博士論文

Essays on Competition, Regulation, and Privatization Policies
(競争政策・規制政策・民営化政策に関する論文集)

佐藤 進

Essays on Competition, Regulation, and Privatization Policies

Susumu Sato

Acknowledgement

I am grateful to my supervisor Toshihiro Matsumura for all kinds of supports, guidance, advice, and conversations, without which I might have strayed from the right path. I also thank the rest of committee members, Susumu Cato, Akifumi Ishihara, Hikaru Ogawa, and Dan Sasaki, for many interactions at workshops and conferences. Outside The University of Tokyo, I benefited from Daisuke Hirata, Hodaka Morita, Noriaki Matsushima, and Yusuke Zennyo in many occasions. Finally, I thank my families and friends.

Contents

Chapter 1

Introduction

Firms play vital roles in a wide range of economy; they produce, improve, and distribute products essential for consumers, through R&D, development of viable business models, and other kinds of marketing activities. However, prominent firms often exploit their market power to raise profits thereby hurting consumers' welfare. The ways firms do so include raising prices (either unilaterally or multilaterally), foregoing investments, consolidating to a small number of firms to reduce competition, among others. Governments therefore often need to intervene with the firms' activities in order to refrain them from exercising market powers. The way the government intervene depends on the market environments.

In most industries, competition authorities monitors firms' activities to make sure that their conducts, such as cartels, mergers, and vertical restraints, do not hurt consumers. This line of government interventions is called competition policy, where the governments' role is to keep the markets competitive. In some industries such as water, gas, and electricity networks, keeping competitive markets is often unfeasible due to the technological constraints such as extremely high (sunk) fixed costs. In such natural monopoly, governments need to tailor the regulation policy and often directly control the firm's decisions such as pricing and investments. This is called regulation policy. In recent years, thanks to the technological progress, the technological constraints imposed on network industry are relaxed, which induces market liberalization, and reduction in the degree of government control on the firms. At the same time, the government's budget constraints has become tighter, which also induces the reduction in the government control by giving up the ownership of regulated enterprises. In this way, the government started to privatize the originally state-owned regulated enterprises. The situation where the state-owned enterprises and private enterprises co-exist is called a mixed oligopoly. In mixed oligopoly, the government need to design the role of the state-owned enterprises, which may result in greater or smaller control over the state-owned enterprises, which is implemented by the design of privatization policies.

In this dissertation, I present three theoretical analysis on each area of competition, regulation, and privatization policies. Below I present the abstracts of respective chapters.

Chapter 2: Horizontal Mergers in the Presence of Network Externalities

Evaluating network effects and two-sidedness is critical for merger control in the digital economy. Using a model of multiproduct-firm oligopoly with firm-level network externalities, I provide several results regarding the impacts of network externalities on consumer-surplus effects of mergers. First, the impacts of network externalities on the welfare properties of mergers depend on the sizes of merging parties relative to the industry; direct network externalities justify mergers between small firms but make the mergers between dominant firms more likely to hurt consumers. Second, when an incumbent tries to acquire an innovative entrant, the incumbent may have a greater incentive to innovate due to the demand-side scale economies, which suggests that too strict merger control may inhibit potential innovations. Finally, I present a merger analysis in two-sided markets and show how the pre-merger sizes of firms on one side can predict the post-merger consumer surplus on the other side. These results provide handy theoretical guidance on merger policy toward platforms.

Chapter 3: Monopoly Regulation in the Presence of Consumer Demand-Reduction

In this chapter, I study a monopoly regulation in the setting where consumers can engage in demand-reducing investments. I first show that, when the regulator ignores the consumers' investments, the excess investment occurs. Next, I analyze the case where the regulator takes consumers' investments into account and compare the optimal policy under asymmetric information with the first-best policy. Optimal policy results in higher average price, higher level of consumer investment, but lower prices for efficient firms, compared to the first-best.

Chapter 4: Dynamic Privatization Policy

This chapter formulates a two-period model of mixed oligopoly in which the government privatizes a state-owned public firm over multiple periods. We introduce the shadow cost of public funding (i.e., the excess burden of taxation). The government is concerned about both the total surplus and the revenue obtained from the privatization of the public firm. We find that the government may or may not increase the degree of privatization over time depending on the competitiveness of the product market and nationality of private competitors. The government increases the degree of privatization over time if the product market is competitive and the foreign ownership share in private firms is low. Although it adjusts its privatization policy over time, this harms welfare. In addition, this distortion in the ex-post incentive leads to too low a degree of privatization in the first period.

Chapter 2

Horizontal Mergers in the Presence of Network Externalities*

2.1 Introduction

In the digital economy, several markets are characterized by network externalities and multi-sidedness, where platform operators generate direct and indirect network externalities by facilitating the interaction among the participants. While network effects benefit consumers in many cases, the positive-feedback effect of network externalities often leads consumers to focus on a few dominant platforms. This "winner-takes-all" feature makes competition authorities raise concern about the potential harm of persistent dominance of particular firms.*

Furthermore, dominant firms in the tech industry often expand their networks through mergers and acquisitions. Such acquisitions include many small and several large ones, such as Google/DoubleClick, Microsoft/LinkedIn, Apple/Shazam, and Facebook/WhatsApp, which were subject to scrutiny by competition authorities. The common issue in these merger reviews is the evaluation of the impact of network effects and two-sidedness on the desirability of mergers. While it is recognized that merger policy must consider network effects and multi-sidedness of platform businesses (Evans and Noel, 2008; Ocello and Sjödin, 2018), the theoretical guidance on these issues is still scarce. This lack of guidance may hurt the society through incorrect decisions on the prospective mergers between platforms.

The aim of this study is to provide a theoretical guidance on merger policy in industries characterized by network externalities. To this end, I develop a model of multiproduct-firm oligopoly with direct and indirect network externalities that allow for an arbitrary number of heterogeneous firms. In particular, this study introduces firm-level network externalities to CES and multinomial-logit demand systems and applies an aggregative-games approach (Nocke and Schutz, 2018b) to characterize the equilibrium in the pricing game among firms with different product portfolios. In this framework, as in Nocke and Schutz (2018b), type-aggregation obtains: all the relevant

*This chapter is based on Sato (2019).

*For example, Australian Competition and Consumer Commission released "Digital Platforms Inquiry" that tries to identify the market powers of digital platforms (https://www.accc.gov.au/focus-areas/inquiries/digital-platforms-inquiry).

information for the pricing of each firm is summarized in some sufficient statistics, named as the firm's "type," the dimension of which equals the number of sides of the markets. Therefore, any merger can be formalized as a change in the profile of types, which enables the characterization of the competitive effects of mergers by analyzing the impact of changes in types. In particular, by defining a merger-specific synergy as an increase in types following a merger, one can characterize the synergies required for mergers to improve consumer surplus. Characterizing such synergies gives conditions under which a merger is likely to benefit or harm consumers.

Using the framework described earlier, this study offers three sets of merger analyses: mergers in the presence of direct network externalities, acquisitions of innovative entrants by incumbents, and merges in two-sided markets. In all the three analyses, the presence of direct and indirect network externalities changes the market-power effects of mergers through demand-side scale economies and changes in subsidization incentives. The directions and magnitudes of such changes depend on the pre-merger sizes of the merging parties relative to the markets.

From the perspective that the objective of competition authorities can be approximated by consumer surplus (Whinston, 2007), I first examine the impact of mergers on consumer surplus in the presence of direct network externalities. To this end, I explore how the size of the synergy required for a merger to improve consumer surplus varies based on the sizes of the merging parties and the magnitude of the network externalities. Network externalities have two countervailing effects on the welfare properties of mergers. While consumers directly benefit from the network externalities arising from mergers because of the expansion of the merged entity's networks, network externalities might also increase the market power of the merged entity, which accompanies the higher markups. The overall impact of network externalities on the competitive effects of mergers depends on the relative magnitudes of these two effects.

Several cases in which one effect dominates the other are identified. For example, when the merging parties are small or when the firms in an industry are symmetric, the synergy required for a merger to improve consumer surplus decreases with the magnitude of network externalities or even become negative. This is because the benefit from network expansion dominates the cost of incremental market power. Accordingly, the consumer surplus is improved by the presence of network externalities in mergers involving non-dominant firms. However, when the merging parties are dominant in the industry, the presence of network externalities increases the synergy required for a merger to improve the consumer surplus. Thus, the merger is likely to hurt consumers. This follows from the fact that when the merging parties are sufficiently large compared with the industry, the presence of network effects enables the merged entity who is equipped with a huge customer base to easily attract consumers without lowering the prices. In this case, the merged entity can easily exert its market power to increase markups, which in turn decreases consumer welfare. In total, network externalities can serve as a justification for mergers involving small firms or mergers in an industry with symmetric firms. However, as the size of merging parties grows relative to the industry, the presence of network externalities requires the more intense scrutiny of merger reviews.

To contribute to the recent discussion on "killer acquisitions" (Cunningham, Ederer and Ma, 2018), I extend the model to incorporate the case where an incumbent tries to acquire an innovative potential entrant. This extension shows that when the size of a potential innovation is small, the incumbent has greater incentive to continue the project than the entrant does, while the opposite may hold when the size of a potential innovation is large. When the size of the innovation is small, the entrant suffers from an inability to expand its network, while the incumbent can leverage its installed base to effectively sell the new product. This demand-side scale economy makes the incumbent more willing to innovate, which is in sharp contrast with the replacement effect argument that the incumbent has smaller incentive to innovate because the new product just "replaces" the existing product. This result suggests that if the authorities adopt a too aggressive merger policy and ban acquisitions of innovative entrants such as tech-startups, investors might stop funding some innovative projects at all. This observation is consistent with the remark put by Bruce Hoffman at the Federal Trade Commission that "[t]o the extent exit strategies for startups involve acquisitions, if such acquisition opportunities are constrained, the capital available for startups may fall. That, in turn, could result in fewer startups."[†] Given the fact that a large fraction of exit strategies adopted by startups are M&A, this is a serious concern.[‡]

Finally, I analyze mergers in the presence of indirect network externalities, with a focus on two-sided markets. In the presence of indirect network externalities, firms typically engage in two-sided pricing: they subsidize consumers on one side of the market by charging lower prices, while collecting revenues from consumers on the other side of the market by charging higher prices at the same time. Such subsidization incentives are interrelated with the size of the firm on each side of the market (Weyl, 2010). Mergers thus affect not only the market power but also the subsidization incentives and the price structure among multiple sides of markets, which leads to a change in the division of surplus between different groups of consumers.

