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Planetesimal Accretion under  
a Realistic Accretion Condition

(現実的な合体条件を考慮した微惑星集積)

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## Abstract

Terrestrial planets, ice giants and the cores of gas giants are thought to be formed by the accumulation of planetesimals. There has been no research on accumulation correctly evaluating the merging criteria at the time of planetesimal collision by using  $N$ -body simulation, while a recent study used the conditions of protoplanet coalescence. In order to properly know the accretion process of planetesimals, it is necessary to clarify the merging criteria of the planetesimal including the rebound.

The rotation of protoplanet in the giant impact stage of planetesimal accretion affects the merging criteria of protoplanets and that becomes the initial condition of the study of terrestrial planet's rotation. The origin of Mars's rotation is revealed by the study of protoplanet's rotation because Mars is the survivor of protoplanets. The study of protoplanet's rotation is limited, in particular, the study using  $N$ -body simulation as first-principle calculation doesn't exist. The method is useful to know the mass, the velocity and the spatial distribution of planetesimals during the accretion process. Because the evolution of the distributions affects the rotation angular momentum of protoplanets, the study using  $N$ -body simulation is needed to know the realistic rotation of protoplanets, including Mars. In addition, the rebound of colliding planetesimals affects the rotation of protoplanets. Thus, the simulation needs to include the realistic merging criteria of planetesimals.

In this study, as the first step, we investigate conditions that determine coalescence vs. rebound (merging criteria) by numerically colliding undifferentiated rocky planetesimals, undifferentiated icy planetesimals, and differentiated icy planetesimals using Smoothed Particle Hydrodynamics (SPH). We vary the total mass, mass ratio, collision speed, and collision angle of the colliding planetesimals. We investigate the critical impact velocity distinguishing coalescence from the rebound by a radical change of the largest remnant's mass represented against impact velocity. The critical impact velocity normalized by the escape velocity depends on the mass ratio of planetesimals and the impact angles. The critical impact velocity normalized by the escape velocity decreases with the target mass increasing relative to impactor mass, and decreases with increasing the impact angle whose maximum value shows us a grazing collision. The critical impact velocity is independent of the total mass of the planetesimals. This condition has a very small dependence on the composition and internal structure of the planetesimals. From the above results, we formulate the critical impact velocity as a variable for the planetesimals' mass ratio and collision angle.

As the second step, we investigate the accretion process of planetesimals to know the formation process and the rotation of protoplanets, including Mars, by using  $N$ -body simulation code named GPLUM (Global PLanetary system simulation code with Mass-dependent cut-off method). We apply to GPLUM the merging criteria necessary to account for the bounce of planetesimals. We set two hundred thousand rocky 100km-sized planetesimals as a narrow ring around the sun and calculate the orbits of the planetesimals by using  $N$ -body simulation.

We compare the results of imperfect coalescence case and the results of perfect coalescence case, where all colliding planetesimals merge. Protoplanets grow similarly in each case but the time for sweeping surrounding planetesimals elongates in the imperfect coalescence case. The mass distribution is bipolarized since the growth of planetesimals is prevented by the rebound. Then, runaway growth and oligarchic growth becomes more prominent than the perfect coalescence case.

For both cases, the spin angular velocity rapidly increases when the collision with massive planetesimal occur. However, the angular velocity decrease with mass increasing by planetesimals accretion from random directions. The mean spin angular velocity of planetesimals and protoplanets under the imperfect coalescence case is 70% and 30% smaller than the perfect coalescence case, respectively. The obliquity of protoplanets distribute in a wide range of angle and they have the peak around  $90^\circ$ , which is parallel to the ecliptic plane of the planetary system. The distribution is almost isotropic, which means the obliquity is decided by the stochastic component of angular momentum.

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This is the general introduction.

Takashi Shibata

# 1

## Introduction

The planets found in the solar system are rich in diversity, and analytical and numerical theoretical studies have been conducted to clarify the formation process. The basic framework of the solar system formation theory was created in the 1980s, and numerical studies using supercomputers are being conducted in addition to classical analytical studies. In addition, since the 1990s,

observation of exoplanets has progressed, and various planets including Hot Jupiter have been found. In order to explain the formation process, a huge amount of efforts is being made to generalize the model of the solar system formation process that has been made so far.

## 1.1 Observation of Planets

### 1.1.1 Solar System

The solar system is composed of several planets, satellites, and the other small objects (e.g., asteroids, trans Neptunian objects and comets). The orbital elements and physical characteristics of all planets in the solar system are shown in Figure 1.1.1.

The planets are categorized into terrestrial planets (Mercury, Venus, Earth, Mars), gas giants (Jupiter, Saturn), ice giants (Uranus, Neptune) and they have a significantly different composition, mass and semi-major axis each other. Terrestrial planets have small semi-major axis ( $a \leq 1.5\text{AU}$ , where AU is the astronomical unit, i.e., the average distance between the sun and the Earth,  $1.5 \times 10^{13}$  cm), small mass (masses  $M \leq M_{\oplus}$ , where  $M_{\oplus}$  is the Earth mass,  $6.0 \times 10^{27}$  g) and are composed of rocky dust. Gas giants are heavy planets ( $M \geq 10^2 M_{\oplus}$ ), mainly composed of H/He gas and locate on  $a \sim 5 - 10$  AU as semi-major axis. Ice giants have the mass  $M \sim 10 M_{\oplus}$ , are composed of  $\text{H}_2\text{O}/\text{CH}_4/\text{NH}_3$  ice and locate on the region far away from Sun ( $a \sim 20 - 30$  AU). The solar system is thought to be formed from a protoplanetary disk surrounding protosun because all of the

planets orbiting around the sun have a coplanar orbit.

The planets have diverse rotation property. The rotation period of terrestrial planets has a wide range from one day to several hundreds days (Bakich 2000). Although the spin axis of Mercury is vertical from own orbital plane, as well as its rotation period, those properties must be affected by the tidal force from the sun and changed significantly from initial condition (Makarov 2012). The spin axis of Venus inclines over  $177^\circ$ , which shows the inverse rotation of the Earth. Two gas giants including Jupiter and Saturn have similar rotation speed. Although ice giants also have similar rotation speed each other, Uranus has parallel spin axis to the orbital plane. The giant impact by Earth sized object and the gentle change during the migration may be able to explain the tilt of Uranus's rotation (Boué and Laskar 2010; Safronov 1966).

### 1.1.2 Exoplanets

Sun is one of the standard G-type star in the galaxy. It is expected that planetary systems can exist around stars typically because planets can be formed during the formation of a star. Although the hunting exoplanets (planets orbiting a star) started from 1940s, we could not detect them until the end of the last century. The first exoplanet called 51 Pegasi b around a main sequence star has been detected at 1995 by using Doppler-shift method that observes the oscillation of main star by the orbital motion of planets (Mayor and Queloz 1995). Although the planet is a gas giant whose mass is  $1 - 100M_\oplus$ , the semi-major axis is 0.05 AU and it orbits around the central star with the orbital period of 4 days. The gas

