ winding number.

Appendix C

Physical interpretation of instanton

To obtain the physical interpretation of the instanton solutions, let us place our system inside a box. Then, the vacuum condition is

$$F^a_{\mu\nu} = 0, \tag{C.1}$$

where this condition is taken outside very large volume and the time of the box. We take the gauge fixing condition,

$$A_0(x) = 0 \tag{C.2}$$

where the A_0 is the time component of the $SU(2)$ gauge field. Under this gauge fixing condition, the gauge transformation of the A_0 by using the $SU(2)$ unitary matrix, $U(x)$,

$$A_0(x) \to A'_0(x) = U^{-1}(x)\partial_0 U(x), \tag{C.3}$$

induces $U^{-1}(x)\partial_0 U(x) = 0$, equivalently $\partial_0 U(x) = 0$. Thus, the vacuum is described by the time-independent space components of the gauge field,

$$A_i(\mathbf{x}) = U^{-1}(\mathbf{x})\partial_i U(\mathbf{x}) \tag{C.4}$$

where $\mathbf{x}$ denotes the label of the space in the three dimensions. To take the remaining gauge freedom of the $U(\mathbf{x})$ as $U(\mathbf{x}) = 1$ at initial time $t \to \infty$, we obtain

$$A_i(\mathbf{x}) = 0. \tag{C.5}$$

From the vacuum condition, this implies

$$0 = F_{0i} = \partial_0 A_i(\mathbf{x}) \tag{C.6}$$

which leads to $A_i(\mathbf{x}) = 0$ throughout the vacuum. The solution under the above condition corresponds to the mapping from the $SU(2)$ gauge group manifold to the $3+1$ space-time with infinities identified as a vacuum. The solution is nothing but the instanton solution discussed in Appendix B. To be more specific, the winding number can be written by,

$$\begin{aligned} n_W &= \int d^4x \partial_\mu K^\mu = \int d^3x (K_0|_{t=\infty} - K_0|_{t=\infty}) && \text{(C.7)} \\ &\equiv \mathcal{K}(t=+\infty) - \mathcal{K}(t=-\infty) \equiv n_+ - n_-, && \text{(C.8)} \end{aligned}$$

where the $\mathcal{K}$ shows the spatial winding number (the pontryagin number) for U,

$$\mathcal{K} = \frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} \, \mathrm{tr}[(U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U)], \tag{C.9}$$

which gives some integer, and $n_+ = \mathcal{K}(t=+\infty)$, $n_- = \mathcal{K}(t=-\infty)$. Therefore, the instanton is the quantum tunneling from the state with n_- to the state with n_+. Notice that the $n_\pm$ are not gauge invariant, but the n is the gauge invariant.

Appendix D

θ-vacuum

As noticed in the previous section, the spatial winding number is changed by the gauge transformation, and thus the true vacuum is expected as a superposition of the n vacuum,

$$|\theta\rangle = \sum_n e^{-in\theta}|n\rangle, \tag{D.1}$$

where $|n\rangle$ denotes the state with the spatial winding number n, and $|\theta\rangle$ is the state which is gauge invariant under the transformation of $n \to n + n_W$. This is called θ-vacuum. The vacuum to vacuum transition amplitude should be given by the Euclidean path integral formalism[1],

$$\begin{aligned} \langle\theta'|e^{-Ht}|\theta\rangle &= \Sigma_{n,m} e^{i(m\theta'-n\theta)}\langle m|e^{-Ht}|n\rangle & \text{(D.2)} \\ &= \Sigma_{m,\nu} e^{im(\theta'-\theta)} \int \mathcal{D}A_\nu \exp[-i\,\nu\,\theta - \int \mathcal{L}\,] & \text{(D.3)} \\ &= \delta(\theta'-\theta)\Sigma_\nu \int \mathcal{D}A_\nu \exp[-i\,\nu\,\theta - \int \mathcal{L}\,] & \text{(D.4)} \end{aligned}$$

where $\int \mathcal{L}$ is *e.g.* the action of $SU(2)$ pure gauge theory. There, ν can be written by the total derivative term in Eq. (B.10).

[1] We omit the normalization factor for simplicity.

Appendix E

Axion-photon coupling

In this section, we calculate the axion-photon coupling. There, the axion mixing with the neutral pion contributes to the axion-photon coupling as well as the triangle anomaly between the PQ symmetry and the electromagnetic gauge symmetry $PQ - U(1)_{\rm em} - U(1)_{\rm em}$.

E.1 Axion-photon coupling in KSVZ model

Let us consider the KSVZ model with one flavor of the KSVZ quark, *i.e* a pair of the fundamental **3** and (anti-)fundamental $\bar{\mathbf{3}}$ quarks in $SU(3)_c$. The KSVZ quarks couple with the complex scalar field ϕ,

$$\mathcal{L} = -\phi \mathbf{3}\bar{\mathbf{3}}, \tag{E.1}$$

where the flavor indices and the coupling constant are omitted for simplicity. The ϕ spontaneously breaks the PQ symmetry by its VEV v_{PQ},

$$\phi = \frac{v_{PQ}}{\sqrt{2}} \exp(ia/v_{PQ}) \tag{E.2}$$

where we only focus on the phase direction and a is the axion. By the transformation of the KSVZ quarks,

$$\mathbf{3} \to \exp(-ia/v_{PQ})\mathbf{3}, \tag{E.3}$$

the axion-gluon coupling is obtained

$$\mathcal{L} = \frac{g^2}{32\pi^2}\frac{a}{v_{PQ}} G\tilde{G}. \tag{E.4}$$

At this stage, let integrate out the heavy quarks except for the up and down quarks, and then redefine the axion a as $\tilde{a}$ to cancel the θ-term. On the other word, only the term,

$$\mathcal{L} = \frac{g^2}{32\pi^2}\frac{\tilde{a}}{v_{PQ}}G\tilde{G}, \tag{E.5}$$

is proportional to $G\tilde{G}$ now. In the following, we label the redefined axion $\tilde{a}$ just as a for simplicity. Let us transform the up and down quarks to erase the axion-gluon coupling,

$$u_R \to \exp(ic_u a/v_{PQ})u_R \quad , \quad d_R \to \exp(ic_d a/v_{PQ})d_R, \tag{E.6}$$

which produces the axion-photon coupling,

$$\mathcal{L} = -\frac{g_{\rm em}^2}{16\pi^2}\frac{a}{v_{PQ}}3\left[c_u\left(\frac{2}{3}\right)^2 + c_d\left(\frac{1}{3}\right)^2\right]F\tilde{F}. \tag{E.7}$$

Below the confinement scale, the axion and pion Lagrangian is as in Eq. (3.22),

$$\mathcal{L} = \frac{1}{2}(\partial a)^2 + \frac{1}{2}(\partial\pi^0)^2 \tag{E.8}$$

$$- m_u B_0 F_\pi^2 \cos\left(c_u\frac{a}{v} + \frac{\pi^0}{F_\pi}\right) - m_d B_0 F_\pi^2 \cos\left(c_d\frac{a}{v} - \frac{\pi^0}{F_\pi}\right). \tag{E.9}$$

The neutral pion-photon coupling from the chiral anomaly of $SU(2)_A$ is

$$\mathcal{L} = \frac{g_{\rm em}^2}{16\pi^2}\frac{\pi^0}{F_\pi}6\left[\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right]F\tilde{F}. \tag{E.10}$$

The neutral pion mixes with the axion, and after the diagonalization of the mass matrix,

$$\pi_{\rm phs} \simeq \pi^0 + \frac{1}{2}\left(\frac{-m_u c_u + m_d c_d}{m_u + m_d}\right)\frac{F_\pi}{v_{PQ}}a \tag{E.11}$$

$$a_{\rm phys} \simeq a. \tag{E.12}$$

Eventually, the axion-photon coupling is given by

$$\mathcal{L} = -\frac{g_{\rm em}^2}{16\pi^2}\frac{a_{\rm phys}}{v_{PQ}}3\left[c_u\left(\frac{2}{3}\right)^2 + c_d\left(\frac{1}{3}\right)^2\right. \tag{E.13}$$

$$+ \left.\left(\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right)\left(\frac{-m_u c_u + m_d c_d}{m_u + m_d}\right)\right]F\tilde{F} \tag{E.14}$$

$$= -\frac{g_{\rm em}^2}{32\pi^2}\frac{a_{\rm phys}}{v_{PQ}}3 \tag{E.15}$$

$$\times \left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 - \left(\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right)\left(\frac{m_u - m_d}{m_u + m_d}\right)\right]F\tilde{F} \tag{E.16}$$

$$= -\frac{g_{\rm em}^2}{32\pi^2}\frac{a_{\rm phys}}{v_{PQ}}\frac{2}{3}\frac{m_u + 4m_d}{(m_u + m_d)}F\tilde{F} \tag{E.17}$$

$$= -\frac{g_{\rm em}^2}{32\pi^2}\frac{2}{3}\frac{m_u + 4m_d}{m_u + m_d}\frac{m_u + m_d}{\sqrt{m_u m_d}}\frac{m_a}{m_\pi F_\pi}a_{\rm phys}F\tilde{F} \tag{E.18}$$

$$= -\frac{g_{\rm em}^2}{32\pi^2}\frac{2}{3}\frac{z + 4}{z + 1}\frac{z + 1}{\sqrt{z}}\frac{m_a}{m_\pi F_\pi}a_{\rm phys}F\tilde{F} \tag{E.19}$$

Here, we took $c_u = c_d = 1/2$ to erase the kinetic mixing between the axion and the pion below the confinement scale. In the fourth line, the KSVZ axion mass in Eq. (3.39) is used. This result is equivalent to $E = 0$ in Eq. (3.41) for

Before going to the ZDFS model, let us consider the KSVZ model with N_f flavor of a fundamental $\mathbf{5}$ and an anti-fundamental $\bar{\mathbf{5}}$ representations in $SU(5)$ GUT. These KSVZ quarks obtain their masses by the coupling with the PQ breaking scalar field ϕ as in Eq. (E.1), *i.e.*

$$\mathcal{L} = -\phi\mathbf{5}\bar{\mathbf{5}}, \tag{E.20}$$

where flavor indices are omitted for simplicity. The ϕ gets the VEV as $\phi = v_{PQ}/\sqrt{2}\exp(ia/v_{PQ})$. By the transformation,

$$\mathbf{5} \to \exp(-ia/v_{PQ})\mathbf{5}, \tag{E.21}$$

the axion-gluon and the axion-photon coupling is obtained,

$$\mathcal{L} = \frac{g^2}{32\pi^2}N_f\frac{a}{v_{PQ}}G\tilde{G} + \frac{g_{\rm em}^2}{16\pi^2}N_f\left[3\left(\frac{1}{3}\right)^2 + (+1)^2\right]\frac{a}{v_{PQ}}F\tilde{F} \tag{E.22}$$

$$= \frac{g^2}{32\pi^2}\frac{a}{F_a}G\tilde{G} + \frac{g_{\rm em}^2}{16\pi^2}\left[3\left(\frac{1}{3}\right)^2 + (+1)^2\right]\frac{a}{F_a}F\tilde{F}. \tag{E.23}$$

In the second line, we define $F_a \equiv v_{PQ}/N_f$. Then, we repeat the discussion below Eq. (E.4). Eventually, the axion-photon coupling is

$$\mathcal{L} = \frac{g_{\rm em}^2}{32\pi^2}\left[\frac{8}{3} - \frac{2}{3}\frac{z + 4}{z + 1}\right]\frac{z + 1}{\sqrt{z}}\frac{m_a}{m_\pi F_\pi}a_{\rm phys}F\tilde{F}. \tag{E.24}$$

This result is equivalent to $E = 8N_f/3,\ N = N_f$ in Eq. (3.41).