Focusing on a relatively simple case where only consumers on one side gain from indirect network externalities, I examine the relation between the pre-merger sizes of the merging parties and the consumer-surplus effects of mergers. I show that the synergies on the two sides of markets required for any merger to leave consumer surplus on both the sides unchanged, named as CS-neutral synergies, can be negative depending on the pre-merger sizes and the price structures of the merging parties. In particular, CS-neutral synergy on the side that benefits from indirect network externalities (subsidizing segment) is negative when the sizes of merging parties are small. This result is analogous to the effect of direct network externalities: when the merging parties are small, the benefit from a network expansion outweighs the cost of an increase in market power. In addition, CS-neutral synergy on the side that generates indirect network externalities (subsidized segment) is negative when the merging parties set negative markups on that side, which occurs when the merging parties are relatively large on the subsidizing segment. This is because the increase in the size on the subsidizing segment following the merger increases the incentive to subsidize the consumers on the subsidized segment. Therefore a merger that involves firms with a large pre-merger share on the subsidizing segment is more likely to benefit consumers on the subsidized segment.

The results of this study provide theoretical guidance on merger policy toward platforms. In the presence of network externalities, mergers between non-dominant firms are more likely to benefit consumers, while mergers that lead to extreme concentration are more likely to hurt consumers. Killer acquisitions are of less concern when the sizes of potential innovations are small. In two-sided

[†]"Competition Policy and the Tech Industry – What's at stake?" Available at (https://www.ftc.gov/system/files/documents/public_statements/1375444/ccia_speech_final_april30.pdf).

[‡]In the annual report at 2016, CB Insights notes that there were 3,358 total tech exits in the year, and that 3,260 of them were M&A exits. See CB Insights, "2016 Global Tech Exits Report," (https://www.cbinsights.com/research/report/tech-exits-2016/).

markets, both pre-merger price structures and sizes provide relevant information on the effects of mergers on consumers.

The remainder of the paper is organized as follows. Section 2.2 discusses the related literature. Section 2.3 presents a model of multiproduct-firm oligopoly with direct network externalities. Sections 2.4 then analyzes the impact of mergers on consumer surplus in the presence of network externalities. Section 2.5 and 2.6 analyze acquisitions of innovative entrants and mergers in two-sided markets. Section 2.7 discusses several issues abstracted in the main analysis. Section 2.8 concludes.

2.2 Related Literature

The present study is related to four strands of literature: network externalities, two-sided markets, innovation, and horizontal mergers.

The first strand of literature analyzes the impact of network externalities on the consumer choice of technologies (Farrel and Saloner, 1986) and firms' pricing in static and dynamic environments (Katz and Shapiro, 1985; Cabral, 2011). To the best of my knowledge, there has been no systematic theoretical study on the relation between the network externalities and horizontal mergers, because of the lack of the tractable framework to analyze the merger in flexible environments. By introducing the network externalities to the framework of multiproduct-firm oligopoly proposed by Nocke and Schutz (2018b), this study provides a systematic theoretical analysis of the impact of network externalities on the welfare properties of mergers.

The second strand of literature studies competition in two-sided markets, with a focus on the determinants of price structures in the presence of indirect network externalities (Rochet and Tirole, 2003, 2006; Armstrong, 2006; Weyl, 2010). Several recent studies theoretically analyze the impact of mergers between two-sided platforms on consumer welfare under single-homing (Baranes, Cortade and Cosnita-Langlais, 2014; Tan and Zhou, 2017; Correia-da-Silva, Jullien, Lefouili and Pinho, 2019), competitive-bottleneck (Anderson and Peitz, 2015), and multi-homing situations (Anderson, Foros and Kind, 2019). This study uses a model with single-homing consumers (Baranes et al., 2014; Tan and Zhou, 2017; Correia-da-Silva et al., 2019). Including the present study, this strand of literature faces a trade-off between analyzing the general market environments and providing clear welfare implications. Using demand systems that enables to adopt an aggregative-games approach developed by Nocke and Schutz (2018b), this study allows for an arbitrary number of heterogeneous firms and obtains a clear characterization of the consumer-surplus effects of mergers with respect to pre-merger sizes and price structures of the merging parties.

The third strand of literature studies the impact of mergers on innovation. (Federico, Langus and Valletti, 2018; Cunningham et al., 2018).[§] Cunningham et al. (2018) use a simple model of the acquisition of innovative entrant by an incumbent and show that due to the replacement effect, the incumbent always has a smaller incentive to innovate than the entrant does. They empirically confirmed this prediction by examining the progress of R&D projects in the pharmaceutical industry accompanying the acquisitions. The present study adopts the same timeline as Cunningham et al. (2018) do and shows that, in the presence of network externalities, incumbents may have greater

[§]See Jullien and Lefouili (2018) for more comprehensive review.

incentives to innovate than the entrant does due to the demand-side scale economies. Thus, the presence of network externalities enriches the predictions made by the existing research.

The last strand of literature studies the welfare properties of horizontal mergers (Williamson, 1968; Farrell and Shapiro, 1990; Nocke and Whinston, 2010, 2013; Nocke and Schutz, 2018b,a). The framework of this study is based on Nocke and Schutz (2018b,a). They show that with a class of demand systems that satisfy IIA properties among the sets of products offered by firms, price competition in multiproduct-firm oligopoly can be expressed as an aggregative game, which largely simplifies the characterization of equilibria. Furthermore, they also show that when the demand system is given by nested multinomial-logit or nested CES demand, all the relevant information for a firm's optimal strategy can be summarized by a scalar-valued "type" and the industry-level aggregator, which enables conducting a tractable merger analysis. This study extends the framework proposed by Nocke and Schutz (2018a) to incorporate firm-level direct and indirect network externalities while simultaneously preserving the aggregative feature of consumer demand. Technically, this study extends their "type-aggregation property" to two-sided markets, where all the relevant information for an optimal strategy of each firm can be summarized in the type that has the dimension corresponding with the number of sides of the markets. This enables the study of the impacts of direct and indirect network externalities on the welfare properties of horizontal mergers by conducting a merger analysis in a similar way as Nocke and Schutz (2018a). Therefore, this study contributes to the existing literature by examining how the results in the standard settings are preserved or altered by the introduction of direct and indirect network externalities. For example, while mergers without synergies hurt the consumer surplus in Nocke and Schutz (2018a)'s environment, such mergers can improve the consumer surplus in the presence of direct or indirect network externalities.

Finally, I mention several studies that provide tools to quantify the effects of horizontal mergers in two-sided markets. Affeldt, Filistrucchi and Klein (2013) propose a simple measure of upward-pricing pressure modified to incorporate the two-sidedness of markets. While the concept of upward-pricing pressure provides insights to policymakers, the presence of network effects and two-sidedness complicates the evaluation of the consumer-surplus effects of mergers solely based on the price levels. In this regard, the present study provides a clear characterization of the consumer-surplus effects of mergers. Jeziorski (2014) provides a merger analysis using a merger simulation based on an estimated structural model. This approach has an advantage in that, once given appropriate data, it can incorporate rich structures and directly quantify how a specific merger would affect consumer welfare. However, the implication of a specific merger simulation is inevitably case-by-case and cannot be easily generalized to the other environments. In this regard, the theoretical implications provided in this study has an advantage in linking the sizes of the merging parties to the welfare effects of mergers and providing simple intuitions that would apply to a range of applications.

2.3 Model with Direct Network Externalities

This section presents a model of multiproduct-firm oligopoly with firm-level direct network externalities. All details of derivation are provided in Section 2.9, which presents the analysis of a generalized model of multiproduct-firm oligopoly in the presence of both direct and indirect network externalities.

Consumer demand

Consider an industry with a set $\mathcal{N}$ of imperfectly substitutable products produced by a set of firms $\mathcal{F}$. Each firm $f \in \mathcal{F}$ produces the set $\mathcal{N}_f$ of products, where $\mathcal{N}_f \cap \mathcal{N}_{f'} = \emptyset$ for $f \neq f'$ and $\bigcup_{f \in \mathcal{F}} \mathcal{N}_f = \mathcal{N}$. There is a mass of consumers who derive firm-level network externalities from the purchase of each product $i \in \mathcal{N}_f$, which depends on the number of consumers n_f who purchase the products of firm f. In particular, each consumer $z \in [0, 1]$ yields the indirect utility from the purchase of product $i \in \mathcal{N}_f$ by

$$\log h_i(p_i) + \alpha \log n_f + \varepsilon_{iz}, \tag{2.1}$$

where $\log h_i(p_i)$ is the stand-alone indirect subutility from product i at price p_i, $\alpha \log n_f$ is the utility from the direct network externalities, and ε_{iz} is an idiosyncratic taste shock that follows an i.i.d. type-I extreme value distribution. Sizes of the direct network externalities depend on the number n_f of consumers who purchase products from firm f and the magnitude of direct network externalities $\alpha \in [0, 1)$. The direct network externalities are based on the number of participants in each firm. Such a form of network externalities can be interpreted as membership externalities (Armstrong, 2006; Weyl, 2010). The assumption that network externalities present at a firm level means that products in a firm's single network have greater compatibility than products in different firms' networks.