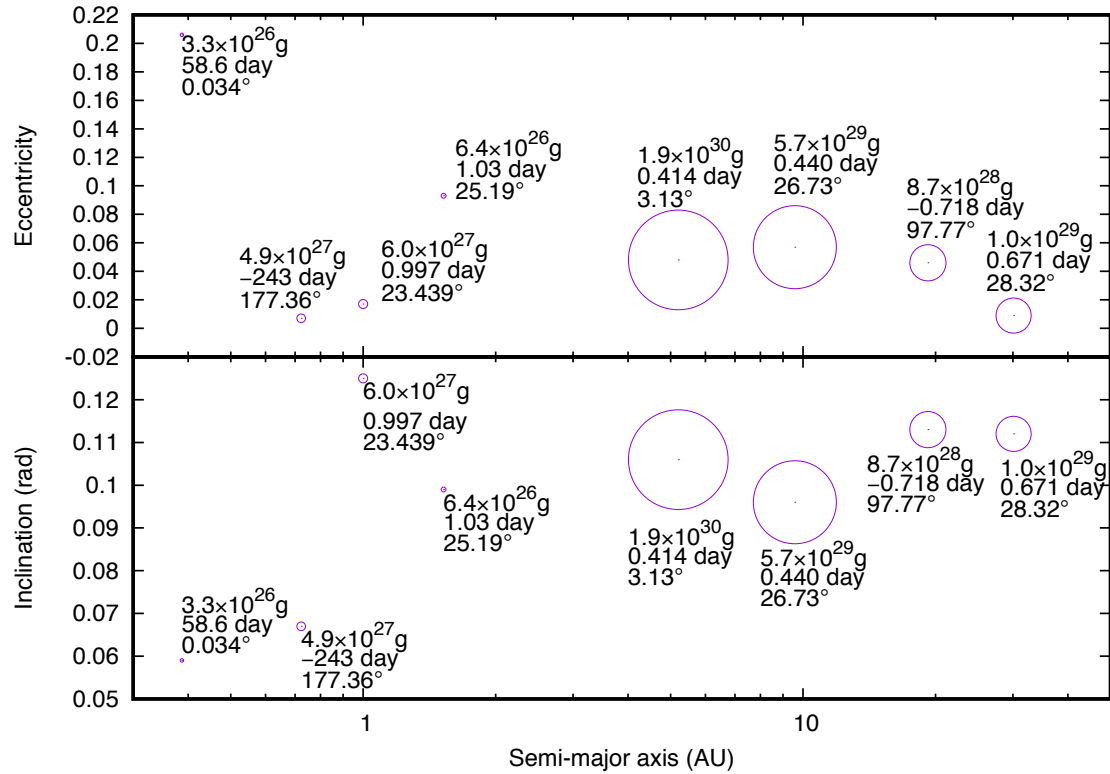


Figure 1.1.1: The orbital elements and the physical properties of planets in the solar system. Eccentricity and Inclination are shown against the planet's semi-major axis. The labels in the figure show mass, rotational period and obliquity based on their orbital plane of the planets. The size of circles is proportional to the radius of planets.



giants close to their central star is called hot Jupiter. Currently, there are huge amounts of exoplanets significantly different from the planets in the solar system, whose orbit is highly elliptic and has high eccentricity (Fig.1.1.2). Since [Rivera et al. \(2005\)](#) found a super Earth ( $M \leq 10M_{\oplus}$ ) ([Valencia et al. 2007](#)) in 2005, it is possible to detect small solid planets including super Neptune ( $20M_{\oplus} \leq M \leq 80M_{\oplus}$ ) close to central star ([Bonomo et al. 2014](#)). In 2017, Kepler space telescope found over 4000 candidates of exoplanets ([Batalha et al. 2013](#); [Thompson et al. 2017](#)). This telescope can detect an eclipse of a star by orbiting planets (Transit method). [Thompson et al. \(2017\)](#) found new 10 candidates of Earth sized planets orbiting inside of habitable zone. We summarize the mass distribution of exoplanets detected before (Fig.1.1.3). The theory construction of the formation process of various planets started from 1960s.

## 1.2 Standard Senario of Planetary Formation

The standard scenario of solar system formation is constructed from 1960s to 1980s ([Hayashi et al. 1985](#)). The standard flow of the standard scenario is below (Fig. 1.2.1):

1. A protoplanetary disk consisting of dust and gas was formed around the primordial sun,
2. The aggregation of dusts produces km-sized planetesimals,

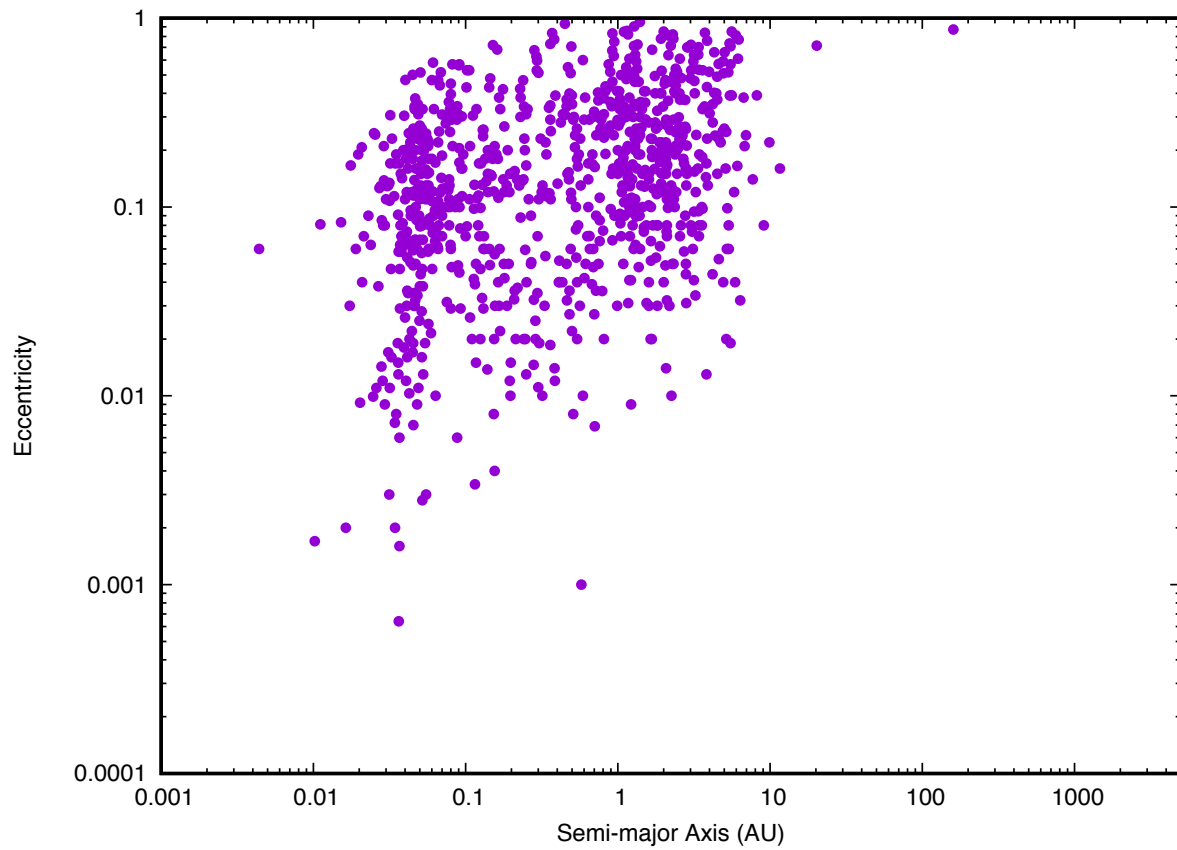


Figure 1.1.2: The eccentricity of exoplanets plotted against their semi-major axis. The data are selected from NASA EXOPLANET ARCHIVE (<https://exoplanetarchive.ipac.caltech.edu/index.html>).

