論文の内容の要旨

Abstract

論文題目

Discrete Convexity in Valued Constraint Satisfaction Problems:
An Approach by Quadratic M-Convexity
（値付き制約充足問題における離散凸性：2次M凸性によるアプローチ）

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The theme of this thesis is valued constraint satisfaction problem (VCSP), discrete convex analysis (DCA), and their interplay, where VCSP and DCA have been developed independently.

The VCSP is the minimization problem of a function represented as the sum of functions with few variables. One of fundamental subclasses of VCSP is binary VCSP, which is VCSP with only unary and binary functions. VCSP and binary VCSP naturally arise from problems in the real world, and have been widely studied in different contexts of computer science. Although the (binary) VCSP is NP-hard in general, some classes of (binary) VCSP instances can be solved in polynomial time. An important line of research is to investigate what restrictions on classes of VCSP instances ensure tractability.

DCA is a theory of convex functions on the integer lattice developed by Murota and his collaborators. In DCA, two kinds of discrete convexity, called L-convexity and M-convexity, play a central role, which appear as underlying mathematical structures in a variety of polynomially solvable problems. Concepts and properties of discrete convexity inspired by convex analysis, such as duality and conjugacy, provide a unified theoretical framework for solving tractable problems efficiently.

In this thesis, we exploit hidden discrete convexity, particularly, quadratic M-convexity, in a tractable class of binary VCSP instances, and derive a new tractable class by utilizing the revealed quadratic M-convexity.

In binary VCSPs, Cooper and Živný classified the tractability in terms of the notion of “triangle,” and showed that the only interesting tractable case in this setting is the

one induced by the joint winner property (JWP). We show that a binary VCSP instance satisfying the JWP can be transformed into the sum of two quadratic M-convex functions. Moreover we can obtain the two M-convex summands in polynomial time. The sum of two M-convex functions, called an M_2-convex function, can be minimized in polynomial time provided the value oracles of the two M-convex functions are available. Hence our result gives a DCA interpretation of the tractability of binary VCSP instances having the JWP.

Following the above DCA interpretation, we consider the problem named TESTING QUADRATIC M_2-REPRESENTABILITY: What binary VCSP instances can be transformed into the sum of two quadratic M-convex functions? We present a polynomial-time algorithm to solve TESTING QUADRATIC M_2-REPRESENTABILITY. To prove the correctness of our proposed algorithm, we establish a sharp characterization of the quadratic M-convexity in terms of a quadratic coefficient. This characterization itself is a fundamental contribution to DCA. The polynomial-time solvability of TESTING QUADRATIC M_2-REPRESENTABILITY implies that the representable binary VCSP instances constitute a tractable class of binary VCSPs. This tractable class is a novel one, which properly contains the JWP class.

Quadratic M-convex functions are closely related to metrics induced by trees. Based on this relation, we reveal that the problem of reconstructing a phylogenetic tree has a similar structure to TESTING QUADRATIC M_2-REPRESENTABILITY. A phylogenetic tree is a graphical representation of an evolutionary history in a set of taxa in which the leaves correspond to taxa and the non-leaves correspond to speciations. One of important problems in phylogenetic analysis is to assemble a global phylogenetic tree from smaller pieces of phylogenetic trees, particularly, quartet trees. Although the problem is NP-hard in general, we introduced two novel classes of quartet systems, named complete multipartitite quartet systems and full multipartite quartet systems, and showed that a variant of our proposed algorithm for TESTING QUADRATIC M_2-REPRESENTABILITY can reconstruct a phylogenetic tree from these quartet systems in polynomial time.