E.2 Axion-photon coupling in ZDFS model

From Eq. (E.25), the axion component is in the neutral component of two Higgs doublets,

$$H_u^0 = \frac{v_u}{\sqrt{2}}\exp\left(i\frac{2x}{x+x^{-1}}\frac{a}{v'}\right),\ H_d^0 = \frac{v_d}{\sqrt{2}}\exp\left(i\frac{2x^{-1}}{x+x^{-1}}\frac{a}{v'}\right), \phi = \frac{v_{\tilde{\rm PQ}}}{\sqrt{2}}\exp\left(i\frac{a}{v'}\right) \tag{E.25}$$

After the decoupling of heavy quarks and leptons except for the up quark, the down quark and the electron, and the transformation of up and down quarks and electron by

$$u_R \to \exp\left(-i\frac{2x}{x+x^{-1}}\frac{a}{v'}\right)u_R,\ d_R \to \exp\left(-i\frac{2x^{-1}}{x+x^{-1}}\frac{a}{v'}\right)d_R, \tag{E.26}$$

$$e_R \to \exp\left(-i\frac{2x^{-1}}{x+x^{-1}}\frac{a}{v'}\right)e_R, \tag{E.27}$$

the axion-gluon and axion-photon coupling at this stage are given by

$$\mathcal{L} = \frac{g^2}{32\pi^2}6\frac{a}{v'}G\tilde{G} \tag{E.28}$$

$$+ \frac{g_{\rm em}^2}{16\pi^2}\frac{a}{v'}\left[9\frac{2x}{x+x^{-1}}\left(\frac{2}{3}\right)^2 + 9\frac{2x^{-1}}{x+x^{-1}}\left(\frac{1}{3}\right)^2 + 3\frac{2x^{-1}}{x+x^{-1}}\right]F\tilde{F}, \tag{E.29}$$

$$= \frac{g^2}{32\pi^2}\frac{a}{F_a}G\tilde{G} \tag{E.30}$$

$$+ \frac{g_{\rm em}^2}{16\pi^2}\frac{3}{2}\frac{a}{F_a}\left[\frac{2x}{x+x^{-1}}\left(\frac{2}{3}\right)^2 + \frac{2x^{-1}}{x+x^{-1}}\left(\frac{1}{3}\right)^2 + \frac{1}{3}\frac{2x^{-1}}{x+x^{-1}}\right]F\tilde{F}. \tag{E.31}$$

In the second line, $F_a \equiv v'/6$ is used. Let us transform the up and the down quarks again to erase the axion-gluon coupling,

$$u_R \to \exp\left[i\left(c_u + \frac{x}{3(x+x^{-1})}\right)\frac{a}{F_a}\right]u_R, \tag{E.32}$$

$$d_R \to \exp\left[i\left(c_d + \frac{x^{-1}}{3(x+x^{-1})}\right)\frac{a}{F_a}\right]d_R \tag{E.33}$$

$$c_u + c_d + 1/3 = 1. \tag{E.34}$$

Then, the axion-photon coupling becomes,

$$\mathcal{L} = \frac{g_{\rm em}^2}{16\pi^2}3\frac{a}{F_a}\left[\left(\frac{x}{x+x^{-1}} - c_u - \frac{x}{3(x+x^{-1})}\right)\left(\frac{2}{3}\right)^2\right. \tag{E.35}$$

$$\left.+\left(\frac{x^{-1}}{x+x^{-1}} - c_d - \frac{x^{-1}}{3(x+x^{-1})}\right)\left(\frac{1}{3}\right)^2 + \frac{1}{3}\frac{x^{-1}}{x+x^{-1}}\right]F\tilde{F} \tag{E.36}$$

$$= \frac{g_{\rm em}^2}{16\pi^2}3\frac{a}{F_a}\left[\left(\frac{2}{3}\frac{x}{x+x^{-1}} - c_u\right)\left(\frac{2}{3}\right)^2\right. \tag{E.37}$$

$$\left.+ \left(\frac{2}{3}\frac{x^{-1}}{x+x^{-1}} - c_d\right)\left(\frac{1}{3}\right)^2 + \frac{1}{3}\frac{x^{-1}}{x+x^{-1}}\right]F\tilde{F}. \tag{E.38}$$

The neutral pion-axion kinetic mixing is erased by choosing $c_u = c_d$. The effective Lagrangian of the neutral pion and axion system is

$$\mathcal{L} = \frac{1}{2}(\partial a)^2 + \frac{1}{2}(\partial\pi^0)^2 \tag{E.39}$$

$$- m_u B_0 F_\pi^2 \cos\left[\left(c_u + \frac{x}{3(x+x^{-1})}\right)\frac{a}{F_a} + \frac{\pi^0}{F_\pi}\right] \tag{E.40}$$

$$- m_d B_0 F_\pi^2 \cos\left[\left(c_d + \frac{x^{-1}}{3(x+x^{-1})}\right)\frac{a}{F_a} - \frac{\pi^0}{F_\pi}\right]. \tag{E.41}$$

After the diagonalization of the mass matrix, the mass eigenstate of the pion and the axion is

$$\pi_{\rm phs} \simeq \pi^0 + \frac{1}{2}\left(\frac{-m_u c_u' + m_d c_d'}{m_u + m_d}\right)\frac{F_\pi}{F_a}a, \tag{E.42}$$

$$a_{\rm phys} \simeq a, \tag{E.43}$$

$$c_u' = c_u + \frac{x}{3(x+x^{-1})}, \tag{E.44}$$

$$c_d' = c_d + \frac{x^{-1}}{3(x+x^{-1})}. \tag{E.45}$$

Due to the π^0 coupling with the photon,

$$\mathcal{L} = \frac{g_{\rm em}^2}{16\pi^2}\frac{\pi^0}{F_\pi}6\left[\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right]F\tilde{F}, \tag{E.46}$$

we eventually obtain the axion-photon coupling,

$$\mathcal{L} = \frac{g_{\rm em}^2}{16\pi^2}3\frac{a}{F_a}\left[\left(\frac{2}{3}\frac{x}{x+x^{-1}} - c_u\right)\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\frac{x^{-1}}{x+x^{-1}} - c_d\right)\left(\frac{1}{3}\right)^2\right. \tag{E.47}$$

$$\left.+ \frac{1}{3}\frac{x^{-1}}{x+x^{-1}} - \left(\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right)\left(\frac{-m_u c_u' + m_d c_d'}{m_u + m_d}\right)\right]F\tilde{F} \tag{E.48}$$

$$= \frac{g_{\rm em}^2}{32\pi^2}\frac{a}{F_a}\frac{2z}{1+z}F\tilde{F}. \tag{E.49}$$

This result can be rewritten as

$$= \frac{g_{\rm em}^2}{32\pi^2} a \left[\frac{8}{3} - \frac{2}{3}\frac{1+4z}{1+z}\right] \frac{1+z}{\sqrt{z}} \frac{m_a}{m_\pi F_\pi} F\tilde{F}, \tag{E.50}$$

$$m_a = \frac{\sqrt{z}}{1+z}\frac{m_\pi F_\pi}{F_a} \tag{E.51}$$

Appendix F

Solitons

Let us review the two types of the topological soliton, so-called the domain wall and the (cosmic) string. Each soliton is further characterized by so-called the topological charge. See *e.g.* Refs. [95, 153] for more details.

F.1 Domain wall solution

Let us consider the following Lagrangian for a real scalar field, ϕ, in $1+1$ dimension,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi), \tag{F.1}$$

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v_D^2)^2, \tag{F.2}$$

where λ is a dimensionless coupling constant, and ϕ obtains the VEV, v_D, for its potential minimum. In the Lagrangian, there is a global discrete Z_2 symmetry, $\phi \leftrightarrow -\phi$. Thus, there are two degenerate vacua as $\langle\phi\rangle = \pm v_D$. The energy from this Lagrangian is given by,

$$E = \int dtdz \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\phi'^2 + V(\phi)\right). \tag{F.3}$$

To obtain the finite energy configuration, we require a boundary condition that the scalar field ϕ must sit in their minimum at the space infinity *i.e.*,

$$\lim_{|z|\to\infty}\phi \to \pm v_D. \tag{F.4}$$

In this condition, let us consider the time-independent equation of the local minimum of the energy,

$$0 = \delta E = \int dtdz \left(\phi''\delta\phi + \frac{\delta}{\delta\phi}V(\phi)\right), \frac{\delta}{\delta\phi}V(\phi) = \lambda(\phi^2 - v_D^2)\phi\delta\phi, \tag{F.5}$$

$$\to \quad \phi'' + \lambda(\phi^2 - v_D^2)\phi = 0. \tag{F.6}$$

The solutions of this equation are given by,

$$\phi_\pm = v_D \tan\left[\pm\frac{\lambda v_D}{\sqrt{2}}(z - z_0)\right], \tag{F.7}$$

where z_0 is the free parameter showing the soliton center called the collective coordinate. These solutions are nothing but the kink (domain wall) solutions.[1]

The two solutions can be characterized by a quantity called the "topological charge", Q, defined by,

$$Q = \frac{1}{v_D}\int dz \partial_z \phi = +2 \text{ or } -2, \tag{F.8}$$

for the ϕ_+ and the ϕ_-, respectively. The difference of the topological charge denotes that the two solutions are topologically inequivalent. Two solutions are to be topologically equivalent if they cannot be continuously deformed into one another without passing through a barrier of infinite action (energy). Let us check that these kink solutions are topologically equivalent or not. We consider the scalar field configuration Φ which connects two equations continuously,

$$\Phi(s, z) = g(s)\phi_+(z) + h(s)\phi_-(z) \quad (0 \le s \le 1), \tag{F.9}$$

where $g(s)$, $h(s)$ are continuous functions with the boundary conditions, $g(0) = h(1) = 1$ and $g(1) = h(0) = 0$. The static action is,

$$\begin{aligned} S(s) &= \int dz \frac{1}{2}\Phi'^2 + \frac{\lambda}{4}(\Phi^2 - v_D)^2, && \text{(F.10)} \\ &= \frac{\lambda v_D^2}{4}\int dz \left[(g-h)^2\frac{1}{\cosh^4(z')} + ((g-h)^2\tanh^2(z') - 1)^2\right], && \text{(F.11)} \end{aligned}$$

where $z' = \lambda v_D(z - z_0)/\sqrt{2}$. The first term becomes finite, but the second term diverges for $|g(t) - h(t)| \neq 1$. Thus, if two solutions are topologically equivalent, we can find the continuous solution for $g(t)$, $h(t)$ satisfying the condition, $|g(t)-h(t)| \neq 1$ for every $0 \le s \le 1$. The boundary condition requires, however, $g(0) - h(0) = 1$, $g(1)-h(1) = -1$. This means that the continuous solution must pass the points of $|g(t)-h(t)| \neq 1$. Therefore, we find that two solutions are topologically inequivalent. On the other word, the quantity,

$$\phi(\infty) - \phi(-\infty), \tag{F.12}$$

must be conserved, which is proportional to the topological charge in Eq. (F.8). The axion generally has the periodic potential with n degenerate vacua, and thus stable domain walls are formed as the solution connecting the adjacent vacua.

[1] The solution in $1+1$ dimension is called kink. For higher space dimension, the solution is called the domain wall. For example in $1 + 3$ dimension, the solution separates two domains as a wall.

F.2 String solution

In this section, we review the string solutions.