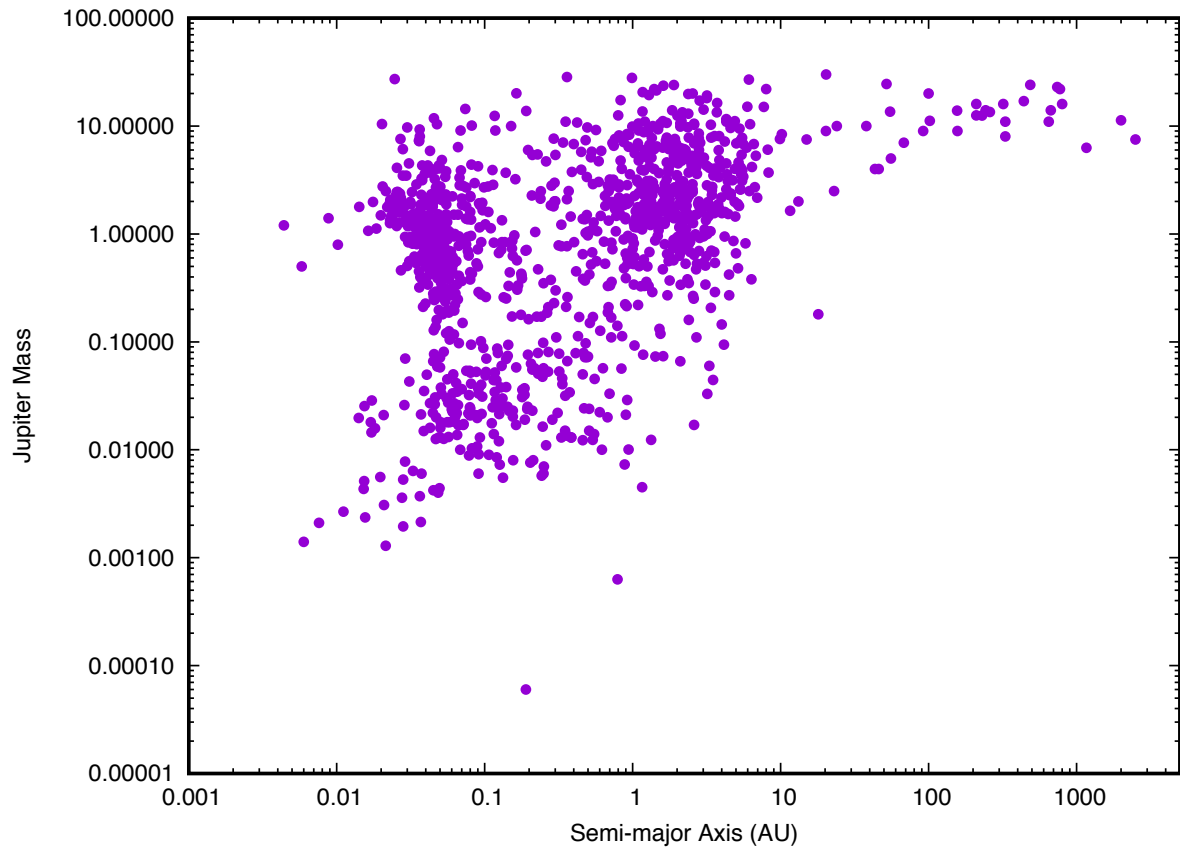


Figure 1.1.3: The mass of exoplanets plotted against their semi-major axis. The data are selected from NASA EXOPLANET ARCHIVE (<https://exoplanetarchive.ipac.caltech.edu/index.html>).

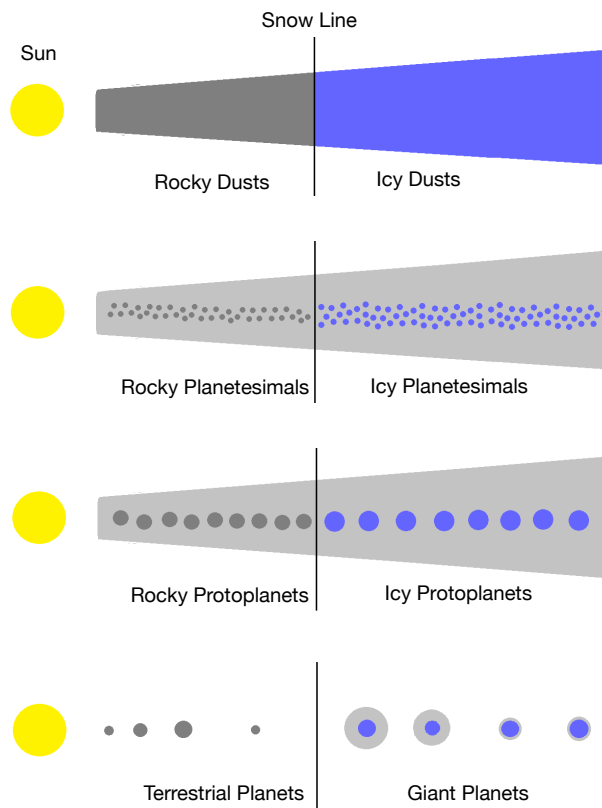


Figure 1.2.1: Standard Senario of Planetary Formation

3. The planetesimal accretion by the mutual gravity forms protoplanet,
4. The intersecting orbits by gravitational scattering leads to giant impact and forms planets.

The process of planetesimal accretion and planetary formation is summarized below.

### 1.2.1 Minimum Mass Solar Nebula

The standard protoplanetary disk model is given based on the minimum mass solar nebula (MMSN) model (Hayashi 1981). MMSN model is constructed by determining the amount of dust according to the amount of solid component existing in the Solar System. The amount of gas in this model is determined by gas to dust ratio, which is assumed from the amount of gas in the Solar System. In the standard disk model, the gas and the solid surface density are assumed as

$$\Sigma_{\text{gas}} = 2400 f_{\text{gas}} \left( \frac{r}{1\text{AU}} \right)^{-3/2} \text{ g/cm}^2, \quad (1.1)$$

$$\Sigma_{\text{dust}} = 10 f_{\text{dust}} \eta_{\text{ice}} \left( \frac{r}{1\text{AU}} \right)^{-3/2} \text{ g/cm}^2, \quad (1.2)$$

where  $r$  is the distance from the sun. The value  $\eta_{\text{ice}}$  is the enrichment factor of the surface density of solid material outside of the snow line, which is the boundary of  $\text{H}_2\text{O}$  condensation.  $\eta_{\text{ice}} = 1$  inside of snow line,  $r < r_{\text{ice}}$ , and  $\eta_{\text{ice}} \simeq 3 - 4$  outside of snow line,  $r > r_{\text{ice}}$ , where  $r_{\text{ice}}$  is the distance of snow line from the sun.

For the Solar System,  $r_{\text{ice}} \simeq 2.7\text{AU}$ . The value  $f_{\text{gas}}$  and  $f_{\text{dust}}$  are scaling factors of gas and solid material, respectively. In the MMSN model,  $f_{\text{gas}} = f_{\text{dust}} = 0.71$ .

### 1.2.2 Planetesimal Formation

The protoplanetary disk surrounding sun contained micron or sub-micron sized solid materials. The small objects are called dust and aggregate by collisions and gravitational interaction. The compressed materials form the km sized planetesimals. The major composition of planetesimals is rock and iron in the close region to the sun, on the other hand, Ice and Rock in the further region from the sun. The difficulty of planetesimal formation due to the gas drag in a protoplanetary disk is pointed out. Currently, although we have several candidates to solve the problems (e.g., [Kataoka et al. 2013](#)), the realistic formation process is unknown.

### 1.2.3 Formation of Protoplanets

The cross section for collisions of growing planetesimals increases by the gravity and catch other planetesimals efficiently. The planetesimal that grows rapidly than other planetesimals by the strong gravity is called protoplanet and the mass ratio between protoplanet and planetesimals increases with time evolution ([Kokubo and Ida 1996](#)). The gravitational scattering from protoplanet increases the velocity dispersion of planetesimals around the protoplanet. It becomes difficult to catch neighbor planetesimals for protoplanets and their growing speed decelerates from the ones close to the sun. The growth rate of

protoplanets that locate on the outer region of protoplanetary disk catches up with that in the inner region. Then, protoplanets grow orderly in the wide region of the disk (Kokubo and Ida 2000). On the other hand, the mass ratio between protoplanet and neighbor planetesimals increases. Orbital repulsion by the gravitational scattering of protoplanets and the dynamical friction from surrounding planetesimals makes the orbital interval of protoplanets wider. The mass of protoplanets that caught all surrounding planetesimals is called isolation mass and the protoplanet ends growing (Lissauer 1993).

