博士論文（要約）

Connectivity in Networks: Arborescence Packing and Bidirected Graphs

（ネットワークの連結性：有向木詰め込みと双向グラフ）

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Abstract

Connectivity of graphs is a fundamental theme in graph algorithms. There are different measures for connectivity of graphs depending on their focus. Edge and vertex-connectivities of undirected graphs are the minimum numbers of edges and vertices whose deletion makes the input graph disconnected, respectively. In directed graphs, rooted-connectivity is the minimum size of an arc set that enters a vertex subset not including the specified root vertex. Partition-connectivity is related to the number of edges between different components for all partitions of the vertex set in undirected graphs.

Tree/arborescence packing is a set of edge/arc-disjoint trees/arborescences. It was shown by Tutte (1961) and Nash-Williams (1961) that partition-connectivity is equal to the maximum number of disjoint spanning trees. Similarly, Edmonds (1973) showed that rooted-connectivity is equal to the maximum number of spanning arborescences rooted at the specified root vertex. Thus, tree and arborescence packing problems are closely related to connectivity of graphs.

In this thesis, we focus on arborescence packing and strong connectivity of bidirected graphs. In particular, we devise algorithms for the following three problems.

The first problem is branching packing with conditions on arc capacity and an polymatroid. For the spanning arborescence packing problem and the spanning branching packing problem, Edmonds (1973) gave a necessary and sufficient condition for the existence of a feasible solution. Later, Durand de Gevigney–Nguyen–Szigeti (2013) gave a necessary and sufficient condition for the existence of a feasible solution, and a polynomial-time algorithm for a generalized problem of the spanning arborescence packing problem with a constraint on matroid bases. We deal with a further generalized problem of this by arc capacity. For a problem called the splittable version, we devise a strongly polynomial-time algorithm.

The second problem is mixed arborescence packing with a condition on reachability. Kamiyama–Katoh–Takizawa (2009) gave a necessary and sufficient condition for the existence of a feasible solution, and a polynomial-time algorithm for the problem of packing rooted-arborescences that span the reachable vertex set from each root. Fortier–Király–Léonard–Szigeti–Talon (2018) mentioned that it had not been shown whether the above result can be extended to the mixed graph case.

We answer this question positively by giving a polynomial-time algorithm, and a necessary and sufficient condition for the existence of a packing.

The last problem is to make bidirected graphs strongly connected by adding arcs. A bidirected graph consists of a vertex set and an arc set each element of which has one sign or two signs. Bidirected graphs are common generalizations of undirected graphs and directed graphs. Eswaran–Tarjan (1976) investigated the problem of making a given directed graph strognly connected by adding arcs. Ando–Fujishige–Nemoto (1996) defined the strong connectivity for bidirected graphs and devised a linear-time algorithm for strongly connected component decomposition for bidirected graphs. We consider two problems of making a given bidirected graph strongly connected by adding arcs: (i) with the minimum number of signs and (ii) with the minimum number of arcs. For the former one, we show a closed formula of the minimum number of additional signs and devise a linear-time algorithm for finding an optimal solution. For the latter problem, we give a linear-time algorithm for finding a feasible solution whose size is more than that of a optimal solution at most by one.