F.2.1 Local string

Let us consider the following Lagrangian for a complex scalar field, Φ, in $1+2$ dimension with $U(1)$ gauge symmetry,

$$\mathcal{L} = \frac{1}{2}|\mathcal{D}_\mu \Phi|^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - V(\Phi), \tag{F.13}$$

$$V(\phi) = \lambda(|\Phi|^2 - v_S)^2, \tag{F.14}$$

where $F_{\mu\nu}$ is the field strength of the $U(1)$, and D_μ is the covariant derivative written by

$$D_\mu = \partial_\mu - in_e e A_\mu, \tag{F.15}$$

where e is the coupling constant of the $U(1)$ gauge symmetry, n_e is the charge of the ϕ in the unit of the e, [2] and A_μ is its gauge field. The $U(1)$ gauge symmetry is spontaneously broken down after the field Φ obtains its VEV as $\langle\Phi\rangle = v_S$. To obtain the finite solution, we require the following boundary condition at infinity of the space under the gauge fixing $A_0 = 0$,

$$|\Phi| \to v_S, \ F_{ij} \to 0, \ \mathcal{D}_i\Phi \to 0, \tag{F.16}$$

where the subscripts i and j denote the space indices. As in the case of the kink solution, we can find the local string solution to solve the equation of the motion from the condition of the local energy minimum. To do this, we take the following ansatz,

$$\Phi = v_S \phi(r) e^{in\theta}, \ A_i = -\epsilon_{ij} x_j \frac{n}{n_e r^2}(1 - f(r)), \tag{F.17}$$

where (r, θ) is circular coordinate in the space $(x, y) = (r\cos\theta, r\sin\theta)$, ϵ_{ij} is the anti-symmetric tensor, n is the integer called the winding number, $\phi(r)$ and $f(r)$ are continuous functions of the radius r with the boundary conditions,

$$\Phi(\infty) \to 1, \ f(\infty) \to 0, \ \phi(0) \to 0, \ f(\infty) \to 1, \tag{F.18}$$

[2] The value of the n_e has the physical meaning if there are the other scalar fields or matter fields which have different $U(1)$ charges in the theory.

where the condition in Eq. (F.16) are satisfied, and the solution becomes non-singular at $r = 0$. From the condition of the local energy minimum, we obtain two non-trivial second-order equations,

$$\frac{d}{dr}\left(\frac{1}{r}\frac{df}{dr}\right) - 2n_e^2 e^2 v_S^2 \frac{\phi^2}{r} f = 0, \tag{F.19}$$

$$-\frac{d}{dr}\left(\frac{1}{r}\frac{d\phi}{dr}\right) + 2\lambda v_S^2 r\phi(\phi^2 - 1) + n^2\frac{\phi}{r}f^2 = 0. \tag{F.20}$$

These equations are numerically solved, which is known as the vortex. The energy of the vortex is roughly given by

$$T \simeq 2\pi v^2 n \tag{F.21}$$

for $en_e/(\sqrt{2\lambda}) \simeq 1$. The integer n in Eq. (F.17) denotes how many Φ of the vacuum configuration returns to its original value $\theta = 0$ while the space angle θ runs $0 \to 2\pi$. The Φ with the winding number n cannot be continuously transformed to the one with the different winding number m ($n \neq m$) keeping the finite energy, and thus the winding number labels the topologically inequivalent solutions. The non-trivial solution as $n \neq 1$ is called the vortex (the string[3]) solution.

F.2.2 Global string

In the case of the global string, there is no gauge field, and then one of the conditions in Eq. (F.16), *i.e.*

$$\mathcal{D}_i\Phi \to 0, \tag{F.22}$$

cannot be satisfied. Due to this property, the energy of the global string is logarithmically divergent at far away from the string center,

$$\int d^2x|\partial_i\Phi|^2 \to 2\pi n^2 v_S^2 \int dr\frac{1}{r}. \tag{F.23}$$

In the case of the cosmic string following the scaling law, the energy of the string is roughly estimated as

$$E(t) \simeq 2\pi v^2(n + n^2\ln(\delta_S/H(t)), \tag{F.24}$$

where $H(t)$ is the Hubble constant, and the δ_S is the width of the string around $\delta_S \simeq 1/(2\sqrt{\lambda}v_S)$.

[3]In $1+3$ dimension, the vortex solution is called the string.

Appendix G

Topological suppression of wormhole effect

To discuss the topological suppression of the wormhole effect, let us consider so-called the Gauss-Bonnet term which exists in the general Euclidean action with gravity,

$$S_{\mathrm{GB}} = -\frac{\gamma}{32\pi^2}\int d^4x\sqrt{g_g}\,(\mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2). \tag{G.1}$$

In the geometry of the wormhole, this term can be written by the $R(\rho)$ as,

$$\begin{aligned} S_{\mathrm{GB}} &= \frac{3\gamma}{4\pi^2}\int d^4x\frac{R''(r)(1-R'^2(r))}{R^3(r)} && \text{(G.2)} \\ &= \frac{3\gamma}{2}\int_0^\infty drR''(r)(1-R'^2(r)) && \text{(G.3)} \\ &= \frac{3\gamma}{2}\int_0^\infty dr\frac{d}{dr}\left(R'(r)-\frac{1}{3}'^3(r)\right) && \text{(G.4)} \\ &= \frac{3\gamma}{2}\left[R'(r)-\frac{1}{3}R'^3(r)\right]_{r=0}^{r\to\infty}. && \text{(G.5)} \end{aligned}$$

From the condition of the wormhole solution, $R'(r) = 0$ and $R'(\infty) = 1$. Thus, the Gauss-Bonnet term is

$$S_{\mathrm{GB}} = \gamma. \tag{G.6}$$

In Eq. (5.34), this Gauss-Bonnet term is included in the action S_0.

Thus, it implies that the global symmetry breaking by the wormhole effect can be suppressed by the large value of γ. Even if we take $\alpha_{wm} = 1$ and $K = M_{\mathrm{pl}}^3\Phi$ (*e.g.* Φ is the PQ breaking scalar field), the action is suppressed by the $e^{-\gamma}$,

$$-S_{eff} \ni \int d^4x\sqrt{g_g}\,M_{\mathrm{pl}}^3\,\Phi\,e^{-\gamma}, \tag{G.7}$$

where the S_{GB} is included in S_0. If this estimation gives the maximum effect of the PQ breaking by the quantum gravity effect, the quality problem can be solved for $\gamma > 190$.

Appendix H

Supersymmetry

In this section, we briefly review supersymmetry.

H.1 Superspace and Lagrangian

Supersymmetry is obtained by using a manifold of the (4-dimensional) spacetime coordinate and four fermionic coordinates.[1] This manifold is called superspace [155, 156] and the coordinates are given as,

$$x^\mu,\ \theta^\alpha,\ \theta^\dagger_{\dot\alpha}, \tag{H.1}$$

where θ^α, $\theta^\dagger_{\dot\alpha}$ are constant complex anticommuting two-component spinors with mass dimension $-1/2$. Any fields on superspace can be expanded in a power series of θ^α, $\theta^\dagger_{\dot\alpha}$ with components of functions of spacetime. Those fields are called superfields. To obtain the supersymmetry transformations, we define the following differential operator acting on superfields,

$$Q_\alpha = i\frac{\partial}{\partial\theta^\alpha} + (\sigma^\mu\theta^\dagger)_\alpha\partial_\mu \quad , \quad Q^\alpha = -i\frac{\partial}{\partial\theta_\alpha} - (\theta^\dagger\bar\sigma^\mu)^\alpha\partial_\mu \tag{H.2}$$

$$Q^{\dagger\dot\alpha} = i\frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} + (\bar\sigma^\mu\theta^\dagger)^{\dot\alpha}\partial_\mu \quad , \quad Q^\dagger_{\dot\alpha} = -i\frac{\partial}{\partial\theta^{\dagger\dot\alpha}} - (\theta^\dagger\sigma^\mu)_{\dot\alpha}\partial_\mu, \tag{H.3}$$

where σ^μ denotes the Pauli matrix and $\sigma^k = -\bar\sigma^k$ for space indices k. Using these operators, the supersymmetry transformation for the superfield $S(x,\theta,\theta^\dagger)$ is given by,

$$-i(\epsilon Q + \epsilon^\dagger Q^\dagger)S = S(x^\mu + i\epsilon\sigma^\mu\theta^\dagger + i\epsilon^\dagger\bar\sigma^\mu\theta, \theta+\epsilon, \theta^\dagger+\epsilon^\dagger) - S(x^\mu,\theta,\theta^\dagger), \tag{H.4}$$

[1] See *e.g.* Ref. [154] for more details.

where ϵ, $\epsilon^\dagger$ denote infinitesimal. This equation shows that the supersymmetry transformation is a translation in superspace. The supersymmetric theories can be described by so-called chiral superfields, vector superfields, and spinor chiral superfields. To obtain these chiral superfields, it is useful to introduce the so-called chiral covariant derivatives,

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu \quad , \quad D^\alpha = -\frac{\partial}{\partial\theta_\alpha} - i(\theta^\dagger\bar{\sigma}^\mu)^\alpha\partial_\mu, \tag{H.5}$$

$$D^{\dagger\dot{\alpha}} = \frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} + i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu \quad , \quad D^\dagger_{\dot{\alpha}} = -\frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}} - i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu, \tag{H.6}$$

which commute with Q_α, $Q^\dagger_{\dot{\alpha}}$, and thus D_α, $D^\dagger_{\dot{\alpha}}$ are supersymmetric covariant. From the chiral covariant derivatives, so-called chiral superfield is defined by,

$$D^\dagger_{\dot{\alpha}}\Phi = 0. \tag{H.7}$$

Changing the coordinate by $y^\mu = x^\mu + i\theta^\dagger\bar{\sigma}^\mu\theta$, the chiral superfield can be expanded as,

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \tag{H.8}$$

where $\phi(y)$ is a complex scalar field, $\psi(y)$ is two-components Weyl spinor field, and $F(y)$ is an auxiliary field. By the supersymmetry transformation, the scalar component $\phi(y)$, for example, changes to the fermion component $\psi(y)$. A vector superfield, V, is defined as $V = V^*$. In Wess-Zumino gauge, a vector superfield is expanded as,

$$V^a = -\theta^\dagger\bar{\sigma}^\mu\theta A^a_\mu + \theta^\dagger\theta^\dagger\theta\lambda^a + \theta\theta\theta^\dagger\lambda^{\dagger a} + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger D^a \tag{H.9}$$

where a is the index of the gauge group generator, A^a_μ is the gauge boson field, λ^a is two-components Weyl spinor field and D_a is an auxiliary field. The spinor chiral superfield $\mathcal{W}^a_\alpha$ is defined from the vector superfield, and given in Wess-Zumino gauge

$$\mathcal{W}^a_\alpha = \lambda^a_\alpha + \theta_\alpha D^a - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\mu\theta)_\alpha F^a_{\mu\nu} - i\theta\theta(\sigma^\mu\nabla_\mu\lambda^{\dagger a})_\alpha, \tag{H.10}$$

where $F^a_{\mu\nu}$ is the field strength, ∇_μ is the gauge covariant derivative. Therefore, the general renormalizable Lagrangian for a supersymmetric gauge theory is written by,

$$\mathcal{L} = \left(\frac{1}{4} - i\frac{g_a^2\theta_a}{32\pi^2}\right)\left(\int d^2\theta\, \mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + c.c\right) \tag{H.11}$$

$$+ \int d^2\theta d^2\theta^\dagger\, [\Phi^{*i}(e^{2g_aT^aV^a})_i^j\Phi_j] + \left(\int d^2\theta W(\Phi_i) + c.c.\right) \tag{H.12}$$

where T^a are generators of the gauge group, g_a are their gauge couplings, the index i denotes the label of different chiral superfields, $W(\Phi_i)$ called the superpotential is the holomorphic function of chiral superfields, θ_a is a CP-violating parameter inducing a total derivative term.[2] In the case of the abelian gauge vector superfield, the term called Fayet-Iliopoulos term which proportional to $\int d^2\theta d^2\theta^\dagger V$ arises in the Lagrangian.

H.2 R-symmetry

Some supersymmetric Lagrangians are invariant under so-called R-symmetry defined by the following transformation of the general superfield $S(x,\,\theta,\,\theta^\dagger)$,

$$S(x,\,\theta,\,\theta^\dagger) \to e^{irs\alpha} S(x,\,e^{-i\alpha}\theta,\,e^{i\alpha}\theta^\dagger), \tag{H.13}$$

where α denotes a phase and r_S is a parameter called R-charge of the superfield S. For the chiral superfield of R-charge r_ϕ, the scalar, the fermion, and the auxiliary fields obtain the charge $r_\phi,\ r_\phi - 1,\ r_\phi - 2$, respectively. For the vector superfields in Wess-Zumino gauge, only the gaugino component obtains the non-zero R-charge, $+1$. Furthermore, $\mathcal{W}^a_\alpha$ carries R-charge $+1$.