#### 1.2.4 Formation of Planets

The circularized orbit of protoplanets by dynamical friction intersects mutually by the gravitational scattering of protoplanets and leads to collisions of protoplanets. These collisions are called giant impact and terrestrial planets are formed from several protoplanets through these collisions. Moon is formed from the circumplanetary disk consisting of the debris of the final collision (Ida et al. 1997). Gas giants are formed on the outside of snow line. There are two stories for the formation of gas giants. The accretion of icy planetesimals forms the core of gas giants whose mass is over  $10M_{\oplus}$  and surrounding gas falls into the core. This model is called core accretion model (Mizuno 1980; Perri and Cameron 1974; Rice and Armitage 2003; Ward 1989). On the other hand, gravitational instability in the protoplanetary disk compress dust and gas, and forms gas giants. This model is called gravitational instability model (Cameron 1978; Kuiper 1951).

## 1.2.5 Problems

Standard scenario of planet formation has serious problems.

### 1.2.5.1 Difficulty of Planetesimal Formation

No one explained the formation process of planetesimals from dust. Although the gravitational instability of dust can be a candidate for the scenario of planetesimal formation (Goldreich and Ward 1973), it is difficult to compress the dust by self-gravity until a large solid body is formed (Cuzzi et al. 1993). Dust aggregate can be another candidate (Benz 2000). Unfortunately, most of the dust must fall down onto the central star by the gas drag of the disk before they grow enough large (Adachi et al. 1976).

### 1.2.5.2 Migration of Planetesimal and Protoplanet

Even if planetesimals are formed from dust, protoplanet constituted of planetesimals (Kokubo and Ida 2002) migrates into the central star, which is called type-I migration (Goldreich and Tremaine 1979; Masset et al. 2006; Tanaka et al. 2002; Ward 1997). In the standard scenario, gas giants also migrate to the position close to the central star in a short period, which is called type-II migration (Trilling et al. 1998, 2002). It makes it difficult to explain the formation of gas giants in the solar system.



### 1.2.5.3 Long-term Formation of Ice Giants

For ice giants, core accretion model cannot explain the formation process because it takes a crucially long time to make the core of ice giants and it is difficult to end the formation before the end of the solar system. Note that there are several candidates as solutions. Ice giants can be formed in a shorter period if they migrate to the outer region of the disk from the inner region (Malhotra 1993, 1995).

Nice model is claimed for the migration of ice giants (Tsiganis et al. 2005). This is a model that predicts that the orbits of ice giants that exist outside Jupiter are expanding toward the outside of the disk in a short period if the four ice giants have narrower orbit intervals than the current ones. If this is possible, it is possible to form the icy planet in a short time at the region close to the central star and then carry it to the current orbit.

Grand tack model is suggested to form four ice giants in a narrow region in the disk (Walsh et al. 2011). In this model, Jupiter migrated to the region of Mars by type-II enters the orbital resonance with Saturn migrated following Jupiter. Jupiter obtains positive orbital angular momentum and migrates to the outside of the disk.

In this case, ice giants can exist with the narrow region and narrow orbital intervals because ice giants formed outside of the orbit of gas giants are pushed to the outside of the disk.

Anyway, this is a candidate for the solution to this problem. We need further studies for the general formation process of planets.

## 1.3 Planetesimal Accretion

Although there are several serious problems in the formation process of planetesimals, we can observe a huge amount of solid bodies such as asteroids. Thus, we assume that a formation process worked and planetesimals were formed.

### 1.3.1 Methods

The accretion process of planetesimals is semi-analytically investigated by using coagulation equation (Wetherill 1990) under particle-in-a-box approximation (called statistical method) (e.g., Barge and Pellat 1991; Greenberg et al. 1978; Nakagawa et al. 1983; Weidenschilling et al. 1997; Wetherill and Stewart 1989).

In this method, the probability of collision as the function of the mass and velocity of planetesimals which are known from the calculation of the evolution of the velocity distribution following dynamical theories is used for the calculation of the mass distribution evolution by the accretion. The advantage of the statistic method is that we can deal with a huge amount of objects.

Coagulation equation does not have the information about the position of planetesimals and assumes uniform spatial distribution. This assumption is unavailable after the nonconstant spacial structure is formed by large planetesimals. It is difficult to investigate the planetesimal accretion process with various spacial structure by using the coagulation equation.

A complementary method of the statistical approach is  $N$ -body simulation.

$N$ -body simulation evaluates the mutual gravity of planetesimals correctly and each orbit are calculated numerically. Because each orbit of planetesimals are revealed, this method is useful for the investigation of the planetesimal accretion with various spacial structure. Unfortunately, the high calculation cost, which increases proportional to the square of the number of particles limits the maximum number of particles.

Today, we have hybrid codes of statistical method and  $N$ -body simulation. Orchestra is parallel C++/MPI hybrid coagulation plus  $N$ -body code that tracks the accretion, fragmentation, and orbital evolution of solid particles ranging in size from a few microns to thousands of kilometers (Bromley and Kenyon 2011; Kenyon and Bromley 2008). Currently, this code is used for the investigation of the accretion process of small bodies in protoplanetary disk (e.g., Bromley and Kenyon 2017). On the other hand, the high performance  $N$ -body codes to reduce the calculation cost are under developed (e.g., PENTACLE from Iwasawa et al. (2017) and GPLUM from Ishigaki et al. (in prep.). They are parallelized by using FDPS (Iwasawa et al. 2016b)).

### 1.3.2 Dynamics

Ida and Makino (1992a,b) investigated the evolution of the velocity distribution of planetesimals by using  $N$ -body simulation. They found that the random velocity of massive planetesimals decreases by dynamical friction efficiently in short timescale relative to the whole planetesimal accretion time. On the other hand, the velocity dispersion of surrounding planetesimals increases with time

evolution due to the mutual gravitational scattering.

Except for the late stage of the formation of planets in which a massive planetesimal disturbs surrounding planetesimals (e.g., [Ida and Makino 1993](#)), it is rare that the relative velocity of planetesimals exceeds the escape velocity on the surface of planetesimals (e.g., [Ida 1990](#)). Thus, most of the collisional outcomes of planetesimals are not rebound and destruction but coalescence.

The mass distribution of planetesimals evolves by the accretion. The cross section of collisions depends on the relative speed of planetesimals. On the other hand, the relative speed depends on the mass distribution. Because mass distribution and the velocity distribution of planetesimals have complex relation, in general, planetesimals grow in non-linear and it is difficult to know the accretion process analytically.

### 1.3.3 Runaway Growth

From the statistical method, on the middle stage of the planetesimals accretion process, the growth mode that larger planetesimals grow faster than others is revealed. In particular, the largest planetesimal grows rapidly that the growth mode is called runaway growth ([Greenberg et al. 1978](#); [Wetherill and Stewart 1989](#)). This is because massive planetesimals catch larger amount of planetesimals by the dynamical friction in the planetesimal system and the gravitational focusing.

In the particle-in-a-box approximation, the growth rate of the massive planetesimal is well known and is described by the two-body approximation

(Greenzweig and Lissauer 1990; Ida and Nakazawa 1989), which is given by

$$\frac{dM}{dt} \simeq C \Sigma_{\text{dust}} \pi R^2 \left( 1 + \frac{v_{\text{esc}}^2}{v_{\text{rand}}^2} \right) \Omega_{\text{K}}, \quad (1.3)$$

where  $M$ ,  $\Sigma_{\text{dust}}$ ,  $R$  is the mass of the massive body, the mass surface density of solid bodies and the radius of the massive body, respectively.