I adopt two specific forms of functions h_i. One is *MNL-class* demand specification, where

$$h_i(p_i) = \exp\left(\frac{a_i - p_i}{\lambda}\right),$$

and the other is *CES-class* demand specification where

$$h_i(p_i) = \begin{cases} a_i p_i^{1-\sigma} & \text{if } p_i > 0 \\ +\infty & \text{if } p_i \leq 0 \end{cases}$$

with $\sigma > 1$. In both the specifications, a_i represents the quality of each product. I mean by "MNL-class" and "CES-class" that, if $\alpha = 0$, the demand system obtained from indirect subutilities $h_i(p_i) = \exp\left(\frac{a_i - p_i}{\lambda}\right)$ and $h_i(p_i) = a_i p_i^{1-\sigma}$ corresponds with that of multinomial-logit and CES demand functions, respectively.

Given the profiles of network sizes $(n_f)_{f \in \mathcal{F}}$ and prices $p := (p_i)_{i \in \mathcal{N}}$, consumers choose one product to purchase and the amount of the purchase of the product. This implies that consumers single-home.[¶] I assume that there is no outside option so that all consumers purchase some product from the set $\mathcal{N}$.[‖] With this specification, the demand system is derived as a rational-expectation equilibrium among consumers. In other words, based on the common expectation over network

[¶]This assumption is not innocuous when we consider digital markets such as social media. I discuss some implications of multi-homing in Section 2.7.

[‖]In Section 2.7 discusses the way to relax this assumption.

sizes, consumers choose their own decisions to maximize the utilities, and the realized network sizes are consistent with the original expectation.

The demand system under the rational expectation equilibrium is derived in the following manner. First, the firm-level and industry-level aggregators are defined as follows:

$$H_f(p_f) = \sum_{i \in \mathcal{N}_f} h_i(p_i),$$

where $p_f := (p_i)_{i \in \mathcal{N}_f}$, and

$$H(p) = \sum_{f \in \mathcal{F}} \left(H_f(p_f)\right)^{\frac{1}{1-\alpha}}.$$

Next, I derive demand for each product conditional on the purchase. Applying Roy's identity, the conditional demand function for product $i \in \mathcal{N}$ conditional on the purchase is given by $-h_i'(p_i)/h_i(p_i)$. I assume that consumers form the correct expectation that all firms have positive network shares. I call the network choice of consumers based on such expectation as an *interior consumption equilibrium*. Applying the Holman and Marley's Theorem, the consumer choice probability s_i of product $i \in \mathcal{N}_f$ given the expectation over network shares $(n_{f'})_{f' \in \mathcal{F}}$ is given by

$$s_i = \frac{h_i(p_i)\left(n_f\right)^\alpha}{\sum_{f' \in \mathcal{F}} \sum_{j \in \mathcal{N}_{f'}} h_j(p_j)\left(n_{f'}\right)^\alpha}. \tag{2.2}$$

In the interior consumption equilibrium, the network shares are consistent with the consumers' behaviors so that the network share n_f of firm f is given by the sum of the choice probability of products produced by firm f:

$$n_f = \sum_{i \in \mathcal{N}_f} s_i. \tag{2.3}$$

Note that there are other trivial equilibria in the consumers' network choices where a set of products is expected to be purchased by no consumers. I exclude such equilibria from consideration.** From equations (2.2) and (2.3), the share of product $i \in \mathcal{N}_f$ in the set of products sold by firm f is given by

$$\frac{s_i}{n_f} = \frac{h_i(p_i)}{H_f(p_f)}. \tag{2.5}$$

**This selection has the following asymptotic justification. Consider the following best-response dynamics. First, fix an initial value of the vector of network shares $(n_f^0)_{f \in \mathcal{F}}$ such that $n_f^0 > 0$ for all $f \in \mathcal{F}$. Next, for each $t > 0$, update the network share based on the value of network share in the previous iteration $t-1$. Then, the sequence of network shares $\left\{(n_f^t)_{f \in \mathcal{F}}\right\}_{t=0,\dots}$ is obtained. Here, for any $t > 0$, we have

$$\frac{n_f^t}{n_g^t} = \frac{H_f}{H_g}\left(\frac{n_f^{t-1}}{n_g^{t-1}}\right)^\alpha = \left(\frac{H_f}{H_g}\right)^{\sum_{\tau=0}^t \alpha^\tau}\left(\frac{n_f^0}{n_g^0}\right)^{\alpha^t} \to \left(\frac{H_f}{H_g}\right)^{\frac{1}{1-\alpha}} \in (0,\infty) \quad \text{as } t \to \infty. \tag{2.4}$$

From this observation, we must have the vector of positive network shares as the limit of the best-response dynamics. Thus, the interior consumption equilibrium is the only equilibrium to which the best-response dynamics from any vector of positive network shares.

As derived in the Section 2.9, the network share n_f of firm f in the interior consumption equilibrium is given by

$$n_f(p) = \frac{\left(H_f(p_f)\right)^{\frac{1}{1-\alpha}}}{H(p)}. \tag{2.6}$$

Equations (2.5) and (2.6) imply that the probability that product $i \in \mathcal{N}_f$ is purchased by a consumer is given by the following equation:

$$s_i(p) = n_f(p)\frac{h_i(p_i)}{H_f(p_f)} = \frac{\left(H_f(p_f)\right)^{\frac{\alpha}{1-\alpha}} h_i(p_i)}{H(p)}. \tag{2.7}$$

Therefore, the demand for the product $i \in \mathcal{N}_f$ given the profile of prices p has the following form:

$$\begin{aligned} D_i(p) &= \hat{D}_i\left(p_i, H_f(p_f), H(p)\right) \\ &= s_i(p) \times \frac{-h_i'(p_i)}{h_i(p_i)} \\ &= -\frac{\left(H_f(p_f)\right)^{\frac{\alpha}{1-\alpha}} h_i'(p_i)}{H(p)}. \end{aligned} \tag{2.8}$$

With the CES-class demand and negative price, Roy's identity cannot be used to derive demand. Thus, to allow for the demand with negative prices, we assume that $D_i(p) = \lim_{p_i \to 0} D_i(p) = +\infty$ for all $p_i < 0$.

Finally, the consumer surplus CS is given by the expected indirect utility of consumers, which is given by

$$\begin{aligned} CS(p) &= \log\left(\sum_{f\in\mathcal{F}} \left(H_f(p_f)\right)^{\frac{1}{1-\alpha}} \frac{1}{(H(p))^{\alpha}}\right) \\ &= (1-\alpha)\log H(p). \end{aligned} \tag{2.9}$$

I put one remark on the form of demand function that is given by equation (2.8). The demand function for the MNL-class demand system has the form

$$D_i(p) = \frac{\left\{\sum_{j\in\mathcal{N}_f} \exp\left(\frac{a_j-p_j}{\lambda}\right)\right\}^{\delta}}{\sum_{f'\in\mathcal{F}}\left\{\sum_{j\in\mathcal{N}_{f'}} \exp\left(\frac{a_j-p_j}{\lambda}\right)\right\}^{\delta}} \frac{\exp\left(\frac{a_i-p_i}{\lambda}\right)}{\sum_{j\in\mathcal{N}_f} \exp\left(\frac{a_j-p_j}{\lambda}\right)}$$

with $\delta := 1/(1-\alpha)$. This demand function can be regarded as a nested-logit demand function with the set of nests $\{\mathcal{N}_f\}_{f\in\mathcal{F}}$. One difference of this demand function from standard nested-logit demand is that this demand function has $\delta = 1/(1-\alpha) \geq 1$, while standard nested-logit demand functions based on discrete-choice model must have a nest coefficient $\delta \leq 1$. This is because every pair of products should be substitutes in the standard discrete-choice models, and if $\delta > 1$, two products in the same nest is complements when $n_f < (\delta-1)/\delta$. In this sense, the demand function given by equation (2.8) allows for complementarity among products and shows that network externalities provide a microfoundation for nested-logit demand functions with nest coefficient δ greater than 1.

Firm pricing and equilibrium

Each product $i \in \mathcal{N}$ has a constant marginal cost $c_i > 0$ of production. Given the demand system, the profit function of each firm $f \in \mathcal{F}$ is written as a function of the profile of the firm's own prices $p_f = (p_i)_{i \in \mathcal{N}_f}$ and the industry-level aggregator H:

$$\Pi_f(p) = \sum_{i \in \mathcal{N}_f} \hat{D}_i \left(p_i, H_f(p_f), H(p)\right)(p_i - c_i). \tag{2.10}$$

A pricing game consists of a demand system $(D_i)_{i \in \mathcal{N}}$, a set of firms $\mathcal{F}$, sets of the products of each firm $(\mathcal{N}_f)_{f \in \mathcal{F}}$, and a profile of marginal costs $(c_i)_{i \in \mathcal{N}}$. In the pricing game, firms simultaneously set the prices $p_f := (p_i)_{i \in \mathcal{N}_f}$ of their products, with the payoff function Π_f defined by equation (2.10). I call a Nash equilibrium of this pricing game as a *pricing equilibrium*. In the following analysis, I often suppress the arguments of functions for the sake of readability.

Arranging the first-order condition for the profit-maximization of each firm f ($\partial \Pi_f / \partial p_i = 0$), the price of product $i \in \mathcal{N}_f$ at firm f's best-response should satisfy the equation

$$\begin{aligned} & -\frac{h_i''(p_i)}{h_i'(p_i)}(p_i - c_i) \\ &= 1 - \underbrace{\frac{\alpha}{1-\alpha}\frac{\Pi_f}{n_f}}_{\text{network-externality discount}} + \underbrace{\frac{1}{1-\alpha}\Pi_f}_{\text{cannibalization terms}} \\ &=: \mu_f \end{aligned} \tag{2.11}$$

for some μ_f. Following Nocke and Schutz (2018b), I call μ_f as the ι*-markup* of firm f. This ι-markup summarizes the pricing incentive of each firm for each product.

As shown in equation (2.11), ι-markup is decomposed into three factors. The first term, 1, in the second line of the equation (2.11) is the baseline ι-markup, which would be set under the monopolistic competition. The second term is the downward-pricing pressure to expand the networks. The third term is the upward-pricing pressure of oligopoly due to the internalization of cannibalization effects of the change in the industry-level aggregator. The relative magnitudes of the second and the third terms determine the price level of each firm.