H.3 Perturbative non-renormalization theorem

Here, let us prove the so-called non-renormalization theorem [157–159]. One rescales the vector fields $g_a V^a \to V^a$, and then consider the following general Lagrangian[3],

$$\mathcal{L} = \frac{1}{4g_a^2}\left[\int d^2\theta \mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + c.c\right] \tag{H.14}$$

$$+ \int d^2\theta d^2\theta^\dagger\,[\Phi^{*i}(e^{2T^aV^a})_i^j\Phi_j] + \left[\int d^2\theta\, W(\Phi_i) + c.c\right], \tag{H.15}$$

where we omit the term proportional to θ_a in Eq. (H.11) because the total derivative terms have no effects on the perturbative theory. To prove the theorem, we introduce two additional external gauge invariant chiral superfields $X_R,\ Y_R$ in the Lagrangian,

$$\mathcal{L} = \frac{1}{4}\left[\int d^2\theta X_R \mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + c.c\right] \tag{H.16}$$

$$+ \int d^2\theta d^2\theta^\dagger\,[\Phi^{*i}(e^{2T^aV^a})_i^j\Phi_j] + \left[\int d^2\theta\, Y_R W(\Phi_i) + c.c.\right], \tag{H.17}$$

[2]The definitions of integrations are in *e.g.* Ref. [154]

[3]For simplicity, we only consider the renormalizable terms. See Ref. [160] for more general proof including non-renormalizable terms.

which corresponds to the original Lagrangian when the scalar components of X_R and Y_R are taken as $1/g_a^2$ and 1, and the other components of the spinor and the auxiliary components are zero. Assuming no gauge symmetry breaking and no supersymmetry breaking, the Wilsonian effective (classical) Lagrangian of an ultra-violet cutoff scale λ_c[4] can be written as

$$\mathcal{L}_{\lambda_c} = \int d^2\theta d^2\theta^\dagger \mathcal{A}_{\lambda_c}(\Phi, \Phi^\dagger, V, X_R, X_R^\dagger, Y_R, Y_R^\dagger) \tag{H.18}$$

$$+ \left[\int d^2\theta \mathcal{B}_{\lambda_c}(\Phi, \mathcal{W}, X_R, Y_R) + c.c\right], \tag{H.19}$$

where $\mathcal{A}_{\lambda_c}$ denotes the gauge invariant function including chiral covariant derivatives (Kähler potential), and $\mathcal{B}_{\lambda_c}$ denotes the gauge invariant holomorphic function (superpotential). The form of $\mathcal{B}_{\lambda_c}$ can be limited by using two symmetries of the Lagrangian in Eq. (H.16). One is the perturbative $U(1)_R$ symmetry, where Y_R has R-charge +2, X_R and Φ's are neutral, $\mathcal{W}$'s are R-charge +1. Due to this symmetry, $\mathcal{B}_{\lambda_c}$ has the form,

$$\mathcal{B}_{\lambda_c} = Y f_{\lambda_c}(\Phi, X_R) + \mathcal{W}^a_\alpha \mathcal{W}^{b\alpha} h_{\lambda_c ab}(\Phi, X_R), \tag{H.20}$$

where both f_{λ_c} and h_{λ_c} are gauge invariant holomorphic functions. Another symmetry in Eq. (H.16) is the translation of X_R by

$$X_R \to X_R + i\xi, \tag{H.21}$$

where ξ is real. Under this imaginary constant transformation, only total derivative terms arise in the Lagrangian of Eq. (H.16), and thus there is no effect in perturbative theory. Due to this symmetry, the form of the $\mathcal{B}_{\lambda_c}$ can be determined as,

$$\mathcal{B}_{\lambda_c} = Y_R f_{\lambda_c}(\Phi) + \mathcal{W}^a_\alpha \mathcal{W}^{b\alpha}(c_{\lambda_c}\delta_{ab} X_R + l_{\lambda_c ab}(\Phi)), \tag{H.22}$$

where c_{λ_c} is a real cut-off dependent constant, and l_{λ_c} is a gauge invariant holomorphic function of Φ's. Here, we label the scalar components of X_R, Y_R as x_r, y_r, and set the other components as zero. Because f_{λ_c} only depends on Φ, the form of f_{λ_c} is invariant for arbitrary values of x_r and y_r. When we take the limits of $x_r \to \infty$ and $y \to 0$, the gauge couplings vanishes as $1/\sqrt{x_r}$ and the Yukawa couplings and the other dimensionful couplings go to zero proportional to y_r. In this limit, the graph proportional to y_r from the effective action is the same with a single vertex from Y_R from the original Lagrangian in Eq. (H.16). Thus,

$$f_{\lambda_c}(\Phi) = W(\Phi). \tag{H.23}$$

[4]Below the cut-off scale, the Wilsonian effective Lagrangian gives the same S-matrix elements as the original Lagrangian.

This shows the non-renormalization theorem in the Wilsonian effective superpotential. The form of the coupling l_{λ_c} can be also determined. l_{λ_c} When we take Y=0, the original Lagrangian in Eq. (H.16) has the symmetry of equal number of Φ, $\Phi^\dagger$, and thus l_{λ_c} cannot depend on only Φ,

$$l_{\lambda_c ad}(\Phi) = L_{\lambda_c}\delta_{ab}, \tag{H.24}$$

where L_{λ_c} has no Φ, $\Phi^\dagger$, X_R, Y_R dependence. For $Y = 0$, the number of powers of x_r, N_x, of any diagrams with no external Φ, $\Phi^\dagger$'s are given by,

$$N_x = 1 - N_L, \tag{H.25}$$

where N_L is a number of loops of a diagram. This relation shows that the couplings c_{λ_c} and L_{λ_c} is determined from the tree level and the one loop diagrams, respectively. Therefore, taking $Y_R = 1$ and $X_R = 1/g_a^2$, the $c_{\lambda_c} = 0$, and L_{λ_c} is given as the one-loop renormalized gauge coupling constant. We will use this result in Appendix H.5.

H.4 Soft supersymmetry breaking

The supersymmetry must be broken in nature, and thus we briefly discuss one type of the "soft" supersymmetry breaking so-called the Planck scale mediated supersymmetry breaking [161–167]. The meaning of "soft" is that the supersymmetry is violated only by terms with positive mass dimension couplings.

The Planck scale mediated supersymmetry breaking is discussed in a theory with a local SUSY called supergravity. To see concretely, let us quote the scalar potential $V_{\rm SG}$ in supergravity theory (See *e.g.* Ref. [168] for more details),

$$V_{\rm SG} = F_j F^{*i} - 3e^{K/M_{\rm pl}^2} W W^*/M_{\rm pl}^2, \tag{H.26}$$

where all chiral superfields are replaced by their scalar components, $K \equiv \phi^{*i}\phi_i$ is called a minimal Kähler potential, $K^i = \delta K/\delta\phi_i$, $K_j = \delta K/\delta\phi^{*j}$, and $F_j = -e^{D/(2M_{\rm pl})}(W_j^* + W^* K_j/M_{\rm pl}^2)$. Here, $W_j^* = \delta W^*/\delta\phi^{*j}$. The local supersymmetry is broken when some F_i obtain the vacuum expectation values, and then the superpartner of the graviton called gravitino obtains a squared mass,

$$m_{3/2}^2 = \langle F_i F^{*i}\rangle/(3M_{\rm pl}^2). \tag{H.27}$$

For an example of the Planck scale mediated SUSY breaking model, let us consider the following superpotential and Kähler potential,

$$W = W_{\rm vis}(\phi) + W_{\rm hid}(X), \ K = \phi^{*i}\phi_i + X^* X \tag{H.28}$$

where W_i is the visible sector superpotential with the visible sector chiral superfields ϕ_i, and $W_{\rm hid}$ is the hidden sector superpotential with the hidden sector chiral superfield X. Here, the visible sector includes the minimal supersymmetric standard model superpotential. We assume that the supersymmetry is broken in the hidden sector by non-zero vacuum expectation value,

$$\langle X\rangle = w_0 M_{\rm pl},\ \ \langle W_{\rm hid}\rangle = w_1 M_{\rm pl}^2,\ \ \langle \delta W_{\rm hid}/\delta X\rangle = w_2 M_{\rm pl}, \tag{H.29}$$

where w_0 is dimensionless quantity, $w_1,\ w_2$ are mass dimension $+1$ quantity. Requiring that the vacuum energy density equals zero, the leading order scalar potential is obtained as[5],

$$\begin{aligned} V_{\rm SG} &= (W^*_{\rm vis})_i (W_{\rm vis})^i + m^2_{3/2}\phi^{*i}\phi_i & \text{(H.30)}\\ &+ e^{|w_0|^2/2}[w_1^*\phi_i(W_{\rm vis})^i + (w_0^* w_2^* + |w_0|^2 w_1^* - 3w^*)W_{\rm vis} + c.c.]. & \text{(H.31)} \end{aligned}$$

where we used the gravitino mass given by

$$m_{3/2} = |\langle F_X\rangle|/\sqrt{3}M_{\rm pl} = e^{|w_0|^2/2}|w_1|. \tag{H.32}$$

The second term in Eq. (H.30) provides the universal soft scalar squared mass, the third and the fourth terms give the holomorphic couplings so-called b-term and a-term, respectively. Due to the supersymmetry breaking, the masses of the superfield components are split.

H.5 Non-perturbative corrections

In this section, let us discuss the non-perturbative effects in the Supersymmetric QCD (SQCD).

H.5.1 Affleck-Dine-Seiberg Superpotential

Consider $SU(N_c)$ SQCD with N_f flavors and $N_c > N_f$. When we assume there are no superpotential terms at tree level, the theory can obtain three global symmetries as in Tab. H.1. In the absence of a superpotential, one has some flat-directions with vanishing D-terms,

$$D_a = g(\Phi^{\dagger jn}(T^a)^m_n \Phi_{mj} - \bar{\Phi}^{jn}(T^a)^m_n \bar{\Phi}^\dagger_{mj}) = 0, \tag{H.33}$$

[5]The superfields are replaced by their scalar fields. Note that we rescale the visible sector superpotential, $W_{\rm vis} \to e^{-|w_0|^2/2} W_{\rm vis}$ to obtain the scalar potential term as given in the global supersymmetry.

Table H.1: Field contents and charge assignments

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)$	$U(1)_R$
Φ	$\square$	$\square$	$\mathbf{1}$	1	$\frac{N_f-N_c}{N_f}$
$\bar{\Phi}$	$\bar{\square}$	$\mathbf{1}$	$\bar{\square}$	-1	$\frac{N_f-N_c}{N_f}$

where g is the gauge coupling constant of $SU(N_c)$, T_a is the generators of $SU(N_c)$, j is a flavor index of N_f flavors, m and n are color indices of $SU(N_c)$. Concretely, the flat-directions can be labeled by the N_f number of VEV's, $v_i\,(i = 1, \ldots, N_f)$ [169],

$$\left\langle \bar{\Phi}^{\dagger} \right\rangle = \langle \Phi \rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}, \tag{H.34}$$

where $\langle \Phi \rangle$ is a $N_c \times N_f$ matrix. At the generic points of $v_i > 0\,(i = 1, \ldots, N_f)$, the $SU(N_c)$ gauge symmetry is broken to $SU(N_c - N_f)$. There, some of $2N_fN_c$ chiral supermultiplets are eaten by the gauge components, *i.e.*

$$(N_c^2 - 1) - ((N_c - N_f)^2 - 1) = 2N_cN_f - N_f^2. \tag{H.35}$$

Thus, N_f^2 chiral supermultiplets remain massless, and this freedom is described by an $N_f \times N_f$ matrix,

$$M_i^j = \bar{\Phi}^{jn}\Phi_{ni}, \tag{H.36}$$

which is gauge invariant. Due to the non-renormalization theorem of the superpotential, those massless freedoms are not affected by the superpotential in perturbative ways. But, it may be possible to lift up such flat-directions by non-perturbative terms in the superpotential.