$v_{\text{esc}} = \sqrt{2G(M+m)/(R+r)}$ , where  $m$  and  $r$  is the mass and radius of the smaller body, is the escape velocity.  $v_{\text{rand}}$  and  $\Omega_{\text{K}}$  is the random velocity of planetesimals and the Kepler angular velocity, respectively.  $C$  is the factor due to the effect of distribution of eccentricities and inclinations of planetesimals.  $\sqrt{1 + v_{\text{esc}}^2/v_{\text{rand}}^2}$  is gravitational focusing factor. If  $v_{\text{rand}}$  is small enough by the gas drag in the disk,  $v_{\text{rand}} \ll v_{\text{esc}}$  and the first part is negligible. Using the mass dependency of  $v_{\text{esc}}$  and  $R$ , the relative growth rate of massive planetesimal becomes

$$\frac{1}{M} \frac{dM}{dt} \propto M^{1/3}. \quad (1.4)$$

The massive planetesimal growth time scale becomes shorter with the planetesimal growth. This shows us the runaway manner of the massive planetesimal growth and the body is called protoplanet.

#### 1.3.4 Oligarchic Growth

Ida and Makino (1993) shows that the growth speed of protoplanets decreases by the difficulty of planetesimal accretion due to that the velocity dispersion of planetesimals that are scattered by a protoplanet gravity increases. The growth

speed of a protoplanet becomes slower than the smaller protoplanets. On the other hand, it keeps faster than the growth speed of surrounding planetesimals. Protoplanets scatter mutually in the wide region of the protoplanetary disk by their gravity. The dynamical friction from surrounding planetesimals circularizes the orbit of protoplanets and keeps orbital separations. By these phenomena, the growth mode of protoplanets becomes orderly. This mode is called oligarchic growth (Kokubo and Ida 1998).

The orbital region where planetesimals, which are caught by a protoplanet, distribute is called feeding zone and the total mass in the region is called isolation mass. The mass is given by

$$M_{\text{iso}} \simeq 2\pi ab \Sigma_{\text{dust}} = 0.16 \left( \frac{\bar{b}}{10} \right)^{3/2} \left( \frac{\Sigma_{\text{dust}}}{10 \text{ gcm}^{-2}} \right)^{3/2} \left( \frac{a}{1 \text{ AU}} \right)^3 \left( \frac{M_*}{M_{\odot}} \right)^{-1/2} M_{\oplus}, \quad (1.5)$$

where  $a$  is the distance from a central star,  $M_*$ ,  $M_{\odot}$  and  $M_{\oplus}$  are the mass of the central star, sun and the Earth, respectively (Kokubo and Ida 2002). By putting orbital separation of protoplanets ( $b$ ),  $\bar{b} = b/r_{\text{H}}$ , where mutual hill radius is give by

$$r_{\text{H}} = \left( \frac{2M}{3M_*} \right)^{1/3} a. \quad (1.6)$$

A protoplanet grows until surrounding planetesimals are exhausted and the mass reaches isolation mass. The growth timescale of the protoplanet is given by

$$T_{\text{grow}} \equiv \frac{M}{dM/dt} \simeq 2 \times 10^4 \left( \frac{e}{h_M} \right)^2 \left( \frac{M}{10^{26} \text{ g}} \right)^{1/3} \left( \frac{\Sigma_{\text{dust}}}{10 \text{ gcm}^{-2}} \right)^{-1} \left( \frac{a}{1 \text{ AU}} \right)^{1/2} \text{ years}, \quad (1.7)$$

where  $h_M$  is the reduced Hill radius of the protoplanet given by  $h_M \simeq (M/3M_\odot)^{1/3}$  and  $e$  is the eccentricity of the protoplanet (Kokubo and Ida 2000). This suggests that the growth timescale of protoplanets at 1AU must be about a million years.

The grown protoplanets scatter mutually and collide each other. This phase is called the giant impact stage, and terrestrial planets are formed by several collisions in several 10 million years around 1AU. The moon is formed in this phase.

### 1.3.5 Collisional Outcomes

From previous numerical experimental studies, it is known that collisions between self-gravity-dominated objects (e.g., planetesimals, protoplanets) of the same size cause hit-and-run and destruction such as the spread of the mantle (Agnor and Asphaug 2004; Asphaug et al. 2006; Benz et al. 2007; Genda et al. 2012; Leinhardt and Stewart 2012; Leinhardt et al. 2010; Marcus et al. 2009, 2010).

Smoothed particle hydrodynamics (SPH) is frequently used for reproducing the collision of protoplanets. According to Agnor and Asphaug (2004), which used SPH, over the half amount of collisions of the same sized protoplanets does not reach to a merger.

Leinhardt and Stewart (2012) evaluated the critical impact energy to distinguish the collisional outcomes by using PKDGRAV considering colliding rubble pile planetesimals. Although equation of state is not taken into account for this study and the realistic difference of materials are not considered, they

systematically categorized the collisional outcomes and revealed the boundaries of each outcomes of the rubble pile planetesimals that belong to the self-gravity-dominated regime.

When we pay attention to the accretion process of planetesimals including giant impact, the typical impact velocity of planetesimals and protoplanets must be similar to escape velocity of the colliding objects. In the low energy collisions, merging and rebound is the most important as collisional outcomes. [Genda et al. \(2012\)](#) investigated the merging criteria of protoplanets. SPH is used for this study and the rock and iron of Tillotson EoS is adopted for the mantle and the core. The criteria as the boundary of merging and rebound are described as the function of impact velocity, the mass ratio of colliding protoplanets and impact angle. Protoplanets bounce easier under the condition of the grazing collision of same sized colliding protoplanets.

### 1.3.6 Fragmentation

[Kobayashi et al. \(2010\)](#) investigated the growth of protoplanet considering the fragments of planetesimal collisions by using the statistical method. They calculated the amount of fragments from the colliding energy of planetesimals and investigated the mass falling into the central star by the gas drag. In addition, they calculated the accumulating mass onto protoplanets and revealed the upper limit of the mass of protoplanets. Because protoplanet's growth is suppressed than the past study that did not consider fragments, the core of gas giants cannot reach the critical mass that is needed for the gas accretion at



outside of snow line in MMSN.

[Chambers \(2013\)](#) investigated the terrestrial planet formation process under the realistic collision considering fragments and rebound by using  $N$ -body simulation. Protoplanet and surrounding planetesimals are distributed in the wide region where terrestrial planets exist today. The collision model (fragmentation and rebound) is based on [Leinhardt and Stewart \(2012\)](#) and [Genda et al. \(2012\)](#). Although the major accretion time of terrestrial planets is not changed from the results with the model in which all colliding objects merge (perfect merger), the time for sweeping up of collision fragments and formation of the full sized terrestrial planets is prolonged. The final mass and eccentricity of terrestrial planets are smaller than the model of the perfect merger.

The destruction of planetesimals by collisions affects the growth speed and the upper limit of the mass of planets. Thus, we need to pay special attention to fragments in the investigation of the accretion process of planetesimals.

### 1.3.7 Spin

In the giant impact stage, planets are formed by the collision and coalescence of the protoplanets. Therefore, in order to investigate the formation process of planets, realistic merging criteria of protoplanets are needed. On the other hand, merging criteria can be affected by the rotation of the protoplanet ([Kokubo and Genda 2010](#)). Thus, we need to investigate the rotation of protoplanets to know the realistic merging criteria. In addition, the spin angular momentum in protoplanets must accumulate on the planets consist of protoplanets. Therefore,

the realistic rotation of protoplanets is regarded as the initial condition for the study of planetary rotation. In addition, Mars is considered to be a survivor of the protoplanet in the formation region of the terrestrial planet. Therefore, clarifying the rotation of the protoplanet reveals the origin of Mars's rotation.

