論 文 の 内 容 の 要 旨

論文題目　Foundations of Algebraic Theories and Higher Dimensional Categories
（代数理論の基礎と高次元圏）

氏　　名　藤井　宗一郎

Universal algebra uniformly captures various algebraic structures, by expressing them as equational theories or abstract clones. The ubiquity of algebraic structures in mathematics and related fields has given rise to several variants of universal algebra, such as symmetric operads, non-symmetric operads, generalised operads, and monads. These variants of universal algebra are called notions of algebraic theory. Although notions of algebraic theory share the basic aim of providing a background theory to describe algebraic structures, they use various techniques to achieve this goal and, to the best of our knowledge, no general framework for notions of algebraic theory which includes all of the examples above was known. Such a framework would lead to a better understanding of notions of algebraic theory by revealing their essential structure, and provide a uniform way to compare different notions of algebraic theory. In the first part of this thesis, we develop a unified framework for notions of algebraic theory which includes all of the above examples. Our key observation is that each notion of algebraic theory can be identified with a monoidal category, in such a way that theories correspond to monoid objects therein. We introduce a categorical structure called metamodel, which underlies the definition of models of theories. The notion of metamodel subsumes not only the standard definitions of models but also non-standard ones, such as graded algebras of symmetric operads and relative algebras of monads on Set introduced by Hino, Kobayashi, Hasuo and Jacobs. We also consider morphisms between notions of algebraic theory, which are a monoidal version of profunctors. Every strong monoidal functor gives rise to an adjoint pair of such morphisms, and provides a uniform way to establish isomorphisms

between categories of models in different notions of algebraic theory. A general structure-semantics adjointness result and a double categorical universal property of categories of models are also shown.

In the second part of this thesis, we shift from the general study of algebraic structures, and focus on a particular algebraic structure: higher dimensional categories. Higher dimensional categories arise in such diverse fields as topology, mathematical physics and theoretical computer science. On the other hand, the structure of higher dimensional categories is quite complex and even their definition is known to be subtle. Among several existing definitions of higher dimensional categories, we choose to look at the one proposed by Batanin and later refined by Leinster. In Batanin and Leinster's approach, higher dimensional categories are defined as models of a certain generalised operad, hence it falls within the unified framework developed in the first part of this thesis. Batanin and Leinster's definition has also been used by van den Berg, Garner and Lumsdaine to describe the higher dimensional structures of types in Martin-Löf intensional type theory. We show that the notion of extensive category plays a central role in Batanin and Leinster's definition. Using this, we generalise their definition by allowing enrichment over any locally presentable extensive category.