Before going to look for non-perturbative terms in the Wilsonian effective superpotential, let us introduce the so-called intrinsic cutoff scale. In the previous section of the non-renormalization theorem, we show that the gauge couplings are only 1-loop renormalized in the effective Wilsonian superpotential. The gauge coupling at some cut-off scale μ is given by,

$$\frac{1}{g^2(\mu)} = -\frac{b}{8\pi} \ln\left(\frac{|\Lambda|}{\mu}\right), \tag{H.37}$$

Table H.2: Field contents and charge assignments including the holomorphic intrinsic scale and $U(1)_A$.

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)$	$U(1)_R$	$U(1)_A$
Φ	$\Box$	$\Box$	$\mathbf{1}$	1	$\frac{N_f-N_c}{N_f}$	1
$\bar{\Phi}$	$\bar{\Box}$	$\mathbf{1}$	$\bar{\Box}$	-1	$\frac{N_f-N_c}{N_f}$	1
Λ^b	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$2N_f$

where $b = 3N_c - N_f$, and the gauge coupling $g(\mu)$ becomes infinity at $\mu = \Lambda$. We can define the holomorphic gauge coupling $\tilde{\tau}$ including the θ-parameter,

$$\tilde{\tau} \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{g^2(\mu)} = \frac{\theta}{2\pi} + \frac{b}{2\pi i}\ln\left(\frac{|\Lambda|}{\mu}\right) = \frac{b}{2\pi i}\ln\left[\left(\frac{|\Lambda|e^{i\theta/b}}{\mu}\right)\right]. \tag{H.38}$$

Then, we can also define the holomorphic intrinsic scale,

$$\Lambda \equiv |\Lambda|e^{i\theta/b} = \mu\, e^{2\pi i\tilde{\tau}/b}. \tag{H.39}$$

We can regard this holomorphic intrinsic scale as an external chiral superfield (no corresponding measure in the path integral), and then we can define a spurious symmetry $U(1)_A$ in Tab. H.2.

Now, let us consider non-perturbative terms in the Wilsonian effective superpotential by following the general method [158]. The superpotential which is invariant under gauge symmetries and $SU(N_f) \times SU(N_f)$ flavor symmetry is described by,

$$W_{\text{eff}} = \Lambda^{bn}(\mathcal{W}^a\mathcal{W}^a)^m(\det M)^p, \tag{H.40}$$

where n, m and p are real parameters, $\det M$ is the only form with M_i^j under $SU(N_f) \times SU(N_f)$ invariance, $\mathcal{W}^a$ is a spinor chiral superfield. From the periodicity of $\theta \to \theta + 2\pi$, n must be an integer. Requiring the $U(1)_A$ and $U(1)_R$ invariance,

$$2N_f p + 2N_f n = 0,\ \ 2p(N_f - N_c) + 2m = 2. \tag{H.41}$$

The solution is,

$$n = -p = \frac{1-m}{N_c - N_f}. \tag{H.42}$$

The theory becomes a free theory if we switch off all couplings. On the other word, we must be able to take weak coupling limit safely. Then, $n \geq 0$ is required, where $\Lambda \to 0$ corresponds to $g(\mu) \to 0$ as some finite μ. In the Wilsonian effective action, all terms must be expanded by positive powers of momentum (If not, the Wilsonian effective action is not valid at low energy.), and thus $m \geq 0$. Therefore,

$$n \geq 0,\ p \leq 0,\ 0 \leq m \leq 1. \tag{H.43}$$

For $m = 1$, $n = p = 0$, and it is just the tree level field strength term. For $m = 1$, we obtain so-called Affleck-Dine-Seiberg (ADS) superpotential [169],

$$W_{\mathrm{ADS}} = C(N_c, N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\mathrm{det}M}\right)^{\frac{1}{N_c-N_f}}, \tag{H.44}$$

where $C(N_c, N_f)$ is the renormalization scheme dependent coupling depending on N_c and N_f. In the $\bar{\mathrm{DR}}$ scheme, $C(N_c, N_f) = N_c - N_f$ [169, 170].[6] It should be noted that the holomorphy of the superpotential means no complex conjugate of the superfields, and thus a negative power of a superfield is allowed.[7] This potential show that the vacuum is only at $\mathrm{det}M \to \infty$ and the flat-directions are lifted in except such points.

For later convenience, let us add the tree level mass term and make all M^i_j components massive,

$$W = W_{\mathrm{ADS}} + \mathrm{tr}(m^j_i M^i_j), \tag{H.45}$$

where m^i_j is a $N_f \times N_f$ mass matrix. Considering the vanishing F-term condition, we obtain the following relation,

$$M^j_i = (m^{-1})^j_i \left(\frac{\Lambda^{3N_c-N_f}}{\mathrm{det}M}\right)^{1/(N_c-N_f)}, \tag{H.46}$$

where m^{-1} is the inverse matrix of m. Here, we used the relation $M^j_i \tilde{M}^k_j = \mathrm{det}M\, \delta_{ik}$, where $\tilde{M}$ is a cofactor matrix of M. Taking the determinant of both sides of this equation, $\mathrm{det}M$ can be written by m, Λ, N_c, N_f. Then, plugging the result into the above equation,

$$M^j_i = (m^{-1})^j_i (\Lambda^{3N_c-N_f}\mathrm{det}m)^{1/N_c}. \tag{H.47}$$

We will use this relation in the next section.

[6] Presicely speaking, the cutoff Λ must be labeled as Λ_{N_c,N_f} because the Λ_{N_c,N_f} is determined in the effective theory with N_c, N_f.

[7] By our definition, we start from the theory with no negative power of superfields. Even under this assumption, the non-perturbative effect gives the ADS potential in the effective theory

H.5.2 Quantum moduli space

Here, let us consider non-perturbative effects in SQCD with $N_f = N_c$ [113, 171].

Before going to discuss the SQCD with $N_f = N_c$, let us check the flat-directions of SQCD with $N_f \geq N_c$. In the absence of the superpotential term, the theory obtains some symmetries in Tab. H.1. Then, the D-flat directions are parametrized as,

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & & & 0 & \dots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & v_{N_c} & 0 & \dots & 0 \end{pmatrix}, \quad \langle \bar{\Phi} \rangle = \begin{pmatrix} \bar{v}_1 & & \\ & \dots & \\ & & \bar{v}_{N_c} \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix}, \tag{H.48}$$

where $|v_i|^2 = |\bar{v}_i|^2 + \rho$ $(i = 1 \dots N_c)$ with an arbitral real parameter ρ. On the points that $SU(N_c)$ gauge symmetry is completely broken, there are left $2N_cN_f - (N_c^2 - 1)$ massless chiral supermultiplets. These massless freedoms can be described by,

$$M_i^j = \bar{\Phi}^{jn}\Phi_{ni}, \tag{H.49}$$

$$B_{i_1,\dots,i_{N_c}} = \Phi_{n_1 i_1} \dots \Phi_{n_{N_c} i_{N_c}} \epsilon^{n_1,\dots,n_{N_c}}, \tag{H.50}$$

$$\bar{B}^{i_1,\dots,i_{N_c}} = \bar{\Phi}^{n_1 i_1} \dots \bar{\Phi}^{n_{N_c} i_{N_c}} \epsilon_{n_1,\dots,n_{N_c}}. \tag{H.51}$$

Generally, there are relations between M, B, and $\bar{B}$. For example of $N_c = N_f$, the constraint (at classical level) is

$$B_{i_1,\dots,i_{N_c}} \bar{B}^{j_1,\dots,j_{N_c}} = N_c! M_{[i_1}^{j_1} \dots M_{i_{N_c}]}^{j_{N_c}}, \tag{H.52}$$

where [ ] shows antisymmetrization. In the following, we simply write this relation as,

$$B\bar{B} - \det M = 0. \tag{H.53}$$

In the following, we call M as meson fields, and B, $\bar{B}$ as baryon fields.

Now, we derive non-perturbative effects of $N_c = N_f$. As we have seen in Eq.(H.53), there is one classical relation between B, $\bar{B}$, M. But, the relation is changed by quantum effects. For example of $N_c = N_f = 2$, the massless degrees of freedom are described by only meson fields, and the classical constraint is

$$\det M = 0. \tag{H.54}$$

Let us remember the relation in Eq. (H.47),

$$M_i^j = (m^{-1})_i^j (\Lambda^{3N_c - N_f} \det m)^{1/N_c}. \tag{H.55}$$

This relation is derived in the case of $N_c > N_f$. But, it seems to be valid even in the case of $N_c = N_f$. Thus, when all quark mass turned on in $N_c = N_f$, we will obtain the constraint,

$$\det M = \Lambda^{N_c}. \tag{H.56}$$

The classical constraint of $\det M = 0$ seems to be changed in $N_f = N_c$. For another example, at $B = \bar{B} = M = 0$, the $SU(N_c)$ gauge symmetry is completely restored, and the massless fields increases. It means the Kähler potential is singular at the origin because the Lagrangian is obtained by taking the field derivative of the Kähler potential. One possibility to avoid this singularity is $B\bar{B} - \det M \neq 0$. From those investigations, it is expected that the classical constraint is changed as,

$$\det M - B\bar{B} = \Lambda^{2N_c}. \tag{H.57}$$

This constraint is described in the superpotential by using the Lagrange multiplier field X,

$$W = X(\det M - B\bar{B} - \Lambda^{2N_c}). \tag{H.58}$$

Note that X has its kinetic term (is not an auxiliary field) and it becomes massive when M or B obtain the VEV. We can check this result by integrating out one flavor. Concretely, we describe the meson fields as,

$$M = \begin{pmatrix} M_i'^j & N^j \\ P_i & Y \end{pmatrix}, \tag{H.59}$$

where $M_i'^j$ is a $(N_f - 1) \times (N_f - 1)$ matrix, both N^j and P_i denote the $N_f - 1$ component vectors. Y is a chiral superfield. Consider the superpotential,

$$W = X(\det M - B\bar{B} - \Lambda^{2N_c}) + mY, \tag{H.60}$$

where m denotes a mass of quarks. From the condition that all F-terms are zero, we obtain the following relation,

$$Y \det M' = \Lambda^{2N}. \tag{H.61}$$

The matching condition[8] between the theory with $N_f = N_c$ and the theory $N_f = N_c - 1$ is

$$m\Lambda^{2N_c} = \Lambda^{2N_c+1}_{N_c,N_c-1}, \tag{H.62}$$

where Λ_{N_c,N_c-1} is the gauge invariant cutoff (the holomorphic intrinsic scale) in $N_f = N_c - 1$. Then, we obtain,

$$W = \frac{\Lambda^{2N_c+1}_{N_c,N_c-1}}{\det\tilde{M}}. \tag{H.63}$$

This is nothing but the ADS superpotential of $N_f = N_c - 1$. Thus, we conclude that the non-perturbative effect in $N_f = N_c$ can be described by

$$W = X(\det M - B\bar{B} - \Lambda^{2N_c}). \tag{H.64}$$

Before closing this section, let us comment on the matching condition in Eq. (H.62). The intrinsic holomorphic scale in $N_f = N_c$ is given by $\Lambda^b_{N_c,N_c} = \mu^b e^{2\pi i\tilde{\tau}(\mu)}$ with $b = 2N_c$, where we explicitly write the μ dependence of the coupling $\tilde{\tau}$ as $\tilde{\tau}(\mu)$. On the other hand, the intrinsic holomorphic scale in $N_f = N_c - 1$ is given by $\Lambda^{b'}_{N_c,N_c-1} = \mu'^{b'} e^{2\pi i\tilde{\tau}'(\mu')}$ with $b' = 2N_c + 1$. Requiring $\tilde{\tau} = \tilde{\tau}'$ as $\mu = \mu' = m$ as the matching condition, we obtain the condition $\Lambda^{2N_c}_{N_c,N_c} m = \Lambda^{2N_c+1}_{N_c,N_c-1}$ as in Eq. (H.62).