We must reveal the protoplanet's rotation. [Dones and Tremaine \(1993\)](#) is a great study that studied the rotation of protoplanets. In this research, they calculated the rotation of the protoplanet by solving the three-body problem of the sun, the protoplanet, and the planetesimal. It has been suggested that the rotational angular momentum accumulated on the protoplanet decreases if the planetesimal's system maintains a large velocity dispersion. It has also been shown that the direction of rotation may be biased if there is a spatial structure such as a gap in the planetesimal's disk ([Lissauer and Kary 1991](#)).

### 1.3.8 Problems

In general, there are several problems in the study of the accretion process of planetesimals by using  $N$ -body simulation. In most of past studies using  $N$ -body simulation, the perfect merger in which all colliding planetesimals merge (e.g., [Kokubo and Ida 1996](#)) is assumed. Although there is a high possibility that the fragment scattering affects the protoplanetary growth process, the effect is difficult to consider in real. In addition, hit-and-run, which is thought to be more basic and actually frequent than destruction, is also not mainly considered. By the hit-and-run, the timescale of planetary formation becomes over two times longer than the case of perfect merger. Therefore, it is necessary to consider

inefficient collisions against accumulations such as hit-and-run and destruction in order to clarify the realistic process of planetesimal accretion.

In [Morishima \(2017\)](#), they used Lagrangian Integrator for Planetary Accretion and Dynamics (LIPAD) ([Levison et al. 2012](#)), which can calculate the dynamical evolution and collision and coalescence of a large number of planetesimals, to investigate the process of planetesimal accretion in a wide area of the protoplanetary disk. However, the merging criteria used in the calculation are those of differentiated rocky protoplanets, and it may be unrealistic for the accumulation process of undifferentiated rocky planetesimals and icy planetesimals.

In order to study the process of accumulation of general planetesimals, it is necessary to investigate realistic merging criteria considering undifferentiated rocky and icy planetesimals, taking into account the equation of state.

In [Dones and Tremaine \(1993\)](#) that studied the rotation of protoplanets, since the space, velocity, and mass distribution of the planetesimals were not calculated simultaneously, the natural accumulation of rotational angular momentum in the process of nonlinear planetesimal growth was not calculated. In order to investigate the rotation of a protoplanet without any assumptions on the distribution of planetesimals, it is necessary to reproduce the accumulation process of the planetesimals on a first principle basis, and calculate the rotation angular momentum that the planetesimals obtain by coalescence. In general, the incomplete accumulation such as rebound and destruction of the planetesimals also affects the rotation of the protoplanet (e.g., [Kokubo and Genda 2010](#)). The

calculation needs to take the incomplete accumulation into account to know the realistic protoplanet's rotation.

Since the rotation of a planetesimal is dependent on its merging criteria, it is necessary first to investigate realistic planetesimal's merging criteria that take into account bounces.

Most of the numerical collision studies have investigated collisions between differentiated bodies. Although [Leinhardt and Stewart \(2012\)](#) investigated collisions between undifferentiated bodies, basically the phase transition based on the equation of state is not considered, so the difference in the collisional outcome depending on the composition may not be correct.

For the investigation of the realistic planetesimal accretion process and the origin of the spin, we need to know the realistic merging criteria of undifferentiated rocky and general icy planetesimals considering collisional outcomes such as rebound and fragmentation with phase transition.

## 1.4 Purpose of This Work

In this study, we investigate the merging criteria of rocky and icy planetesimals using SPH. The merging criteria is applied to  $N$ -body simulation of planetesimal accretion, and the realistic formation process of protoplanets including accumulation of rotation angular momentum is clarified.

In the first half, we study the realistic merging criteria of the planetesimals. Rocky and icy planetesimals are assumed to be 100 km in size, and they maintain their shape under self-gravity, that is, we ignore the material strength. It

is said that the frictional force can not be ignored on a planetesimal of this size (Jutzi 2015), but the model of the force is under studied. In order to eliminate this ambiguity, in this study we assume a fluid that deforms without friction as a planetesimal. When friction is considered, it tends to be difficult to deform at the time of a collision. The energy dissipation due to the friction cannot be ignored. So it is expected that in reality, it will be more likely to merge than the results of this research. The composition is expressed by switching the coefficients of Tillotson equation of state (Tillotson 1962). We change the impact speed, impact angle, mass ratio, and total mass of the planetesimals. Here, the impact velocity, which is the boundary where the collision bodies merge or not (critical impact velocity), is determined. The critical impact velocity is expressed as a function of the mass ratio of the planetesimals and the collision angle, and it is easy to use for  $N$ -body simulation as merging criteria considering hit-and-run.

In the second half,  $N$ -body simulation of planetesimal accretion of undifferentiated rocks are performed using merging criteria considering hit-and-run. Undifferentiated rocky planetesimals are distributed in a ring around 1 AU from the sun. By integrating the trajectories of individual rocky planetesimals, the spatial distribution, velocity distribution, and mass distribution of rocky planetesimals are calculated on first-principles. A protoplanet grows to an isolated mass. The code called GPLUM, which we used, outputs the velocity and angle at the impact of rocky planetesimals. Our analysis code can calculate the rotation angular momentum of the planetesimal as a post process. In this way, we investigate the formation process of protoplanets and

also the rotation evolution simultaneously.

Since Uranus is thought to be an icy planet composed of icy planetesimals, the rotation of Uranus is also discussed based on the comparison of the merging criteria of rocky and icy planetesimals.

In Chapter 2, we derive the merging criteria as a function of the impact parameters and compare the results with those of previous studies. Using the derived criteria, we present the accretional evolution of rocky planetesimals and the spin properties of rocky protoplanets corresponding to Mars in Chapter 3. We then summarize the formation process of rocky protoplanets and discuss the spin properties of protoplanets including Mars and ice giants in general summary (Chapter 4).

We investigate merging criteria for planetesimal collisions using Smoothed Particle Hydrodynamics.

Takashi Shibata

# 2

## Merging Criteria for Planetesimal Collisions

### 第2章

本章については、5年以内に雑誌等で刊行予定のため、非公開。

We investigate the accretion process and the spin of protoplanets under considering the rebound (hit-and-run) of colliding planetesimals.

Takashi Shibata

# 3

## Rocky Planetesimal Accretion with a Realistic Accretion Condition

### 第3章

本章については、5年以内に雑誌等で刊行予定のため、非公開。



This is general summary.

Takashi Shibata

# 4

## Summary

### 4.1 The Merging Criteria of Planetesimals

Terrestrial planets, ice giants and the cores of gas giants are thought to be formed by the accumulation of planetesimals. There has been no research on accumulation correctly evaluating the merging criteria at the time of planetesimal collision by using  $N$ -body simulation, while recent study used the

conditions of protoplanet coalescence. In order to know the accretion process of planetesimals, it is necessary to clarify the merging criteria of the planetesimal including the rebound. There is no study, which investigated the merging criteria of undifferentiated rocky planetesimals and icy planetesimals. We investigate conditions that determine coalescence vs. rebound by numerically colliding undifferentiated rocky planetesimals, undifferentiated icy planetesimals, and differentiated icy planetesimals using Smoothed Particle Hydrodynamics (SPH). We vary the total mass, mass ratio, collision speed, and collision angle of the colliding planetesimals. We investigate the critical impact velocity distinguishing coalescence from the rebound by a radical change of the largest remnant's mass represented against impact velocity. The critical impact velocity normalized by the escape velocity depends on the mass ratio of planetesimals and the impact angles. The critical impact velocity normalized by escape velocity decreases with the target mass increasing relative to impactor mass, and decreases with increasing the impact angle whose maximum value shows us a grazing collision. Evaporation does not occur in this study using 100 km size planetesimals. Thus, the critical impact velocity is mostly independent of the total mass of the planetesimals, and this condition has a very small dependence on the composition and internal structure of the planetesimals. The result of GKI12 is consistent with this study. From the above results, we formulate the critical impact velocity for undifferentiated rocky, icy, and differentiated icy planetesimals as a variable for the planetesimals' mass ratio and collision angle. This is the new achievements in the world. The merging criteria for

undifferentiated planetesimals allow us to know the realistic planetesimal accretion process with a small initial mass of planetesimals. Now we can investigate the formation process of ice giants since we have the merging criteria of icy planetesimals.