We have $-h_i''(p_i)/h_i'(p_i) = 1/\lambda$ and thus $p_i = c_i + \lambda \mu_f$ in the case of MNL-class demand systems, and $-h_i''(p_i)/h_i'(p_i) = \sigma / p_i$ and thus $p_i = c_i/(1 - \mu_f/\sigma)$ in the case of CES-class demand systems. Using these functional forms, the formula for the firm-level aggregators and profit functions are given by

$$H_f = \begin{cases} T_f \exp(-\mu_f) & \text{in the case of MNL-class demand,} \\ T_f \left(1 - \frac{\mu_f}{\sigma}\right)^{\sigma - 1} & \text{in the case of CES-class demand,} \end{cases} \tag{2.12}$$

and

$$\Pi_f = \begin{cases} n_f \mu_f & \text{in the case of MNL-class demand,} \\ \frac{\sigma - 1}{\sigma} n_f \mu_f & \text{in the case of CES-class demand,} \end{cases} \tag{2.13}$$

where $T_f := \sum_{i \in \mathcal{N}_f} \exp\left(\frac{a_i - c_i}{\lambda}\right)$ for MNL-class and $T_f := \sum_{i \in \mathcal{N}_f} a_i c_i^{1-\sigma}$ for CES-class demand. T_f is called as the "type" of firm f that equals the value of the firm-level aggregator of firm f when it engages in marginal cost pricing. Inserting equations (2.3), (2.12), and (2.13) into the first-order condition (2.11) shows that the ι-markup μ_f and the network share n_f depends only on its type T_f and the value of the indusry-level aggregator H. Specifically, the first-order condition can be rewritten as

$$1 = \frac{\mu_f}{1-\alpha}\left(1 - \frac{\gamma(T_f)}{H}\exp\left(-\frac{\mu_f}{1-\alpha}\right)\right) \tag{FOC-MNL}$$

and

$$1 = \frac{\mu_f}{\sigma(1-\alpha)}\left(\sigma - \alpha - (\sigma-1)\frac{\gamma(T_f)}{H}\left(1 - \frac{\mu_f}{\sigma}\right)^{\frac{\sigma-1}{1-\alpha}}\right) \tag{FOC-CES}$$

where $\gamma(x) = x^{\frac{1}{1-\alpha}}$, is the function that amplifies each firm's type through network externalities.

Solving the equation (FOC-MNL) for MNL-class and (FOC-CES) for CES-class demand systems, the ι-markup is obtained as $\mu_f = m(\gamma(T_f)/H, \alpha)$. This implies that all the relevant information for each firm's ι-markup and thus each firm's pricing is summarized in a unidimensional type T_f. This property is called as the "type-aggregation property" (Nocke and Schutz, 2018b), which simplifies the equilibrium and subsequent merger analysis. Using this ι-markup function, we further obtain the network share n_f as

$$n_f = N\left(\frac{\gamma(T_f)}{H}, \alpha\right) := \frac{\gamma(T_f)}{H}\exp\left(-\frac{m\left(\frac{\gamma(T_f)}{H}, \alpha\right)}{1-\alpha}\right), \tag{Share-MNL}$$

and

$$n_f = N\left(\frac{\gamma(T_f)}{H}, \alpha\right) := \frac{\gamma(T_f)}{H}\left(1 - \frac{m\left(\frac{\gamma(T_f)}{H}, \alpha\right)}{\sigma}\right)^{\frac{\sigma-1}{1-\alpha}}, \tag{Share-CES}$$

respectively. It turns out that the function $N(\cdot, \alpha)$ is increasing in the first argument. Thus, a firm f has a large network share n_f either when it has a high type T_f or the value of industry-level aggregator H is small.

Finally, the equilibrium condition for the industry-level aggregator H is that the sum of network shares equals one, that is,

$$\sum_{f \in \mathcal{F}} N\left(\frac{\gamma(T_f)}{H}, \alpha\right) = 1. \tag{2.14}$$

Solving this equation, the equilibrium value of the industry-level aggregator, H^*, is obtained.

The following lemma summarizes this discussion.

Lemma 1. *For any MNL-class or CES-class demand system, there exists a unique pricing equilibrium where each firm $f \in \mathcal{F}$ sets its price profile $p_f^* = (p_i^*)_{i \in \mathcal{N}_f}$ such that*

$$p_i^* = \begin{cases} c_i + \lambda m\left(\frac{\gamma(T_f)}{H^*}, \alpha\right) & \text{in the case of MNL-class demand,} \\ \frac{c_i}{1 - \frac{m\left(\frac{\gamma(T_f)}{H^*}, \alpha\right)}{\sigma}} & \text{in the case of CES-class demand,} \end{cases}$$

where H^* is the solution to equation (2.14).

In Section 2.9, I provide a more generalized result for the existence and the uniqueness of the pricing equilibrium that incorporates both direct indirect network externalities (Proposition 8). Lemma 1 is one special case of that result.

Before discussing the welfare implication of the mergers in the presence of network externalities, it is worth noting that the definition of the type in this study differs from that in Nocke and Schutz (2018a) in the following sense. In Nocke and Schutz (2018a), type $\mathcal{T}_f^{NS}$ is defined as

$$\mathcal{T}_f^{NS} := \left(\sum_{j \in \mathcal{N}_f} h_j(c_j)\right)^{\delta},$$

where $\delta \in [0, 1]$ is the nest coefficient. Nocke and Schutz (2018a) include this nest coefficient in the definition of types because this emerges from the distribution of the consumers' preference for products, which are unaffected by mergers. By contrast, the definition of the type in this study is $T_f := \sum_{j \in \mathcal{N}_f} h_j(c_j)$ but not $\left(\sum_{j \in \mathcal{N}_f} h_j(c_j)\right)^{1/(1-\alpha)}$ because the nest coefficient $1/(1-\alpha)$ emerges from network effects, which mergers affect. This difference of the definition of types changes the welfare implication for the mergers without synergies, as shown in the next section.

2.4 Merger in the Presence of Direct Network Externalities

Based on the equilibrium analysis in the previous section, I proceed to analyze the conditions under which a merger between two firms improves or hurts the consumer surplus. To highlight the impact of network externalities, α is assumed to be strictly positive in the following analysis.

Suppose that two firms f and g with the types T_f and T_g merge to create a new firm M with type T_M. The merger exhibits some technological synergy, which is captured by $\Delta := T_M - T_f - T_g$. The source of technological synergies may be cost reduction, quality improvement, or introduction of new products. It is assumed that such synergies are exogenous primitives of the merger rather than an endogenous choice of the merged entity.

A merger is said to be *CS-increasing* if it increases the equilibrium consumer surplus. Given that the consumer surplus with equilibrium aggregator H^* is given by $(1-\alpha)\log H^*$, the change in consumer surplus resulting from any particular merger can be calculated by the change in the value of equilibrium aggregator H^*. Under this specification of mergers, a merger between two firms is

CS-increasing if and only if

$$N\left(\frac{\gamma(T_f+T_g+\Delta)}{H^*},\alpha\right) \geq N\left(\frac{\gamma(T_f)}{H^*},\alpha\right) + N\left(\frac{\gamma(T_g)}{H^*},\alpha\right) \tag{2.15}$$

holds, where H^* is the pre-merger equilibrium aggregator.[††] Thus, the merger is CS-increasing if and only if the post-merger network share of the merged entity exceeds the pre-merger total network share of merging parties.

From the condition (2.15) and the fact that the network share function is increasing in the type, the merger between two firms is CS-increasing if and only if $\Delta \geq \hat{\Delta}$, where $\hat{\Delta}$ satisfies

$$N\left(\frac{\gamma(T_f+T_g+\hat{\Delta})}{H^*},\alpha\right) = N\left(\frac{\gamma(T_f)}{H^*},\alpha\right) + N\left(\frac{\gamma(T_g)}{H^*},\alpha\right). \tag{2.16}$$

This value $\hat{\Delta}$ is the *CS-neutral technological synergy* required for a merger between two firms. Let $\hat{T}_M := T_f + T_g + \hat{\Delta}$ be the type of the merged entity with CS-neutral technological synergy. The larger the CS-neutral technological synergies are, the more likely it is that mergers with a given value of technological synergy reduce consumer surplus. In this sense, the value of CS-neutral technological synergy can be interpreted as a criterion for approving mergers.

The subsequent sections examine the welfare properties of mergers in the presence of direct network externalities using the notion of CS-neutral technological synergies

.

Merger involving a small firm

First, I show that in the presence of firm-level network effects, acquisition of a sufficiently small firm is CS-increasing without technological synergy, that is, $\hat{\Delta} < 0$.

Proposition 1. *(Merger involving a small firm) Consider a merger between firm f and firm g. If one of the merging parties is sufficiently small, the merger is CS-increasing without technological synergy. That is, there exists $\bar{T}_f$ such that for any $T_f < \bar{T}_f$, $\hat{\Delta} < 0$.*

Proof. In Section 2.10. □

[††]This condition is derived as follows. Suppose that equation (2.15) holds with strict inequality, then, we have

$$\sum_{f' \in \mathcal{F}} N\left(\frac{\gamma(T_{f'})}{H^*},\alpha\right) + N\left(\frac{\gamma(T_f+T_g+\Delta)}{H^*},\alpha\right) > 1.$$

Since the function $N(x,\alpha)$ is decreasing in x, the value of post-merger equilibrium aggregator is greater than H^*. Conversely, if equation (2.15) does not hold, we have

$$\sum_{f' \in \mathcal{F}} N\left(\frac{\gamma(T_{f'})}{H^*},\alpha\right) + N\left(\frac{\gamma(T_f+T_g+\Delta)}{H^*},\alpha\right) < 1.$$

Thus, the value of post-merger equilibrium aggregator is smaller than H^*. Thus, a merger improves consumer surplus if and only if equation (2.15) holds.