[8] See the following discussion for more details.

Bibliography

[1] C. A. Baker *et al.*, "An Improved experimental limit on the electric dipole moment of the neutron," *Phys. Rev. Lett.* **97** (2006) 131801, `arXiv:hep-ex/0602020 [hep-ex]`.

[2] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Instantons," *Phys. Rev. Lett.* **38** (1977) 1440–1443.

[3] S. W. Hawking, "Quantum Coherence Down the Wormhole," *Phys. Lett.* **B195** (1987) 337.

[4] G. V. Lavrelashvili, V. A. Rubakov, and P. G. Tinyakov, "Disruption of Quantum Coherence upon a Change in Spatial Topology in Quantum Gravity," *JETP Lett.* **46** (1987) 167–169. [Pisma Zh. Eksp. Teor. Fiz.46,134(1987)].

[5] S. B. Giddings and A. Strominger, "Loss of Incoherence and Determination of Coupling Constants in Quantum Gravity," *Nucl. Phys.* **B307** (1988) 854–866.

[6] S. R. Coleman, "Black Holes as Red Herrings: Topological Fluctuations and the Loss of Quantum Coherence," *Nucl. Phys.* **B307** (1988) 867–882.

[7] S. R. Coleman, "Why There Is Nothing Rather Than Something: A Theory of the Cosmological Constant," *Nucl. Phys.* **B310** (1988) 643–668.

[8] G. Gilbert, "WORMHOLE INDUCED PROTON DECAY," *Nucl. Phys.* **B328** (1989) 159–170.

[9] T. Banks and N. Seiberg, "Symmetries and Strings in Field Theory and Gravity," *Phys. Rev.* **D83** (2011) 084019, `arXiv:1011.5120 [hep-th]`.

[10] S. M. Barr and D. Seckel, "Planck scale corrections to axion models," *Phys. Rev.* **D46** (1992) 539–549.

[11] J. E. Kim, "Weak Interaction Singlet and Strong CP Invariance," *Phys. Rev. Lett.* **43** (1979) 103.

[12] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, "Can Confinement Ensure Natural CP Invariance of Strong Interactions?," *Nucl. Phys.* **B166** (1980) 493–506.

[13] H. Fukuda, M. Ibe, M. Suzuki, and T. T. Yanagida, "A "gauged" $U(1)$ Peccei–Quinn symmetry," *Phys. Lett.* **B771** (2017) 327–331, arXiv:1703.01112 [hep-ph].

[14] J. E. Kim, "Invisible Axion and Neutrino Oscillation in SU(11)," *Phys. Rev.* **D24** (1981) 3007.

[15] H. M. Georgi, L. J. Hall, and M. B. Wise, "Grand Unified Models With an Automatic Peccei-Quinn Symmetry," *Nucl. Phys.* **B192** (1981) 409–416.

[16] S. Dimopoulos, P. H. Frampton, H. Georgi, and M. B. Wise, "AUTOMATIC INVISIBLE AXION WITHOUT DOMAIN WALLS," *Phys. Lett.* **B117** (1982) 185.

[17] P. H. Frampton, "Simultaneous Solution of Strong CP and Flavor Problems," *Phys. Rev.* **D25** (1982) 294.

[18] K. Kang, I.-G. Koh, and S. Ouvry, "The Strong CP Problem and Axion Invisibility," *Phys. Lett.* **B119** (1982) 361.

[19] G. Lazarides and Q. Shafi, "Axion Models with No Domain Wall Problem," *Phys. Lett.* **B115** (1982) 21–25.

[20] M. Kamionkowski and J. March-Russell, "Planck scale physics and the Peccei-Quinn mechanism," *Phys. Lett.* **B282** (1992) 137–141, arXiv:hep-th/9202003 [hep-th].

[21] R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, "Solutions to the strong CP problem in a world with gravity," *Phys. Lett.* **B282** (1992) 132–136, arXiv:hep-ph/9203206 [hep-ph].

[22] M. Dine, "Problems of naturalness: Some lessons from string theory," in *Conference on Topics in Quantum Gravity Cincinnati, Ohio, April 3-4, 1992*, pp. 157–169. 1992. arXiv:hep-th/9207045 [hep-th]. http://inspirehep.net/record/32647/files/arXiv:hep-th_9207045.pdf.

[23] A. G. Dias, V. Pleitez, and M. D. Tonasse, "Naturally light invisible axion in models with large local discrete symmetries," *Phys. Rev.* **D67** (2003) 095008, arXiv:hep-ph/0211107 [hep-ph].

[24] L. M. Carpenter, M. Dine, and G. Festuccia, "Dynamics of the Peccei Quinn Scale," *Phys. Rev.* **D80** (2009) 125017, arXiv:0906.1273 [hep-th].

[25] K. Harigaya, M. Ibe, K. Schmitz, and T. T. Yanagida, "Peccei-Quinn symmetry from a gauged discrete R symmetry," *Phys. Rev.* **D88** no. 7, (2013) 075022, arXiv:1308.1227 [hep-ph].

[26] K. Harigaya, M. Ibe, K. Schmitz, and T. T. Yanagida, "Peccei-Quinn Symmetry from Dynamical Supersymmetry Breaking," *Phys. Rev.* **D92** no. 7, (2015) 075003, arXiv:1505.07388 [hep-ph].

[27] M. Redi and R. Sato, "Composite Accidental Axions," *JHEP* **05** (2016) 104, arXiv:1602.05427 [hep-ph].

[28] L. Di Luzio, E. Nardi, and L. Ubaldi, "Accidental Peccei-Quinn symmetry protected to arbitrary order," *Phys. Rev. Lett.* **119** no. 1, (2017) 011801, arXiv:1704.01122 [hep-ph].

[29] E. Witten, "Some Properties of O(32) Superstrings," *Phys. Lett.* **B149** (1984) 351–356.

[30] R. Kallosh, A. D. Linde, D. A. Linde, and L. Susskind, "Gravity and global symmetries," *Phys. Rev.* **D52** (1995) 912–935, arXiv:hep-th/9502069 [hep-th].

[31] P. Svrcek and E. Witten, "Axions In String Theory," *JHEP* **06** (2006) 051, arXiv:hep-th/0605206 [hep-th].

[32] H.-C. Cheng and D. E. Kaplan, "Axions and a gauged Peccei-Quinn symmetry," arXiv:hep-ph/0103346 [hep-ph].

[33] K. I. Izawa, T. Watari, and T. Yanagida, "Higher dimensional QCD without the strong CP problem," *Phys. Lett.* **B534** (2002) 93–96, arXiv:hep-ph/0202171 [hep-ph].

[34] C. T. Hill and A. K. Leibovich, "Natural theories of ultralow mass PNGB's: Axions and quintessence," *Phys. Rev.* **D66** (2002) 075010, arXiv:hep-ph/0205237 [hep-ph].

[35] A. Fukunaga and K. I. Izawa, "Warped QCD without the strong CP problem," *Phys. Lett.* **B562** (2003) 251–256, `arXiv:hep-ph/0301273 [hep-ph]`.

[36] K. I. Izawa, T. Watari, and T. Yanagida, "Supersymmetric and CP-symmetric QCD in higher dimensions," *Phys. Lett.* **B589** (2004) 141–146, `arXiv:hep-ph/0403090 [hep-ph]`.

[37] K.-w. Choi, "A QCD axion from higher dimensional gauge field," *Phys. Rev. Lett.* **92** (2004) 101602, `arXiv:hep-ph/0308024 [hep-ph]`.

[38] B. Grzadkowski and J. Wudka, "Note on the strong CP problem from a 5-dimensional perspective," *Phys. Rev.* **D77** (2008) 096004, `arXiv:0705.4307 [hep-ph]`.

[39] H. Fukuda, M. Ibe, M. Suzuki, and T. T. Yanagida, "Gauged Peccei-Quinn symmetry — A case of simultaneous breaking of SUSY and PQ symmetry," *JHEP* **07** (2018) 128, `arXiv:1803.00759 [hep-ph]`.

[40] M. Ibe, M. Suzuki, and T. T. Yanagida, "$B - L$ as a Gauged Peccei-Quinn Symmetry," *JHEP* **08** (2018) 049, `arXiv:1805.10029 [hep-ph]`.

[41] S. Weinberg, "The U(1) Problem," *Phys. Rev.* **D11** (1975) 3583–3593.

[42] G. 't Hooft, "How Instantons Solve the U(1) Problem," *Phys. Rept.* **142** (1986) 357–387.

[43] R. D. Peccei and H. R. Quinn, "Constraints Imposed by CP Conservation in the Presence of Instantons," *Phys. Rev.* **D16** (1977) 1791–1797.

[44] S. Weinberg, "A New Light Boson?," *Phys. Rev. Lett.* **40** (1978) 223–226.

[45] F. Wilczek, "Problem of Strong p and t Invariance in the Presence of Instantons," *Phys. Rev. Lett.* **40** (1978) 279–282.

[46] W. A. Bardeen, R. D. Peccei, and T. Yanagida, "CONSTRAINTS ON VARIANT AXION MODELS," *Nucl. Phys.* **B279** (1987) 401.

[47] Y. Asano, E. Kikutani, S. Kurokawa, T. Miyachi, M. Miyajima, Y. Nagashima, T. Shinkawa, S. Sugimoto, and Y. Yoshimura, "Search for a Rare Decay Mode K+ —> pi+ Neutrino anti-neutrino and Axion," *Phys. Lett.* **107B** (1981) 159. [,411(1981)].

[48] A. R. Zhitnitsky, "On Possible Suppression of the Axion Hadron Interactions. (In Russian)," *Sov. J. Nucl. Phys.* **31** (1980) 260. [Yad. Fiz.31,497(1980)].

[49] M. Dine, W. Fischler, and M. Srednicki, "A Simple Solution to the Strong CP Problem with a Harmless Axion," *Phys. Lett.* **B104** (1981) 199–202.

[50] J. E. Kim, "A COMPOSITE INVISIBLE AXION," *Phys. Rev.* **D31** (1985) 1733.

[51] K. Choi and J. E. Kim, "DYNAMICAL AXION," *Phys. Rev.* **D32** (1985) 1828.

[52] G. Grilli di Cortona, E. Hardy, J. Pardo Vega, and G. Villadoro, "The QCD axion, precisely," *JHEP* **01** (2016) 034, `arXiv:1511.02867 [hep-ph]`.

[53] G. G. Raffelt, "Astrophysical axion bounds," *Lect. Notes Phys.* **741** (2008) 51–71, `arXiv:hep-ph/0611350 [hep-ph]`.

[54] G. G. Raffelt, "Particle physics from stars," *Ann. Rev. Nucl. Part. Sci.* **49** (1999) 163–216, `arXiv:hep-ph/9903472 [hep-ph]`.

[55] K. S. Hirata *et al.*, "Observation in the Kamiokande-II Detector of the Neutrino Burst from Supernova SN 1987a," *Phys. Rev.* **D38** (1988) 448–458.

[56] **IMB** Collaboration, C. B. Bratton *et al.*, "Angular Distribution of Events From Sn1987a," *Phys. Rev.* **D37** (1988) 3361.

[57] E. N. Alekseev, L. N. Alekseeva, V. I. Volchenko, and I. V. Krivosheina, "Possible Detection of a Neutrino Signal on 23 February 1987 at the Baksan Underground Scintillation Telescope of the Institute of Nuclear Research," *JETP Lett.* **45** (1987) 589–592. [,739(1987)].

