博士論文(要約)

Geometric Numerical Integration of Evolutionary Differential Equations with Constraints

(拘束条件つき発展方程式に対する幾何学的数値計算法)

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Abstract

In this thesis, we deal with geometric numerical integration of evolutionary differential equations with constraints. Geometric numerical integration methods, which are also referred to as structure-preserving methods, are specific purpose methods inheriting some geometric properties, such as energy conservation law, of original evolutionary differential equations. They often provide us with qualitatively better numerical solutions in comparison with general purpose methods, in particular, for numerically tough problems. Therefore, in response to the trend toward more realistic numerical simulations involving model equations with increasing complexity, the demand for geometric numerical integration methods has rapidly increased. As a result of intensive studies toward this direction in the last half-century, geometric numerical integration methods for ordinary differential equations (ODEs) have been well developed. Moreover, the studies have been extended to evolutionary partial differential equations (PDEs) over the recent decades. Nevertheless, surprisingly, extension to constrained cases, i.e., differential algebraic equations (DAEs), has been quite limited so far. In the present thesis, we address the necessity of further extensions to constrained cases, and establish some foundations.

First, we develop a new framework of numerical methods for an emerging class of evolutionary PDEs implicitly having a constraint. There, an essential observation is that such PDEs are actually DAEs on function spaces, whereas standard ones are often regarded as ODEs. This viewpoint not only facilitates numerical computations but also encourage further PDE-theoretical studies. Second, we deal with geometric numerical integration methods for finite-dimensional DAEs with a conservation or dissipation law, which often appear as a result of spatial discretization of PDEs with constraints. There, since even the concept of conservation law itself has not yet been discussed, the study begins with developing several foundations in continuous level. Finally, we slightly change the direction of research and deal with multiple invariants preservation in standard (i.e., unconstrained) evolutionary PDEs. There, we propose a strategy of reformulating a given PDE into a constrained system by regarding some of the conserved quantities as constraints. The last contribution implies that the development of numerical methods for constrained systems has also an impact on unconstrained cases.