In the presence of network externalities, a small firm is disadvantaged in selling the products because of a lack of installed base. When this firm is acquired by a large firm, the acquirer can leverage its own network to sell the small firm's products, which also benefits consumers who wish to access the small firm's products. Further, the contribution of the acquisition of this small firm to the increase in the market power of the merged entity is negligible. Thus, overall, the former benefit from an increase in the access to the products of the small firm dominates, and the consumer surplus increases. This result is in sharp contrast with the result of Nocke and Schutz (2018a) that any merger without synergy reduces the consumer surplus.

Merger from symmetric oligopoly

Next, I examine the welfare effects of mergers in an originally symmetric oligopoly. The symmetric environment is well suited to illustrate the trade-off between the benefit from network expansion and the cost of increased market power in a simplest manner.

Consider a symmetric oligopoly with $|\mathcal{F}|$ firms with the same types T. In the symmetric oligopoly, the equilibrium market share of each firm is given by $1/|\mathcal{F}|$. Thus, the value of the equilibrium aggregator is derived by solving

$$N\left(\frac{\gamma(T)}{H^*}, \alpha\right) = \frac{1}{|\mathcal{F}|}$$

Based on the value of the pre-merger equilibrium aggreagtor H^*, consider a merger between two firms that does not generate any synergy (i.e., $\Delta = 0$). The merger improves the consumer surplus if and only if

$$N\left(\frac{\gamma(2T)}{H^*}, \alpha\right) \geq \frac{2}{|\mathcal{F}|} \tag{2.17}$$

The right-hand side of this inequality does not depend on the value of α. Further, the market share of the merged entity increases with α because the merged entity has the largest network. These jointly imply that the merger is CS-increasing if α is above a certain critical value. The next proposition formalizes this discussion.

Proposition 2. *(Merger from symmetric oligopoly) Suppose that all firms are symmetric so that $T_f = T$ for all $f \in \mathcal{F}$. Then;*

1. *there exists a critical value of the magnitude of direct network externalities above which a merger between two firms are CS-increasing without technological synergy. That is, there exists $\hat{\alpha}$ such that $\hat{\Delta} < 0$ if and only if $\alpha > \hat{\alpha}$;*
2. *$\hat{\alpha}$ decreases with the number of firms $|\mathcal{F}|$.*

Proof. In Section 2.10. □

As discussed earlier, when the magnitude of network effects is greater than some critical value, the merger between two firms improves the consumer surplus. Further, as the number of firms increases, the critical value of the magnitude of network effects reduces, because the increase in the market power due to the merger becomes less important as the market is more competitive before the merger, which decreases the magnitude of the network effects required to offset the consumer harm due to market power.

Overall, as long as the firms are symmetric, the presence of network externalities provides a justification of mergers. In this sense, to some extent, the network externalities can be regarded as a form of synergies accompanying mergers.

Technological synergies and network effects

In the preceding analyses, I have discussed the cases where mergers without technological synergies can be CS-increasing. These results indicate that network externalities can be regarded as a form of synergies accompanying mergers. This section analyzes the relation between the CS-neutral technological synergies and the magnitude of network externalities and examines the extent to which the network externalities can be regarded as a form of synergies.

For the tractability of analysis, I confine the analysis to the case with MNL-class demand systems. The MNL-class demand systems have an advantage that the market share function $N((\gamma(T_f)/H, \alpha)$ depends on α only through the value of $\gamma(T_f)/H$. Specifically, the network share function can be written as $N((\gamma(T_f)/H, \alpha) = N_0((\gamma(T_f)/H))$ where $N_0(x) := N(x, 0)$.[‡‡] From this observation, equation (2.16) can be simplified as

$$N_0\left(\frac{\gamma(T_f + T_g + \hat{\Delta})}{H^*}\right) = N_0\left(\frac{\gamma(T_f)}{H^*}\right) + N_0\left(\frac{\gamma(T_g)}{H^*}\right). \tag{2.18}$$

Using this condition, I analyze how CS-neutral technological synergies vary with respect to firm sizes and the magnitude of direct network externalities.

First, I examine how the size of merging parties affects the technological synergies required to justify mergers. As shown in Proposition 1, when one of the merging parties is small, then the merger makes it easier to sell the product to consumers through the expansion of networks, which improves the consumer surplus. However, as the merging parties become large, such benefit from network expansions are offset by the accompanying increase in the market power, which hurts consumers. These jointly lead to the conjecture that the CS-neutral synergies are negative as long as the merging parties are small, and then become positive when the sizes of the merging parties exceeds certain a threshold. The following proposition formalizes such intuition, and Figure 2.1 illustrates this result.

Proposition 3. *(Firm sizes and technological synergies) For any merger between firm f and g with pre-merger network shares N_f and N_g and pre-merger equilibrium aggregator H^*, there exists a critical value of pre-merger joint network share $\tilde{N}$ such that*

[‡‡]To see this, note that from equation FOC-MNL, $\mu_f/(1-\alpha)$ is determined only through $\gamma(T_f)/H$. Letting this value denoted as $\mu_f/(1-\alpha) =: \tilde{m}_0(\gamma(T_f)/H)$, we have the network share function by

$$N\left(\frac{\gamma(T_f)}{H}, \alpha\right) = \frac{\gamma(T_f)}{H}\exp\left\{-\tilde{m}_0\left(\frac{\gamma(T_f)}{H}\right)\right\} = N\left(\frac{\gamma(T_f)}{H}, 0\right).$$

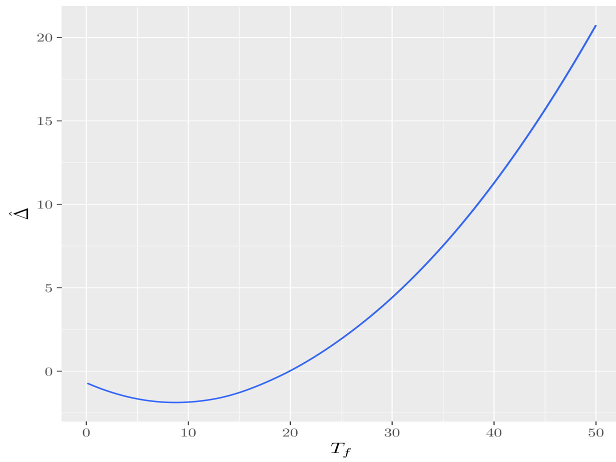


Figure 2.1: The relation between the size of merging parties and the size of technological synergies required for a merger to be CS-increasing.

1. *If* $N_f + N_g \leq \tilde{N}$*,* $\hat{\Delta}$ *decreases with* T_f*;*
2. *If* $N_f \geq \tilde{N}$*,* $\hat{\Delta}$ *increases with* T_f*;*
3. *If* $N_f < \tilde{N} < N_f + N_g$*, then* $\hat{\Delta}$ *increases with* T_f *if and only if* N_g *is greater than some threshold* $\tilde{N}_g(N_f)$*, which decreases with* N_f*.*

Proof. In Section 2.10 □

Proposition 3 and the fact that $\hat{\Delta} = 0$ for $T_f = 0$ jointly lead to the following corollary.

Corollary 1. *Consider a merger between firm* f *and* g *with pre-merger network shares* N_f *and* N_g*. Given the value of the pre-merger equilibrium aggregator, there exists* $\bar{N}(N_g)$ *such that* $\hat{\Delta} < 0$ *if and only if* $N_f < \bar{N}_f(N_g)$*.*

This corollary implies that as long as the pre-merger network shares of the merging parties are below certain critical values, the merger between them is CS-increasing without technological synergies, which generalizes Proposition 1.

Next, I examine how the magnitude of network externalities affects the technological synergies required to improve the consumer surplus. To this end, I introduce some terminologies to describe the firms' sizes, based on the relation between the magnitude of network externalities and the pre-merger network shares. Using the Implicit Function Theorem, we have the following relation:

$$\frac{d}{d\alpha}\left(\frac{\gamma(T_f)}{H^*}\right) = \frac{1}{(1-\alpha)^2}\frac{\gamma(T_f)}{H^*}\frac{\sum_{f'\in\mathcal{F}}(\log T_f - \log T_{f'})N_0'\left(\frac{\gamma(T_{f'})}{H^*}\right)}{\sum_{f'\in\mathcal{F}}N_0'\left(\frac{\gamma(T_{f'})}{H^*}\right)} \tag{2.19}$$

From this equation, the following lemma is obtained.

Lemma 2. *(Network effects and market share) For any type profile $\{T_f\}_{f\in\mathcal{F}}$, there is a threshold value T^* such that*

$$\frac{d}{d\alpha}\left(\frac{\gamma(T_f)}{H^*}\right) \geq 0 \tag{2.20}$$

if and only if $T_f \geq T^*$. Consequently, $N_0(\gamma(T_f)/H^*)$ increases with α if and only if $T_f \geq T^*$.

This result is an example of familiar positive-feedback effects of network externalities. Network effects expand the market shares of firms with greater market shares and shrink that of firms with smaller market share. The threshold type T^* stands for the critical value defining the direction in which the positive feedback effects influence the market shares. I call the firms with $T_f > T^*$ as *strong firms* and the firms with $T_f < T^*$ as *weak firms*.

Based on the definitions of strong firms and weak firms, I analyze the impact of network effects on CS-neutral technological synergies. Using the Implicit Function Theorem, the change in $\hat{\Delta}$ according to the change in α can be written as

$$\frac{d\hat{\Delta}}{d\alpha} = \left\{\frac{d}{d\Delta}N_0\left(\frac{\gamma(\hat{T}_M)}{H^*}\right)\right\}^{-1}\left\{\frac{d}{d\alpha}N_0\left(\frac{\gamma(T_f)}{H^*}\right) + \frac{d}{d\alpha}N_0\left(\frac{\gamma(T_g)}{H^*}\right) - \frac{d}{d\alpha}N_0\left(\frac{\gamma(\hat{T}_M)}{H^*}\right)\right\} \tag{2.21}$$

The following proposition characterizes how the impacts of network externalities on CS-neutral technological synergies vary with the sizes of merging parties.