[58] G. G. Raffelt, "Astrophysical methods to constrain axions and other novel particle phenomena," *Phys. Rept.* **198** (1990) 1–113.

[59] J. Engel, D. Seckel, and A. C. Hayes, "Emission and detectability of hadronic axions from SN1987A," *Phys. Rev. Lett.* **65** (1990) 960–963.

[60] P. W. Graham, I. G. Irastorza, S. K. Lamoreaux, A. Lindner, and K. A. van Bibber, "Experimental Searches for the Axion and Axion-Like Particles," *Ann. Rev. Nucl. Part. Sci.* **65** (2015) 485–514, `arXiv:1602.00039 [hep-ex]`.

[61] I. G. Irastorza and J. Redondo, "New experimental approaches in the search for axion-like particles," *Prog. Part. Nucl. Phys.* **102** (2018) 89–159, arXiv:1801.08127 [hep-ph].

[62] P. Sikivie, "Experimental Tests of the Invisible Axion," *Phys. Rev. Lett.* **51** (1983) 1415–1417. [,321(1983)].

[63] P. Sikivie, "Detection Rates for 'Invisible' Axion Searches," *Phys. Rev.* **D32** (1985) 2988. [Erratum: Phys. Rev.D36,974(1987)].

[64] **ADMX** Collaboration, S. J. Asztalos *et al.*, "A SQUID-based microwave cavity search for dark-matter axions," *Phys. Rev. Lett.* **104** (2010) 041301, arXiv:0910.5914 [astro-ph.CO].

[65] **ADMX** Collaboration, N. Du *et al.*, "A Search for Invisible Axion Dark Matter with the Axion Dark Matter Experiment," *Phys. Rev. Lett.* **120** no. 15, (2018) 151301, arXiv:1804.05750 [hep-ex].

[66] **CAST** Collaboration, K. Zioutas *et al.*, "First results from the CERN Axion Solar Telescope (CAST)," *Phys. Rev. Lett.* **94** (2005) 121301, arXiv:hep-ex/0411033 [hep-ex].

[67] **CAST** Collaboration, S. Andriamonje *et al.*, "An Improved limit on the axion-photon coupling from the CAST experiment," *JCAP* **0704** (2007) 010, arXiv:hep-ex/0702006 [hep-ex].

[68] M. Yamaguchi, J. Yokoyama, and M. Kawasaki, "Numerical analysis of formation and evolution of global strings in (2+1)-dimensions," *Prog. Theor. Phys.* **100** (1998) 535–545, arXiv:hep-ph/9808326 [hep-ph].

[69] M. Yamaguchi, M. Kawasaki, and J. Yokoyama, "Evolution of axionic strings and spectrum of axions radiated from them," *Phys. Rev. Lett.* **82** (1999) 4578–4581, arXiv:hep-ph/9811311 [hep-ph].

[70] M. Yamaguchi, J. Yokoyama, and M. Kawasaki, "Evolution of a global string network in a matter dominated universe," *Phys. Rev.* **D61** (2000) 061301, arXiv:hep-ph/9910352 [hep-ph].

[71] T. Hiramatsu, M. Kawasaki, T. Sekiguchi, M. Yamaguchi, and J. Yokoyama, "Improved estimation of radiated axions from cosmological axionic strings," *Phys. Rev.* **D83** (2011) 123531, arXiv:1012.5502 [hep-ph].

[72] T. Hiramatsu, M. Kawasaki, K. Saikawa, and T. Sekiguchi, "Production of dark matter axions from collapse of string-wall systems," *Phys. Rev.* **D85** (2012) 105020, arXiv:1202.5851 [hep-ph]. [Erratum: Phys. Rev.D86,089902(2012)].

[73] T. Hiramatsu, M. Kawasaki, and K. Saikawa, "Evolution of String-Wall Networks and Axionic Domain Wall Problem," *JCAP* **1108** (2011) 030, arXiv:1012.4558 [astro-ph.CO].

[74] D. J. Gross, R. D. Pisarski, and L. G. Yaffe, "QCD and Instantons at Finite Temperature," *Rev. Mod. Phys.* **53** (1981) 43.

[75] O. Wantz and E. P. S. Shellard, "Axion Cosmology Revisited," *Phys. Rev.* **D82** (2010) 123508, arXiv:0910.1066 [astro-ph.CO].

[76] O. Wantz and E. P. S. Shellard, "The Topological susceptibility from grand canonical simulations in the interacting instanton liquid model: Chiral phase transition and axion mass," *Nucl. Phys.* **B829** (2010) 110–160, arXiv:0908.0324 [hep-ph].

[77] M. S. Turner, "Cosmic and Local Mass Density of Invisible Axions," *Phys. Rev.* **D33** (1986) 889–896.

[78] K. J. Bae, J.-H. Huh, and J. E. Kim, "Update of axion CDM energy," *JCAP* **0809** (2008) 005, arXiv:0806.0497 [hep-ph].

[79] V. F. Mukhanov and G. V. Chibisov, "Quantum Fluctuations and a Nonsingular Universe," *JETP Lett.* **33** (1981) 532–535. [Pisma Zh. Eksp. Teor. Fiz.33,549(1981)].

[80] S. W. Hawking, "The Development of Irregularities in a Single Bubble Inflationary Universe," *Phys. Lett.* **115B** (1982) 295.

[81] A. A. Starobinsky, "Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations," *Phys. Lett.* **117B** (1982) 175–178.

[82] A. H. Guth and S. Y. Pi, "Fluctuations in the New Inflationary Universe," *Phys. Rev. Lett.* **49** (1982) 1110–1113.

[83] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, "Spontaneous Creation of Almost Scale - Free Density Perturbations in an Inflationary Universe," *Phys. Rev.* **D28** (1983) 679.

[84] **Planck** Collaboration, P. A. R. Ade *et al.*, "Planck 2015 results. XX. Constraints on inflation," arXiv:1502.02114 [astro-ph.CO].

[85] S. B. Giddings and A. Strominger, "Axion Induced Topology Change in Quantum Gravity and String Theory," *Nucl. Phys.* **B306** (1988) 890–907.

[86] K.-M. Lee, "Wormholes and Goldstone Bosons," *Phys. Rev. Lett.* **61** (1988) 263–266.

[87] L. F. Abbott and M. B. Wise, "Wormholes and Global Symmetries," *Nucl. Phys.* **B325** (1989) 687–704.

[88] S. R. Coleman and K.-M. Lee, "WORMHOLES MADE WITHOUT MASSLESS MATTER FIELDS," *Nucl. Phys.* **B329** (1990) 387–409.

[89] S. W. Hawking and R. Laflamme, "Baby Universes and the Nonrenormalizability of Gravity," *Phys. Lett.* **B209** (1988) 39–41.

[90] L. Susskind, "Trouble for remnants," arXiv:hep-th/9501106 [hep-th].

[91] N. Arkani-Hamed, L. Motl, A. Nicolis, and C. Vafa, "The String landscape, black holes and gravity as the weakest force," *JHEP* **06** (2007) 060, arXiv:hep-th/0601001 [hep-th].

[92] L. Randall, "Composite axion models and Planck scale physics," *Phys. Lett.* **B284** (1992) 77–80.

[93] M. Kawasaki and K. Nakayama, "Axions: Theory and Cosmological Role," *Ann. Rev. Nucl. Part. Sci.* **63** (2013) 69–95, arXiv:1301.1123 [hep-ph].

[94] M. Kawasaki, E. Sonomoto, and T. T. Yanagida, "Cosmologically allowed regions for the axion decay constant F_a," arXiv:1801.07409 [hep-ph].

[95] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects*. Cambridge University Press, 2000. http://www.cambridge.org/mw/academic/subjects/physics/theoretical-physics-and-mathematical-physics/cosmic-strings-and-other-topological-defects?format=PB.

[96] K.-I. Izawa and T. Yanagida, “Dynamical supersymmetry breaking in vector - like gauge theories,” *Prog. Theor. Phys.* **95** (1996) 829–830, `arXiv:hep-th/9602180 [hep-th]`.

[97] K. A. Intriligator and S. D. Thomas, “Dynamical supersymmetry breaking on quantum moduli spaces,” *Nucl. Phys.* **B473** (1996) 121–142, `arXiv:hep-th/9603158 [hep-th]`.

[98] I. Affleck, M. Dine, and N. Seiberg, “Dynamical Supersymmetry Breaking in Four-Dimensions and Its Phenomenological Implications,” *Nucl. Phys.* **B256** (1985) 557–599.

[99] A. E. Nelson and N. Seiberg, “R symmetry breaking versus supersymmetry breaking,” *Nucl. Phys.* **B416** (1994) 46–62, `arXiv:hep-ph/9309299 [hep-ph]`.

[100] L. M. Krauss and F. Wilczek, “Discrete Gauge Symmetry in Continuum Theories,” *Phys. Rev. Lett.* **62** (1989) 1221.

[101] J. Preskill and L. M. Krauss, “Local Discrete Symmetry and Quantum Mechanical Hair,” *Nucl. Phys.* **B341** (1990) 50–100.

[102] J. Preskill, S. P. Trivedi, F. Wilczek, and M. B. Wise, “Cosmology and broken discrete symmetry,” *Nucl. Phys.* **B363** (1991) 207–220.

[103] T. Banks and M. Dine, “Note on discrete gauge anomalies,” *Phys. Rev.* **D45** (1992) 1424–1427, `arXiv:hep-th/9109045 [hep-th]`.

[104] L. E. Ibanez and G. G. Ross, “Discrete gauge symmetry anomalies,” *Phys. Lett.* **B260** (1991) 291–295.

[105] L. E. Ibanez, “More about discrete gauge anomalies,” *Nucl. Phys.* **B398** (1993) 301–318, `arXiv:hep-ph/9210211 [hep-ph]`.

[106] C. Csaki and H. Murayama, “Discrete anomaly matching,” *Nucl. Phys.* **B515** (1998) 114–162, `arXiv:hep-th/9710105 [hep-th]`.

[107] K. Kurosawa, N. Maru, and T. Yanagida, “Nonanomalous R symmetry in supersymmetric unified theories of quarks and leptons,” *Phys. Lett.* **B512** (2001) 203–210, `arXiv:hep-ph/0105136 [hep-ph]`.

[108] H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg, and P. K. S. Vaudrevange, “A unique $\mathbb{Z}_4^R$ symmetry for the MSSM,” *Phys. Lett.* **B694** (2011) 491–495, arXiv:1009.0905 [hep-ph].

[109] M. Fallbacher, M. Ratz, and P. K. S. Vaudrevange, “No-go theorems for R symmetries in four-dimensional GUTs,” *Phys. Lett.* **B705** (2011) 503–506, arXiv:1109.4797 [hep-ph].

[110] J. L. Evans, M. Ibe, J. Kehayias, and T. T. Yanagida, “Non-Anomalous Discrete R-symmetry Decrees Three Generations,” *Phys. Rev. Lett.* **109** (2012) 181801, arXiv:1111.2481 [hep-ph].

[111] K. I. Izawa and T. Yanagida, “R invariant natural unification,” *Prog. Theor. Phys.* **97** (1997) 913–920, arXiv:hep-ph/9703350 [hep-ph].

[112] K. Harigaya, M. Ibe, and M. Suzuki, “Mass-splitting between haves and have-nots — symmetry vs. Grand Unified Theory,” *JHEP* **09** (2015) 155, arXiv:1505.05024 [hep-ph].

[113] N. Seiberg, “Exact results on the space of vacua of four-dimensional SUSY gauge theories,” *Phys. Rev.* **D49** (1994) 6857–6863, arXiv:hep-th/9402044 [hep-th].

[114] M. Ibe, K. I. Izawa, Y. Nakayama, Y. Shinbara, and T. Yanagida, “Conformally sequestered SUSY breaking in vector-like gauge theories,” *Phys. Rev.* **D73** (2006) 015004, arXiv:hep-ph/0506023 [hep-ph].