## 4.2 The Accretion Process of Planetesimals with Imperfect Coalescence Case

The previous studies for the planetesimal accretion process using  $N$ -body simulation mainly assumed perfect coalescence and the small number of initial planetesimals such as 10,000 particles. The mass, velocity and spatial distribution of planetesimals may be inaccurate from this assumption. We also do not know the rotation of protoplanets as the initial rotation at the giant impact stage of protoplanets,

We investigate a realistic accretion process of rocky planetesimals with small initial mass under the new merging condition for the rebound of planetesimals. We use  $N$ -body simulation code named GPLUM and distribute 200,000 uniform mass planetesimals as a ring around the sun with a narrow width. The number of particles is one of the largest numbers of particles in the world. The initial velocity is decided from equilibrium eccentricity and inclination due to gas drag. We adopt 50% massive disk model of MMSN. We apply the merging criteria to evaluate whether rebound or merge, which is obtained in Chapter 2, to GPLUM and consider the rebound of colliding planetesimals.

Although the rebounding planetesimals prevent their growth and the number of middle size planetesimals is fewer than the perfect merging case, the protoplanets grow similarly in each case. The similar surface mass density and the evolution of eccentricity and inclination lead to the same mass evolution of protoplanets. The time to sweep up planetesimals by protoplanets is elongated in the case of imperfect coalescence since the number of planetesimals does not decrease efficiently by rebound. From this growth mode, the mass distribution is bipolarized and runaway growth and oligarchic growth become prominent in the imperfect coalescence case. This is the new realistic planetesimal accretion process and these results contribute to the investigation for the realistic elementary processes of planetary formation.

We also investigate the rotation of planetesimals and protoplanets during the accretion process. Protoplanets experience impacts with massive planetesimals several times during the growth. The collision makes stochastic component of spin angular momentum dominant. The collision increases the spin angular velocity rapidly and changes obliquity randomly. The accretion of planetesimals from random direction carry the mass on protoplanets but cancel the angular momentum carried by accreting planetesimals. The increasing mass decreases spin angular velocity relatively. The spin angular velocity of planetesimals and protoplanets becomes 70% and 30% smaller in the rebounding planetesimals case than the perfect merging case, respectively. The impact parameter of merging planetesimals becomes smaller since the grazing planetesimals does not merge but rebound. Since the largest spin angular momentum must be carried

on planetesimals and protoplanets by such grazing planetesimals, the accreting angular momentum becomes smaller than the perfect accretion case. The spin angular velocity of protoplanets in this study is consistent with the rotation of Mars. Although the rotation of Mars today is not the same with the initial situation, the tendency to know the origin of Martian rotation is suggested. The obliquity of planetesimals and protoplanets distribute in a wide range of angle and mostly become isotropic since the colliding massive planetesimals from random directions change the obliquity drastically and randomly. The peak of obliquity distribution is around  $90^\circ$ , in which the spin axis is parallel to the ecliptic plane as well as Uranus. While the stochastic component of the spin angular momentum decides the obliquity distribution, the planetesimals from the edge of a gap of the planetesimal disk around a protoplanet may carry the angular momentum for prograde rotation (Lissauer and Kary 1991). Higher resolution  $N$ -body simulation is needed to see the effect of the gap. This study reveals that the spin angular velocity decreases from that of the perfect coalescence case by the rebound of planetesimals. This is new knowledge to know the realistic spin of protoplanets for the investigation of the giant impact stage.

### 4.3 Future Works

The fragments formed by planetesimals collisions must change the mass distribution and affects the accretion process of planetesimals. This code allows us to deal with fragments because of the low calculation cost. We need to know the planetesimal accretion process with the effect of fragments and rebound in

the wide region of a protoplanetary disk.

We will investigate the rotation of Uranus by using GPLUM since the accretion process might be different from that inside of snow line. The accretion of icy planetesimals over the snow line needs the merging criteria of icy planetesimals and takes high computational cost. Our merging criteria for icy planetesimals and GPLUM solve the problems. From this study, the obliquity of protoplanets distributes mostly isotropic. The merger criteria do not change significantly between rocky and icy planetesimals and do not depend on the total mass of colliding planetesimals. If Uranus is formed by planetesimal accretion, the obliquity can be described by the same process of this study. On the other hand, the spin angular velocity decided by the collision speed has to change. Now, we can study the spin of Uranus and compare the spin properties to this study.

## Appendix.1: Tillotson EoS

Tillotson equation of state (EoS) have three formulas for the pressure for each cases. They depend on the density  $\rho$  and the specific internal energy  $u$ . The fast formula is for condensed ( $\rho > \rho_o$ ) or cold state ( $u < u_{iv}$ ). Here, Tillotson EoS can be written in

$$p_{co} = \left( a + \frac{b}{\frac{u}{u_o \eta^2} + 1} \right) \rho u + A\mu + B\mu^2, \quad (1)$$

where  $\eta = \rho/\rho_o$ ,  $\mu = \eta - 1$ . The second formula is for expanded hot state ( $\rho < \rho_o$  and  $u > u_{cv}$ ). Tillotson EoS is written in

$$p_{ex} = a\rho u + \left[ \frac{b\rho u}{\frac{u}{u_o \eta^2} + 1} + A\mu \exp \left\{ -a \left( \frac{1}{\eta} - 1 \right) \right\} \right] \times \exp \left\{ -\beta \left( \frac{1}{\eta} - 1 \right)^2 \right\}. \quad (2)$$

The third formula is for intermediate region ( $u_{iv} < u < u_{cv}$  and  $\rho < \rho_o$ ). Here, a smooth transition between the above states occurs. From this reason, as [Benz et al. \(1986\)](#) showed, we interpolate the formulas ( $p_{co}$  and  $p_{ex}$ ) such as

$$p_{tr} = \frac{(u - u_{iv})p_{ex} + (u_{cv} - u)p_{co}}{u_{cv} - u_{iv}}. \quad (3)$$

Here,  $\rho_o$ ,  $u_o$ ,  $a$ ,  $b$ ,  $A$ ,  $B$ ,  $u_{cv}$ ,  $u_{iv}$ ,  $a$  and  $\beta$  are material parameters. We use the parameter sets of basalt and ice for planetesimals, which are listed on TABLE II of page 7 in [Benz and Asphaug \(1999\)](#).

