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Optimal phase estimation of ultrasonic phased array based on Bayesian optimization
(ベイズ最適化に基づく超音波フェーズドアレイの最適位相推定)

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1. Introduction

An ultrasound phased array is an array of ultrasound transducers whose phases can be individually controlled. The ultrasound waves emitted from these transducers interfere with each other and form a sound field in space.

In general, it's hard to solve the problem of generating complex sound fields intervening obstacles. The greedy algorithm [1] can optimize the phase on such situation but it cost too much operation time. Time reversal method [2] is known to regenerate the focus field even in inhomogeneous medium, however it is difficult to perform if there is no source signal.

We propose a method using Bayesian Optimization (BO) to address the problem above. We make the process by which the phased array generates a field a black box function and apply BO. With this schema, we only have to observe the field, then calculate the error to the target field.

In this research, we attempted to generate the focus field in homogeneous and inhomogeneous medium to analyzed the performance of BO. Besides, we will compare the result with physical calculation, greedy algorithm, and time reversal method (only in inhomogeneous medium).

All the experiments and the proposed method are based on the simulation to estimate the optimal phase.

2. Method

2.1 Focus generation

Let the phase of the i-th transducer be ϕ_i and the position be $\boldsymbol{r}_i$. Assuming the sound emitted from the transducer is a spherical wave, the complex sound field $p(\boldsymbol{r})$ generated by n transducers at position $\boldsymbol{r}$ is expressed as:

$$p(\boldsymbol{r};\phi_1,..,\phi_n) = \sum_{i=1}^{n} \mathrm{e}^{-\mathrm{j}(k|\boldsymbol{r}-\boldsymbol{r}_i|)}\mathrm{e}^{\mathrm{j}\phi_i} \quad (1)$$

where k is a wavenumber. To create a focus at a position $\boldsymbol{r}_f$, we should set ϕ_i as follows:

$$\phi_i^* = k|\boldsymbol{r}_f - \boldsymbol{r}_i|$$

In this paper, we set the sound field p^* generated by this ϕ_i^* as the target. That is, we optimize the following function $f(\phi_1,..,\phi_n)$.

$$f(\phi_1,..,\phi_n) = \sqrt{\frac{1}{V}\int_V d\boldsymbol{r}\,|p^* - p(\boldsymbol{r};\phi_1,..,\phi_n)|^2} \quad (2)$$

We used 16 transducers arranged in a 4 by 4 grid. Note that we use eq. (1) only for simulation, and this is not inherently necessary for BO.

2.2 Bayesian Optimization

Assuming there is a black box model $f(x)$, the purpose of BO is to find an input x^* to make this model get the minimum (or maximum) , that is $x^* = \mathrm{argmin}_{x\in X} f(x)$. In our study, the phases of 16 transducers are the input, and the error between the observed field and target field is the output.

This black box model is usually assumed to be a Gaussian process (GP) for optimization. We have to determine the mean function

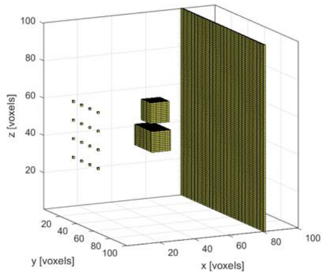

Figure 1 position of obstacles

(usually set to 0) and the covariance function of the Gaussian process. We chose Matérn 5/2 kernel, which makes less restrictive smoothness assumptions.

Then, we chose the acquisition function, MPI, to observe the GP.

Finally, in BO algorithm, repeat the following steps within the specified number of iteration times t.

1. Find x_t by optimizing the acquisition function over the GP: $x_t = argmax_x \mu(x|D_{1:t-1})$.
2. Sample the objective function: $y_t = f(x_t)$.
3. Augment $D_{1:t} = \{D_{1:t-1}, (x_t, y_t)\}$ and update GP.

3. Experiment

3.1 Homogeneous medium

We set the speed of sound to 340m/s and the air density to 1.293 $\mathrm{kg/m^3}$. And we set the range of the observation surface as follows: x-axis: -50mm to 50mm, y-axis: -50mm to 50mm, z-axis is set at a height of 80mm. The simulation space is a 100mm × 100mm × 100mm cube.