Proposition 4. *(Network externalities and technological synergies) Consider a merger between firms f and g with types T_f and T_g that lead to pre-merger equilibrium network shares N_f and N_g.*

1. *If both f and g are weak, then $\hat{\Delta}$ decreases with α.*

2. *If f is strong and g is weak, then there exists $\hat{N} \in (0,1)$ such that if $N_f + N_g < \hat{N}$, then $\hat{\Delta}$ decreases with α.*

3. *If both f and g are strong and $N_f + N_g$ is close to* 1*. Then $\hat{\Delta}$ increases with α.*

Proof. In Section 2.10. □

The underlying intuition of the first part of Proposition 4 is similar to that of Proposition 1. Weak firms fail to leverage the network effects due to their small market shares. The greater the magnitude of the network effects, the more serious is the failure of the weak firms to internalize the network externalities. In such cases, the merger softens this failure to internalize the network externalities. Therefore, the merger between weak firms is more desirable as the magnitude of network effects increases. A similar argument partially extends to the case where one firm in the merging party is strong and the other is weak, which leads to the second part of Proposition 4.

The impact of network effects on mergers between strong firms is ambiguous. When both of the merging parties are strong, the demand-side scale economy may not be sufficient to compensate an increase in the markups caused by a greater concentration. When the joint market share of the two firms is too large, the merged entity has an extremely strong market power. An increase in the consumer gain from a further increase in the magnitude of network effects is offset by an increase in the markup of the merged entity. Therefore, when the joint market share is sufficiently large, a greater magnitude network effects requires greater synergies for mergers between large firms to improve consumer welfare. This result is consistent with the concern raised by EU competition authority regarding the potential harm to the competition in the merger between Facebook and WhatsApp.[§§] Figure 2.2 shows numerical examples of Proposition 4 in an industry with 12 firms, one of which, f' has $T_{f'} = 25$, another has $T_{f'} = 20$, and the remaining 10 firms have $T_{f'} = 5$.

In summary, the presence of network externalities makes mergers more likely to improve consumer surplus when the merging parties are small, or the firms in the industry are symmetric. However, as the size of the merging parties become larger relative to the size of the industry, the presence of network externalities makes mergers more likely to harm consumers.

2.5 Acquisition of Innovative Entrants

This section considers an acquisition of an innovative entrant by an incumbent and compares the innovation incentives of the incumbents and the entrant, which is studied by Cunningham et al. (2018). In the theoretical part of their analysis, Cunningham et al. (2018) show that the incumbent always has a smaller incentive to continue the innovative projects than the entrant does because of the “replacement effect" but might nevertheless have an incentive to acquire the innovative entrant just to eliminate the future competitor. Such an acquisition is called *killer acquisition*.

By contrast, the presence of network externalities modifies the above argument. Specifically, when the scale of the innovative entrant is not large, the incumbent has a greater incentive to incur costs to continue the project because of the demand-side scale economies. Then, there is a case where the incumbent would continue the project, but the entrant would not, in which case the only way to continue the project is to approve the acquisition of the entrant.

This leads to a policy implication regarding the merger control and innovation. Too stringent merger control may reduce the entrepreneurs' incentive to start new projects, because a merger may be the sole method to make a profit. Indeed, for many startups, one of their final objectives is to be acquired by large tech companies in return for a large payment.

Consider the following game. There is a set $\mathcal{F}$ of incumbents. The demand system is given by MNL-class demand. Each incumbent $f \in \mathcal{F}$ has its type T_f. There is also an innovative potential entrant E who has a project that succeeds with probability $\rho \in (0, 1)$. If the project succeeds, it generates a new product with type T_E. Among the incumbents, there is one incumbent $I \in \mathcal{F}$ interested in acquiring the project of the potential entrant E.

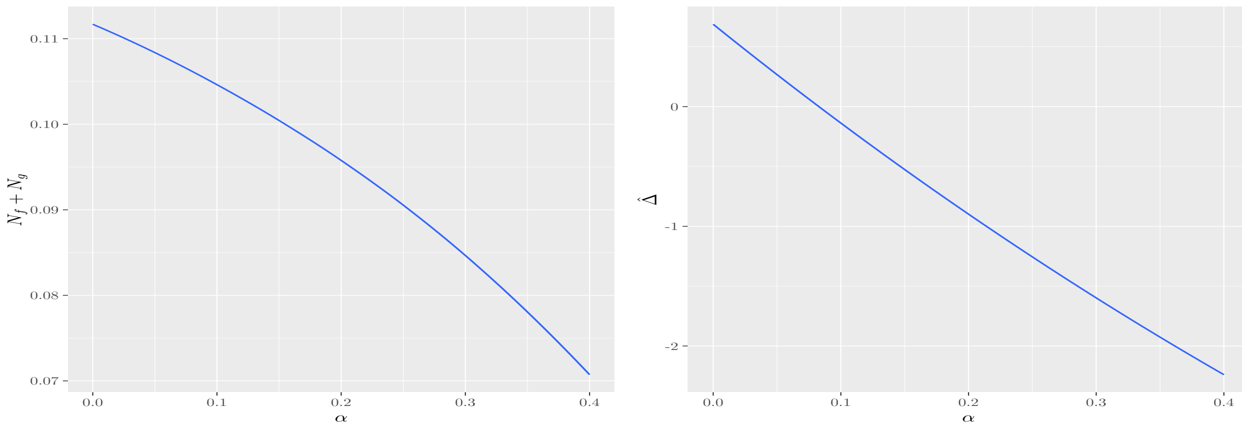
Given this environment, consider the following three-stage game.

1. Whenever feasible, firm I offers a take-it-or-leave-it offer to firm E that specifies the payment to acquire the project. If offered, firm E chooses whether to accept it.

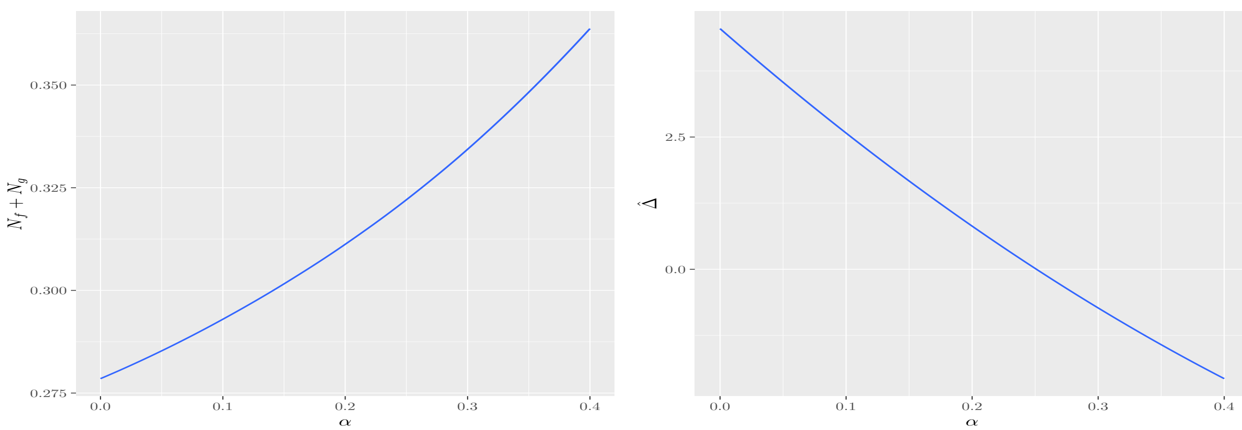
2. The firm that owns the project decides whether to continue or liquidate the project.

   - If the project is continued, it takes cost K and succeeds with probability ρ.
   - If the project is liquidated, it generates the liquidation value L.

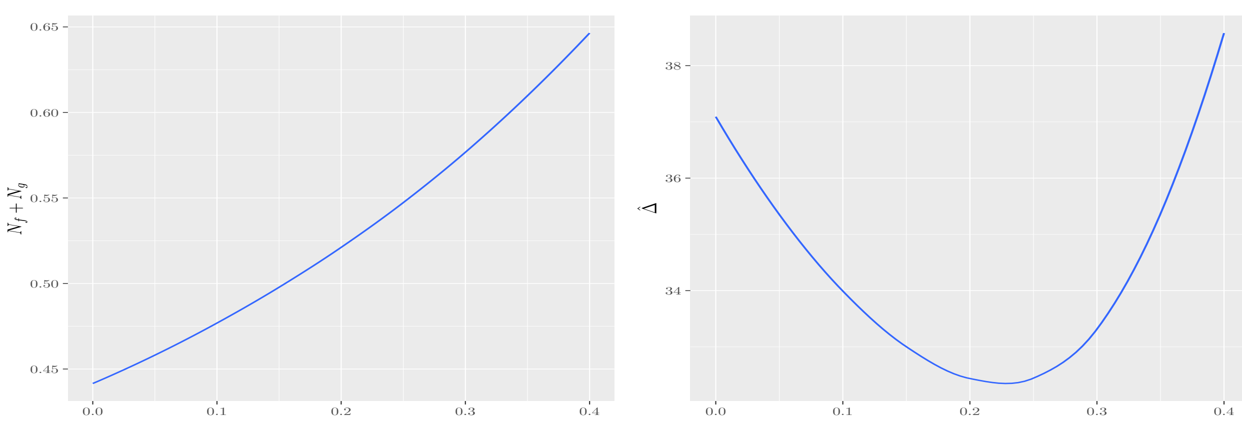
[§§] Case No COMP/M.7217 - FACEBOOK/ WHATSAPP



(a) Weak firms ($T_f = T_g = 5$)



(b) Strong and weak firms ($T_f = 25$, $T_g = 5$)



(c) Strong firms ($T_f = 25$, $T_g = 20$)

Figure 2.2: The effects of direct network externalities α on the joint network share $N_f + N_g$ and the required technological synergies $\hat{\Delta}$ for merging parties f and g. In this example, there are one firm f' with $T_{f'} = 25$, one firm with $T_{f'} = 20$, and 10 firms with $T_{f'} = 5$.