## Appendix.2: Smoothed Particle Hydrodynamics

We show the bases of standard Smoothed Particle Hydrodynamics (SPH). The equation of motion of the  $a$ -th particle of SPH is

$$\frac{d\mathbf{v}_a}{dt} = - \sum_b^{\text{neighbor}} \mathbf{F}_{ab} - \sum_b^{\text{all}} \mathbf{g}_{ab}, \quad (4)$$

where  $\mathbf{v}_a$  is the velocity of the  $a$ -th SPH particle.  $t$  is time.  $\mathbf{F}_{ab}$  and  $\mathbf{g}_{ab}$  are the pressure gradient and mutual gravity terms between the  $a$ -th and  $b$ -th particles, respectively. In SPH, several equations for pressure gradient are known. We adopt the formula below, which is shown in [Monaghan \(1992\)](#) and now commonly used.

$$\mathbf{F}_{ab} = m_b \left( \frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab} \right) \nabla_a W(r_{ab}, h_{ab}), \quad (5)$$

where  $m_b$ ,  $P_b$ , and  $\rho_b$  are mass, pressure, and density of the  $b$ -th particle, respectively.  $\Pi_{ab}$  is the artificial viscosity.  $W$  is the kernel function.  $r_{ab}$  is the distance between the  $a$ -th and  $b$ -th particles.  $h_{ab}$  is the average smoothing length of the  $a$ -th and  $b$ -th particles. For the kernel function  $W$ , we adopt the Wendland  $C^6$  kernel shown in [Dehnen and Aly \(2012\)](#). Mutual gravity is written as

$$\mathbf{g}_{ab} = G \sum_b m_b \frac{\mathbf{r}_a - \mathbf{r}_b}{r_{ab}^3}, \quad (6)$$

where  $G$  is the gravitational constant and  $m_b$  is the mass of the  $b$ -th particle. We



represent the specific internal energy change. The specific internal energy of  $a$ -th particle is set as  $u_a$ ,

$$\frac{du_a}{dt} = \frac{1}{2} \sum_b m_b \left( \frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab} \right) \mathbf{v}_{ab} \cdot \nabla_a W(\mathbf{r}_{ab}, h_{ab}), \quad (7)$$

where  $\mathbf{v}_{ab} = \mathbf{v}_a - \mathbf{v}_b$ .

From [Monaghan \(1997\)](#), artificial viscosity is below.

$$\Pi_{ab} = \begin{cases} -\frac{\alpha_{ab}^{\text{AV}} \mathbf{v}_{ab}^{\text{sig}} w_{ab}}{2 \rho_{ab}}, & \text{if } w_{ab} < 0, \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

Here,

$$\alpha_{ab}^{\text{AV}} = \frac{\alpha_a^{\text{AV}} + \alpha_b^{\text{AV}}}{2}. \quad (9)$$

In commonly use,  $\alpha^{\text{AV}} = 1$  When we set the position of  $a, b$ -th particles as  $\mathbf{r}_a, \mathbf{r}_b$ , we can see

$$w_{ab} = \frac{(\mathbf{r}_b - \mathbf{r}_a) \cdot (\mathbf{v}_b - \mathbf{v}_a)}{|\mathbf{r}_b - \mathbf{r}_a|}. \quad (10)$$

From [Hosono et al. \(2016\)](#), sound speed of  $a, b$ -th particles is  $c_a, c_b$ , then,

$$\mathbf{v}_{ab}^{\text{sig}} = c_a + c_b - 3w_{ab}, \quad (11)$$

and we know

$$\rho_{ab} = \frac{\rho_a + \rho_b}{2}. \quad (12)$$

We use leap-frog method for the time integration here. For the  $a$ -th particle, local

time step can be defined with  $b$ -th particle. The simple formula of the time step is below,

$$\delta t = \frac{\sigma h}{v_{\text{sig}}(a, b)}. \quad (13)$$

We set the smoothing length of a particle as  $h = h_a$ . Here,  $h_a$  and  $h_b$  is the smoothing length of  $a$ -th and  $b$ -th particles, respectively. By setting  $\mathbf{j} = (\mathbf{r}_a - \mathbf{r}_b) / |\mathbf{r}_a - \mathbf{r}_b|$ ,  $v_{\text{sig}}(a, b) = c_a + c_b - 3\mathbf{v}_{ab} \cdot \mathbf{j}$ . From previous numerical studies, we can set  $\sigma \sim 0.3$ . To know the time step used for actual step, we calculate the local time step for all SPH particles and we choose the minimum one (Monaghan 1997). Here, the smoothing length is

$$h_a = \eta \left( \frac{m_a}{\rho_a} \right)^{1/3}, \quad (14)$$

where  $m, \rho$  is the mass of SPH particle and the density, respectively. Here,  $\eta = 1.2$ .

### Appendix.3: Particle-Particle Particle-Tree Method

Particle-Particle Particle-Tree method (P<sup>3</sup>T) is the hybrid method of Hermite forth-order integrator and tree method for  $N$ -body simulation (Oshino et al. 2011). It allows  $N$ -body simulation to become faster to the order  $O(N \log N)$  from  $N^2$ , where  $N$  is the number of particles. This method is used in PENTACLE (Iwasawa et al. 2017) and GPLUM (Ishigaki et al. in prep).

In P<sup>3</sup>T, Hamiltonian can be divided into two parts,  $H_{\text{soft}}$  and  $H_{\text{hard}}$  are described as

$$H = H_{\text{soft}} + H_{\text{hard}}, \quad (15)$$

$$H_{\text{soft}} = - \sum_i \sum_{j>i} \frac{Gm_i m_j}{r_{ij}} W(r_{ij}; r_{\text{out}}), \quad (16)$$

$$H_{\text{hard}} = \sum_i \left( \frac{|\mathbf{p}_i|^2}{2m_i} - \frac{GM_* m_i}{r_i} \right) - \sum_i \sum_{j>i} \frac{Gm_i m_j}{r_{ij}} (1 - W(r_{ij}; r_{\text{out}})), \quad (17)$$

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j. \quad (18)$$

Here,  $G$  is gravitational constant,  $m_i, \mathbf{p}_i, \mathbf{r}_i$  are the mass, momentum, position of  $i$ -th particle, respectively.  $M_*$  is the mass of the central star.  $W(r_{ij}; r_{\text{out}})$  is the changeover function of Hamiltonian, which is the function of the distance of particles ( $r_{ij}$ ) and cut off radius ( $r_{\text{out}}$ ). This function is 1 at  $r_{ij} > r_{\text{out}}$  and decreases to 0 with  $r_{ij}$  decreasing at  $r_{ij} < r_{\text{out}}$ . This allows Hamiltonian to be divided into  $H_{\text{soft}}$  and  $H_{\text{hard}}$  for the calculations of further particles and close particles, respectively. Equation of motion using Hamiltonian is

$$\frac{d\omega}{dt} = \{\omega, H\}. \quad (19)$$

$\omega$  is canonical variable,  $\{, \}$  is Poisson's bracket expression. The solution is

$$\omega(t + \Delta t) = e^{\Delta t \{, H\}} \omega(t). \quad (20)$$

In P<sup>3</sup>T, the solution can be described as

$$\omega(t + \Delta t) = e^{\Delta t/2 \{, H_{\text{soft}}\}} e^{\Delta t \{, H_{\text{hard}}\}} e^{\Delta t/2 \{, H_{\text{soft}}\}} \omega(t). \quad (21)$$

This allows  $H_{\text{soft}}$  and  $H_{\text{hard}}$  to be integrated, individually. In P<sup>3</sup>T, the gravitational interaction in  $H_{\text{soft}}$  is calculated by tree method and that in  $H_{\text{hard}}$  is integrated by Hermite 4th order integrator. For details, see [Oshino et al. \(2011\)](#). Ishigaki et al. (in prep) defines new method to decide the cut off radius for GPLUM. Cut off radius is generally decided as

$$r_{\text{out},i} = R_{\text{cut}}r_{\text{H},i}, \quad (22)$$

where  $R_{\text{cut}}$  is a parameter. In GPLUM, cut off radius is decided by  $i$ -th and  $j$ -th particles as

$$r_{\text{out},ij} = \max(r_{\text{out},i}, r_{\text{out},j}). \quad (23)$$

This is useful for the planetesimal system, in which runaway and oligarchic growth occur and have significant mass ratio of planetesimals.

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