We let the phase as input and determine the bounds of the exploration. In general, the phase has periodicity, so the range is set to $[0,2\pi)$. Because of the periodicity, the sound field produced is not changed if all the phases of the phased array have a uniform offset. This can lead to poor convergent of the BO optimization. Thus, we set the phase of one of the transducers to 0. We set the iteration number at 100 times to ensure it will converge.

In this part, beside the physical calculation, we also apply greedy algorithm to compare the result with BO.

3.2 Inhomogeneous medium

In the inhomogeneous medium, the transducers and the observation surface are as same as the homogeneous medium situation. The difference is that we placed 4 cubes with a length of 10mm between the transducers and the observation surface and let them overlap as a whole as obstacles. The density of each cube is 100kg/m^3, and the speed of sound is 340m/s, as shown in Figure 1.

In this situation, except the physical calculation, greedy algorithm, and BO, we perform time reversal as the ground truth.

4. Result

4.1 Homogeneous medium

In Figure 2, from left to right are the results of physical calculation, greedy algorithm, and BO. The top three figures are the 16 transducer phases obtained by each method, and the figures on the bottom are the field generated by the phases. The result of BO is one out of a hundred experiments. The maximum pressure of physical calculation is 0.0119, and greedy algorithm is 0.0118. The average of the maximum pressure of BO is 0.0106. The evaluation graph of 100 times experiments of

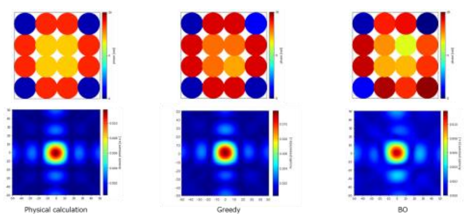

Figure 2 result of homogeneous

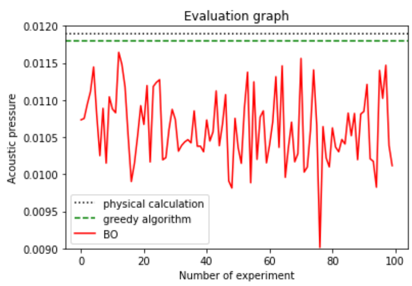

Figure 3 Evaluation graph

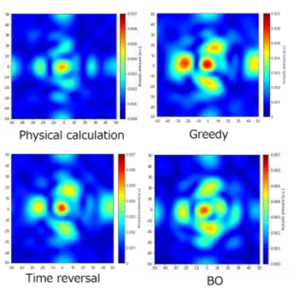

Figure 4 field of inhomogeneous

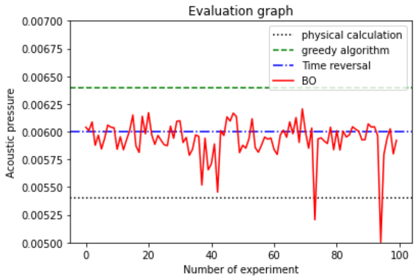

Figure 5 Evaluation graph

BO with other methods is in figure 3. And all the experiment generated focus successfully. For the operation time, physical calculation is 1, greedy algorithm is 256 (16 transducers ×16 equally divided phases), and BO is averaged at 68.68 (iteration times).

4.2 Inhomogeneous medium

We reused the phase of physical calculation in homogeneous that generated the focus, and make it transmit ultrasonic waves through obstacles and form a field on the observation surface as at the top left of Figure 4. The maximum pressure is 0.0054. Greedy algorithm optimized the phase again, and the maximum pressure is 0.0064 (top right). Time reversal result is at the bottom left, and its pressure is 0.0060. Finally, one of the BO result is shown at the bottom right. The average maximum pressure of 100 times experiment is 0.0059. As the operation time, physical calculation is 1, greedy algorithm is 256, time reversal is 2, and BO is averaged at 68.21. Evaluation graph is in figure 5.

5. Conclusion

In homogeneous medium, BO optimized the phase and generate the focus field successfully, however, the pressure of the field is weaker than physical calculation and greedy algorithm. In inhomogeneous medium, physical calculation could not generate a focus field. BO optimized the phase and field again, and those fields have almost same pressure as time reversal did. Greedy still preform best while costed the most operation time.

In summary, whether in homogeneous or inhomogeneous conditions, it is feasible to use Bayesian Optimization to optimize the phase and preform beam focusing. If the phase of ultrasound phased array and the objective field need to be observed, and optimized the phase in less time, BO is a good choice.

References

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