[115] M. Ibe, K. I. Izawa, Y. Nakayama, Y. Shinbara, and T. Yanagida, “More on conformally sequestered SUSY breaking,” *Phys. Rev.* **D73** (2006) 035012, arXiv:hep-ph/0509229 [hep-ph].

[116] M. Ibe, Y. Nakayama, and T. T. Yanagida, “Conformal Gauge Mediation,” *Phys. Lett.* **B649** (2007) 292–298, arXiv:hep-ph/0703110 [HEP-PH].

[117] Z. Chacko, M. A. Luty, and E. Ponton, “Calculable dynamical supersymmetry breaking on deformed moduli spaces,” *JHEP* **12** (1998) 016, arXiv:hep-th/9810253 [hep-th].

[118] M. Ibe, Y. Nakayama, H. Murayama, and T. T. Yanagida, “Nambu-Goldstone Dark Matter and Cosmic Ray Electron and Positron Excess,” *JHEP* **04** (2009) 087, arXiv:0902.2914 [hep-ph].

[119] M. Ibe, R. Sato, T. T. Yanagida, and K. Yonekura, "Gravitino Dark Matter and Light Gluino in an R-invariant Low Scale Gauge Mediation," *JHEP* **04** (2011) 077, arXiv:1012.5466 [hep-ph].

[120] M. Dine and J. Mason, "Gauge mediation in metastable vacua," *Phys. Rev.* **D77** (2008) 016005, arXiv:hep-ph/0611312 [hep-ph].

[121] J. A. Casas and C. Munoz, "A Natural solution to the mu problem," *Phys. Lett.* **B306** (1993) 288–294, arXiv:hep-ph/9302227 [hep-ph].

[122] **ATLAS** Collaboration, G. Aad *et al.*, "Search for production of vector-like quark pairs and of four top quarks in the lepton-plus-jets final state in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector," *JHEP* **08** (2015) 105, arXiv:1505.04306 [hep-ex].

[123] **ATLAS** Collaboration, G. Aad *et al.*, "Analysis of events with b-jets and a pair of leptons of the same charge in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector," *JHEP* **10** (2015) 150, arXiv:1504.04605 [hep-ex].

[124] **ATLAS** Collaboration, G. Aad *et al.*, "Search for pair and single production of new heavy quarks that decay to a Z boson and a third-generation quark in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector," *JHEP* **11** (2014) 104, arXiv:1409.5500 [hep-ex].

[125] **ATLAS** Collaboration, G. Aad *et al.*, "Search for vector-like B quarks in events with one isolated lepton, missing transverse momentum and jets at $\sqrt{s} = 8$ TeV with the ATLAS detector," *Phys. Rev.* **D91** no. 11, (2015) 112011, arXiv:1503.05425 [hep-ex].

[126] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, "Gaugino mass without singlets," *JHEP* **12** (1998) 027, arXiv:hep-ph/9810442 [hep-ph].

[127] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking," *Nucl. Phys.* **B557** (1999) 79–118, arXiv:hep-th/9810155 [hep-th].

[128] B. Bhattacherjee, B. Feldstein, M. Ibe, S. Matsumoto, and T. T. Yanagida, "Pure gravity mediation of supersymmetry breaking at the Large Hadron Collider," *Phys. Rev.* **D87** no. 1, (2013) 015028, arXiv:1207.5453 [hep-ph].

[129] M. Ibe, T. Moroi, and T. T. Yanagida, "Possible Signals of Wino LSP at the Large Hadron Collider," *Phys. Lett.* **B644** (2007) 355–360, arXiv:hep-ph/0610277 [hep-ph].

[130] M. Ibe and T. T. Yanagida, "The Lightest Higgs Boson Mass in Pure Gravity Mediation Model," *Phys. Lett.* **B709** (2012) 374–380, arXiv:1112.2462 [hep-ph].

[131] M. Ibe, S. Matsumoto, and T. T. Yanagida, "Pure Gravity Mediation," *Phys. Rev.* **D85** (2012) 095011, arXiv:1202.2253 [hep-ph].

[132] G. F. Giudice and A. Romanino, "Split supersymmetry," *Nucl. Phys.* **B699** (2004) 65–89, arXiv:hep-ph/0406088 [hep-ph]. [Erratum: Nucl. Phys.B706,487(2005)].

[133] N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice, and A. Romanino, "Aspects of split supersymmetry," *Nucl. Phys.* **B709** (2005) 3–46, arXiv:hep-ph/0409232 [hep-ph].

[134] J. D. Wells, "PeV-scale supersymmetry," *Phys. Rev.* **D71** (2005) 015013, arXiv:hep-ph/0411041 [hep-ph].

[135] N. Arkani-Hamed, A. Gupta, D. E. Kaplan, N. Weiner, and T. Zorawski, "Simply Unnatural Supersymmetry," arXiv:1212.6971 [hep-ph].

[136] K. Harigaya, M. Ibe, and T. T. Yanagida, "A Closer Look at Gaugino Masses in Pure Gravity Mediation Model/Minimal Split SUSY Model," *JHEP* **12** (2013) 016, arXiv:1310.0643 [hep-ph].

[137] E. Calabrese *et al.*, "Cosmological Parameters from pre-Planck CMB Measurements: a 2017 Update," *Phys. Rev.* **D95** no. 6, (2017) 063525, arXiv:1702.03272 [astro-ph.CO].

[138] G. F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, "Towards a complete theory of thermal leptogenesis in the SM and MSSM," *Nucl. Phys.* **B685** (2004) 89–149, arXiv:hep-ph/0310123 [hep-ph].

[139] W. Buchmuller, R. D. Peccei, and T. Yanagida, "Leptogenesis as the origin of matter," *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 311–355, arXiv:hep-ph/0502169 [hep-ph].

[140] S. Davidson, E. Nardi, and Y. Nir, "Leptogenesis," *Phys. Rept.* **466** (2008) 105–177, arXiv:0802.2962 [hep-ph].

[141] J. H. Chang, R. Essig, and S. D. McDermott, "Supernova 1987A Constraints on Sub-GeV Dark Sectors, Millicharged Particles, the QCD Axion, and an Axion-like Particle," arXiv:1803.00993 [hep-ph].

[142] Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, "Are There Real Goldstone Bosons Associated with Broken Lepton Number?," *Phys. Lett.* **98B** (1981) 265–268.

[143] D. H. Lyth, "Axions and inflation: Sitting in the vacuum," *Phys. Rev.* **D45** (1992) 3394–3404.

[144] S. Borsanyi *et al.*, "Calculation of the axion mass based on high-temperature lattice quantum chromodynamics," *Nature* **539** no. 7627, (2016) 69–71, arXiv:1606.07494 [hep-lat].

[145] G. Ballesteros, J. Redondo, A. Ringwald, and C. Tamarit, "Standard Model—axion—seesaw—Higgs portal inflation. Five problems of particle physics and cosmology solved in one stroke," *JCAP* **1708** no. 08, (2017) 001, arXiv:1610.01639 [hep-ph].

[146] A. Ringwald, "Axion mass in the case of post-inflationary Peccei-Quinn symmetry breaking," 2018. arXiv:1805.09618 [hep-ph].

[147] E. W. Kolb and M. S. Turner, "The Early Universe," *Front. Phys.* **69** (1990) 1–547.

[148] M. Kawasaki, K. Kohri, and T. Moroi, "Big-Bang nucleosynthesis and hadronic decay of long-lived massive particles," *Phys. Rev.* **D71** (2005) 083502, arXiv:astro-ph/0408426 [astro-ph].

[149] K. Jedamzik, "Big bang nucleosynthesis constraints on hadronically and electromagnetically decaying relic neutral particles," *Phys. Rev.* **D74** (2006) 103509, arXiv:hep-ph/0604251 [hep-ph].

[150] M. Kawasaki, K. Kohri, T. Moroi, and Y. Takaesu, "Revisiting Big-Bang Nucleosynthesis Constraints on Long-Lived Decaying Particles," *Phys. Rev.* **D97** no. 2, (2018) 023502, arXiv:1709.01211 [hep-ph].

[151] M. Kawasaki, K. Nakayama, and M. Senami, "Cosmological implications of supersymmetric axion models," *JCAP* **0803** (2008) 009, arXiv:0711.3083 [hep-ph].

[152] A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Yu. S. Tyupkin, "Pseudoparticle Solutions of the Yang-Mills Equations," *Phys. Lett.* **B59** (1975) 85–87. [,350(1975)].

[153] M. Shifman, *Advanced topics in quantum field theory.* Cambridge Univ. Press, Cambridge, UK, 2012. http://www.cambridge.org/mw/academic/subjects/physics/theoretical-physics-and-mathematical-physics/advanced-topics-quantum-field-theory-lecture-course?format=AR.

[154] S. P. Martin, "A Supersymmetry primer," arXiv:hep-ph/9709356 [hep-ph]. [Adv. Ser. Direct. High Energy Phys.18,1(1998)].

[155] A. Salam and J. A. Strathdee, "Supergauge Transformations," *Nucl. Phys.* **B76** (1974) 477–482.

[156] S. Ferrara, J. Wess, and B. Zumino, "Supergauge Multiplets and Superfields," *Phys. Lett.* **51B** (1974) 239.

[157] M. T. Grisaru, W. Siegel, and M. Rocek, "Improved Methods for Supergraphs," *Nucl. Phys.* **B159** (1979) 429.

[158] N. Seiberg, "Naturalness versus supersymmetric nonrenormalization theorems," *Phys. Lett.* **B318** (1993) 469–475, arXiv:hep-ph/9309335 [hep-ph].

[159] S. Weinberg, *The quantum theory of fields. Vol. 3: Supersymmetry.* Cambridge University Press, 2013.

[160] S. Weinberg, "Nonrenormalization theorems in nonrenormalizable theories," *Phys. Rev. Lett.* **80** (1998) 3702–3705, arXiv:hep-th/9803099 [hep-th].

[161] A. H. Chamseddine, R. L. Arnowitt, and P. Nath, "Locally Supersymmetric Grand Unification," *Phys. Rev. Lett.* **49** (1982) 970.

[162] R. Barbieri, S. Ferrara, and C. A. Savoy, "Gauge Models with Spontaneously Broken Local Supersymmetry," *Phys. Lett.* **119B** (1982) 343.

[163] L. E. Ibanez, "Locally Supersymmetric SU(5) Grand Unification," *Phys. Lett.* **118B** (1982) 73–78.

[164] L. J. Hall, J. D. Lykken, and S. Weinberg, "Supergravity as the Messenger of Supersymmetry Breaking," *Phys. Rev.* **D27** (1983) 2359–2378.

[165] N. Ohta, "GRAND UNIFIED THEORIES BASED ON LOCAL SUPERSYMMETRY," *Prog. Theor. Phys.* **70** (1983) 542.

[166] J. R. Ellis, D. V. Nanopoulos, and K. Tamvakis, "Grand Unification in Simple Supergravity," *Phys. Lett.* **121B** (1983) 123–129.

[167] L. Alvarez-Gaume, J. Polchinski, and M. B. Wise, "Minimal Low-Energy Supergravity," *Nucl. Phys.* **B221** (1983) 495.

[168] J. Wess and J. Bagger, *Supersymmetry and supergravity.* Princeton University Press, Princeton, NJ, USA, 1992.

[169] I. Affleck, M. Dine, and N. Seiberg, "Dynamical Supersymmetry Breaking in Supersymmetric QCD," *Nucl. Phys.* **B241** (1984) 493–534.

[170] S. F. Cordes, "The Instanton Induced Superpotential in Supersymmetric QCD," *Nucl. Phys.* **B273** (1986) 629–648.

[171] N. Seiberg, "Electric - magnetic duality in supersymmetric nonAbelian gauge theories," *Nucl. Phys.* **B435** (1995) 129–146, `arXiv:hep-th/9411149 [hep-th]`.