3. Depending on the project's success and the owner, three cases emerge.

    - If the project fails, the set $\mathcal{F}$ of firms play the corresponding pricing game. Each firm $f \in \mathcal{F}$ has type T_f.
    - If the project succeeds and firm I owns the project, the set $\mathcal{F}$ of firms play the corresponding pricing game. Each firm $f \in \mathcal{F} \setminus \{I\}$ has type T_f, and firm I has type $T_I + T_E$.
    - If the project succeeds and firm E owns the project, the set $\mathcal{F} \cup \{E\}$ of firms play the corresponding pricing game. Each firm $f \in \mathcal{F} \cup \{E\}$ has type T_f.

Let H_0, H_I, and H_E be the equilibrium values of the aggregators in the cases where (1) project has failed, (2) project has succeeded and I owns it, and (3) project has succeeded and E owns it.

The expected profit of the incumbent from continuing the project is given by

$$\rho N_0\left(\frac{\gamma(T_I+T_E)}{H_I}\right) m\left(\frac{\gamma(T_I+T_E)}{H_I},0\right) + (1-\rho)N_0\left(\frac{\gamma(T_I)}{H_0}\right) m\left(\frac{\gamma(T_I)}{H_0},0\right) - K,$$

whereas the expected profit of the incumbent from liquidating the project is given by

$$N_0\left(\frac{\gamma(T_I)}{H_0}\right) m\left(\frac{\gamma(T_I)}{H_0},0\right) + L.$$

Thus, the incumbent continues the project if and only if

$$\Delta\pi^I := \rho\left\{N_0\left(\frac{\gamma(T_I+T_E)}{H_I}\right) m\left(\frac{\gamma(T_I+T_E)}{H_I},0\right) - N_0\left(\frac{\gamma(T_I)}{H_0}\right) m\left(\frac{\gamma(T_I)}{H_0},0\right)\right\} \geq L+K \qquad (2.22)$$

The expected profit of the entrant from continuing the project is given by

$$\rho N_0\left(\frac{\gamma(T_E)}{H_E}\right) m\left(\frac{\gamma(T_E)}{H_E},0\right) - K,$$

whereas the expected profit of the entrant from liquidating the project is given by L. Thus, the incumbent continues the project if and only if

$$\Delta\pi^E := \rho N_0\left(\frac{\gamma(T_E)}{H_E}\right) m\left(\frac{\gamma(T_E)}{H_E},0\right) \geq L+K \qquad (2.23)$$

The following proposition highlights that, when T_E is small, $\Delta\pi^I > \Delta\pi^E$ can hold, in which case for some values of K, the project is continued only if it is owned by the incumbent.

Proposition 5. *(Innovation incentives) The following statements hold:*

1. *If T_E is sufficiently small, then the incumbent has a greater incentive to continue the project, that is, $\Delta\pi^I > \Delta\pi^E$ holds.*

2. *If T_E is sufficiently large, then the incumbent has a smaller incentive to continue the project, that is, $\Delta\pi^I < \Delta\pi^E$ holds.*

Proof. In Section 2.10. □

If the size of innovation is small relative to the industry, the reverse of killer acquisition may hold. An entrant with small innovation cannot break-even by just entering into the market, but the incumbent can gain by purchasing such an innovation. This is because the entrant cannot attract consumers to the new product because of the lack of the network, while the incumbent can leverage its installed base to attract consumers. Formally, there exist a pair of innovation cost K and liquidation value L such that the incumbent continues the innovation, but the entrant does not.

However, if both the entrant and incumbent are large relative to the industry, the killer acquisition may again emerge by the same logic as Cunningham et al. (2018), which is shown in the second part of the proposition.

For completeness, I close the model when T_E is sufficiently small. When T_E is sufficiently small, $\Delta\pi^I > \Delta\pi^E$ holds by Proposition 5. In such cases, firm I offers a payment $\rho\Delta\pi^E - K - L$ and acquires the project whenever feasible. However, if the merger is expected to be blocked and $K + L \in (\rho\Delta\Pi^E, \rho\Delta\pi^E)$ holds, firm E will not continue the project, and thus the innovation is not completed.

2.6 Merger in Two-Sided Markets

Finally, I analyze the mergers in the presence of indirect network externalities, with particular interest in two-sided markets. The focus is how structures of firm's sizes on two sides of markets affect the welfare properties of mergers. I first examine how two-sidedness affects the price structures and welfare properties of equilibria. Then, based on the equilibrium analysis, I show how two-sidedness affects the welfare properties of mergers. A detailed derivation of equilibrium is relegated to the general analysis in Section 2.9.

Equilibrium analysis

Setup Consider an industry with two sides of markets A and B with a set $\mathcal{N}^J$ of imperfectly substitutable products on side $J \in \{A, B\}$ produced by the set of firms $\mathcal{F}$. Each firm $f \in \mathcal{F}$ produces the set $\mathcal{N}^J_f$ of products on side J, where $\mathcal{N}^J_f \cap \mathcal{N}^J_{f'} = \emptyset$ for $f \neq f'$ and $\bigcup_{f\in\mathcal{F}} \mathcal{N}^J_f = \mathcal{N}^J$.

There is a mass of consumers on side $J \in \{A, B\}$ who derive firm-level indirect network externalities from the purchase of each product $i \in \mathcal{N}^J_f$, which depends on the number of consumers on the other side who purchase the products of firm f. Specifically, each consumer $z \in [0, 1]$ on side $J \in \{A, B\}$ yields the indirect subutility from the purchase of product $i \in \mathcal{N}^J_f$ by

$$\log h^J_i(p_i) + \beta_J \log n^I_f + \varepsilon^J_{iz} \tag{2.24}$$

where $\log h^J_i(p_i)$ is the stand-alone indirect subutility, β_J is the magnitude of the indirect network externalities, n^I_f is the number of consumers on side $I \neq J$ who purchase the product of firm

f, and ε^J_{iz} is an idiosyncratic taste shock that follows an i.i.d. type-I extreme-value distribution. In this section, I assume that demand system is MNL-class, $h^J_i(p_i) = \exp((a_i - p_i)/\lambda^J)$, and that only consumers on side A enjoy indirect network externalities from consumers on side B, that is, $\beta_A = \beta \in (0,1)$, and $\beta_B = 0$. The situation that suits this assumption would be media platforms where firms are newspapers or online content services, consumers on side A are advertisers who benefit from the viewers who are consumers on side B. In some industries, advertisements are considered a nuisance but sometimes helpful to consumers. Another example is an industry where firms are advertising networks, consumers on side A are advertisers, and consumers on side B are publishers who allow advertising networks to display ads. The insights from the results in this setting would apply to the setting where consumers on both sides of the market derive indirect network externalities as long as there is asymmetry in network externalities between two sides of the markets.

Consumers on each side have no outside option and single-home so that they purchase one product that gives the highest level of utilities. Defining firm-level subaggregator on side J as $H^J_f(p_f) = \sum_{i\in\mathcal{N}^J} h^J_i(p_i)$, consumers' optimal choice of products leads to the network shares

$$n^A_f = \frac{H^A_f\left(p^A_f\right)\left(H^B_f\left(p^B_f\right)\right)^\beta}{H^A(p)}, \tag{2.25}$$

and

$$n^B_f = \frac{H^B_f\left(p^B_f\right)}{H^B(p)}, \tag{2.26}$$

where

$$H^A(p) = \sum_{f\in\mathcal{F}} H^A_f\left(p^A_f\right)\left(H^B_f\left(p^B_f\right)\right)^\beta$$

is the industry-level aggregator on side A, and

$$H^B(p) = \sum_{f\in\mathcal{F}} H^B_f\left(p^B_f\right)$$

is the industry-level aggregator on side B. Finally, the demand for product $i \in \mathcal{N}^J_f$ is given by

$$\hat{D}^J\left(p_i, H^J_f\left(p^J_f\right), H^I_f\left(p^I_f\right), H^J(p)\right) = n^J_f \times \frac{-(h^J_i)'(p_i)}{H^J_f(p^J_f)}. \tag{2.27}$$

Given the above demand function, each product $i \in \mathcal{N}^J$ has a constant marginal cost $c_i > 0$ of production, and firm f's profit $\Pi_f(p)$ is given by the sum of the profit on two sides of the markets $\Pi_f(p) = \Pi^A_f(p) + \Pi^B_f(p)$, where the profit on side J is given by

$$\Pi^J_f(p) = \sum_{i\in\mathcal{N}^J_f} \hat{D}^J\left(p_i, H^J_f\left(p^J_f\right), H^I_f\left(p^I_f\right), H^J(p)\right)(p_i - c_i). \tag{2.28}$$

In the pricing game, each firm f chooses its price profile $p_f = (p_i)_{i\in\mathcal{N}^A_f\cup\mathcal{N}^B_f}$ to maximize $\Pi_f(p)$. The Nash equilibrium of the pricing game is called pricing equilibrium.

Equilibrium and welfare analysis Given the significance of the properties of equilibrium pricing and welfare, I examine these properties first.

As in the case with direct network externalities, there is the common ι-markup property; at the optimum, each firm's pricing of each product on each side $J \in \{A, B\}$ is summarized by an ι-markup μ_f^J which is given by

$$-\frac{\left(h_i^J\right)''(p_i)}{\left(h_i^J\right)'(p_i)}(p_i - c_i) = \mu_f^J \text{ for all } i \in \mathcal{N}_f^J,$$

for each $J \in \{A, B\}$. In the case of MNL-class demand system, it turns out that ι-markups μ_f^A and μ_f^B are given by the solution to the following system of equations

$$1 - \left(1 - n_f^A\right)\mu_f^A = 0, \tag{2.29}$$

and

$$1 - \left(1 - n_f^B\right)\mu_f^B - \beta\frac{n_f^A}{n_f^B} = 0, \tag{2.30}$$

where network shares n_f^A and n_f^B are given by

$$n_f^A = \frac{T_f^A\left(T_f^B\right)^\beta}{H^A}\exp\left(-\mu_f^A - \beta\mu_f^B\right), \quad n_f^B = \frac{T_f^B}{H^B}\exp\left(-\mu_f^B\right), \tag{2.31}$$

and $T_f^J := \sum_{f \in \mathcal{N}_f^J} h_i^J(c_i)$ is the type of firm $f \in \mathcal{F}$ on side $J \in \{A, B\}$. Let m^A and m^B be the solution to this system of equations, as functions of (T_f^A, T_f^B, H^A, H^B). Then, plugging $\mu_f^J = m^J$ into equation (2.31), the network share functions $N^A(T_f^A, T_f^B, H^A, H^B)$ and $N^B(T_f^A, T_f^B, H^A, H^B)$ are obtained. Finally, the equilibrium condition for the industry-level aggregators H^A, H^B is given by

$$\sum_{f \in \mathcal{F}} N^J\left(T_f^A, T_f^B, H^A, H^B\right) = 1, \text{ for } J \in \{A, B\}. \tag{2.32}$$

Section 2.9 shows the uniqueness of pricing equilibrium in this environment. Let H^{A*} and H^{B*} be the equilibrium value of the aggregators.

Before the merger analysis, it is useful to understand the nature of price structure in the two-sided markets in this framework. One particular interest is how an increase in the type on one side affects the pricing incentive on the other side, and the resulting market share.

First, I introduce some terminology to describe the price structure of each firm. Equations (2.29) and (2.30) show that given the fixed network shares, each firm has an incentive to lower prices on side B than on side A because the firms can attract more consumers and charge higher prices on side A by providing indirect network externalities by attracting consumers on side B. Thus, consumers on side B are "subsidized" through lower prices, while consumers on side A are "subsidizing" through paying relatively higher prices. Based on this observation, I call side A as *subsidizing segment* and side B as *subsidized segment*, which follows the terminology used by Rochet and Tirole (2003).

The following lemma illustrates the relationship between the type of a firm on each side and its ι-markups and network shares on two sides of markets.

Lemma 3. *The following statements hold:*

1. *Each firm's price level on subsidizing segment increases with the firm's type on both segments. That is,* m^A *is increasing in* T_f^A *and* T_f^B.

2. *Each firm's price level on subsidized segment increases with the firm's type on that segment, but decreases with the firm's type on the subsidizing segment. That is,* m^B *is decreasing in* T_f^A *and increasing in* T_f^B.

3. *Each firm's network share on each segment increases with the firm's type on both segments. That is, both* N^A *and* N^B *are increasing in* T_f^A *and* T_f^B.

Proof. In Section 2.10. □

This lemma characterizes how the price structure and the network shares of each firm on each side are related to its types on two sides of the markets. The first part of the lemma shows how the ι-markup on subsidizing segment, side A, varies with its type on each side. Given that the consumers on side A benefit from the indirect network externalities from side B, increases in T_f^A and T_f^B both expand the network share of firm f on side A through an increase in either the productivity or the benefit of indirect network externalities. Such an increase in network size leads the firm to set a higher markup on side A. The second part of the lemma shows how the ι-markup on subsidized segment, side B, varies with T_f^A and T_f^B. An increase in T_f^B simply increases the market share on side B and thus increases the markup on side B. By contrast, an increase in T_f^A enlarges the firm's customer base and increases the markup on side A, which increases the firm's incentive to further expand the customer base on side A. Therefore, the firm lowers the ι-markups on side B to further subsidize consumers on side B. Finally, the changes in the network shares depend on the direct effect of a change in types and the indirect effect of a change in ι-markup. It turns out that the former dominates the latter whenever they conflict, and thus the network shares on both sides increase with firm's type on each side.

Next, as a preliminary for examining the welfare effects of mergers, consider the relationship between equilibrium consumer surplus and firms' types on each side. The equilibrium consumer surplus on each side is given by

$$CS^{A*} = \log H^{A*} - \beta \log H^{B*} \text{ and } CS^{B*} = \log H^{B*}. \tag{2.33}$$

The aggregate consumer surplus is given by

$$CS^{*} = CS^{A*} + CS^{B*} = \log H^{A*} + (1-\beta) \log H^{B*}. \tag{2.34}$$

These equations show that the equilibrium consumer surplus on each side can be calculated using the values of equilibrium aggregators H^{A*} and H^{B*}. Thus, by characterizing the relation between the equilibrium aggregators and the primitives such as firms' types on the two sides of markets, the determinants of equilibrium consumer surplus can be analyzed. The following result shows the relation between equilibrium aggregators and firms' types.

Proposition 6. *(Firm types and equilibrium aggregators) Consider a pricing equilibrium with equilibrium aggregators H^{A*}, H^{B*}, and equilibrium network shares $(n_f^J)_{f\in\mathcal{F}}$ for $J = A, B$. The following statements hold.*

1. *Both H^{A*} and H^{B*} increase with T_f^B.*
2. *H^{A*} increases with T_f^A.*
3. *There exists $\bar{n}_f^A$ such that H^{B*} increases with T_f^A if $n_f^A \geq \bar{n}_f^A$. Otherwise, H^{B*} may decrease with T_f^A.*

Proof. In Section 2.10 □

This proposition states that both H^{A*} and H^{B*} increase with the type T_f^B of firm f on subsidized segment (side B), which implies that the aggregate consumer surplus that is defined by the sum of consumer surplus on the two sides increases with T_f^B. However, an increase in type T_f^A of firm f on the subsidizing segment (side A) might decreases H^{B*}, while it always increases H^{A*}. This occurs in the following scenario. Suppose that the type T_f^A of firm f on side A increases. This increases the share of firm f on side A, which leads to a lower price on side B to attract more consumers on side A. However, it decreases the share of other firms on side A, which leads to higher prices on side B due to the reduced incentives of firms to subsidize side B. The consumer surplus on side B may decrease if the former gain is not large enough to offset the latter loss. When firm f is small, the latter loss can dominate and eventually hurt consumers on side B in aggregate.

This proposition leads to the following implications for merger controls. Unlike the one-sided markets, synergies may not always benefit consumers on every side, especially when such synergies are generated on the subsidizing segments, such as advertisers' side in media platforms. By contrast, synergies generated on subsidized segments, such as eyeballs, are likely to improve consumer surplus in aggregate. In this sense, it may be more conservative to focus on the synergies generated on subsidized segments when faced with a merger between two-sided platforms.

CS-neutral synergies for mergers in two-sided markets

Based on the equilibrium analysis, I analyze mergers in two-sided markets. Characterizing the consumer-surplus-oriented merger policy with respect to the synergies is challenging, because the consumer surplus can decrease with the type of firm, as shown in Proposition 6. Thus, it is often difficult to characterize the consumer-surplus effects of mergers in two-sided markets using simple threshold types. Nonetheless, Proposition 6 implies that some threshold property in the synergies generated on subsidized segment can be obtained as follows.

Consider a merger between firm f and firm g. Let M be the merged entity with the type $(T_M^A, T_M^B) = (T_f^A + T_g^A + \Delta^A, T_f^B + T_g^B + \Delta^B)$. I call Δ^A and Δ^B as the technological synergy on side A and side B, respectively. A merger is *CS-increasing* if the equilibrium value of CS is greater than that before the merger. The following result shows, as a corollary of Proposition 6, that for any merger with given synergy on side B, the merger is CS-increasing if and only if the synergy on side B is above certain threshold.

Corollary 2. *Consider a merger between firms f and g. For any technological synergy Δ^A on side A, there exists $\bar{\Delta}^B(\Delta^A)$ such that the merger is CS-increasing if and only if $\Delta^B > \bar{\Delta}^B(\Delta^A)$.*

Unfortunately, characterizing the condition under which mergers improve the consumer surplus, that is, the shape of $\bar{\Delta}^B(\Delta^A)$, is complicated because the synergies on side A might reduce the consumer surplus on side B. By contrast, it turns out that characterizing the mergers that leave consumer surplus *on both sides* unchanged is relatively easy, which gives a clean insight on the welfare properties of mergers in two-sided markets. I say that a merger is *strictly CS-neutral* if the equilibrium value of consumer surplus on both sides, CS^{A*} and CS^{B*}, are unchanged as a result of the merger. A merger is strictly CS-neutral if and only if two equations holds;

$$\begin{aligned} N^A\left(T_M^A, T_M^B, H^{A*}, H^{B*}\right) &= N^A\left(T_f^A, T_f^B, H^{A*}, H^{B*}\right) + N^A\left(T_g^A, T_g^B, H^{A*}, H^{B*}\right), \\ N^B\left(T_M^A, T_M^B, H^{A*}, H^{B*}\right) &= N^B\left(T_f^A, T_f^B, H^{A*}, H^{B*}\right) + N^B\left(T_g^A, T_g^B, H^{A*}, H^{B*}\right), \end{aligned} \tag{2.35}$$

where H^{A*} and H^{B*} are the pre-merger aggregators.

The next lemma shows that there is a unique pair of technological synergies that is strictly CS-neutral.

Lemma 4. *For any merger between firms f and g, there exists a unique pair of technological synergies $(\hat{\Delta}^A, \hat{\Delta}^B)$ such that the merger is strictly CS-neutral and thus satisfies the condition (2.35).*

Proof. In Section 2.10. □

The pair of technological synergies $(\hat{\Delta}^A, \hat{\Delta}^B)$ defined in equation (2.35) has a clear and tractable characterization, which is similar to the condition in one-sided markets shown in (2.16). Further, this pair of synergies has the following normative interpretation. Whenever $(\Delta^A, \Delta^B) < (\hat{\Delta}^A, \hat{\Delta}^B)$, consumers surplus on at least one side of the markets is reduced as a result of merger. Thus, the technological synergies for a strictly CS-neutral merger can be treated as a benchmark criterion for the scrutiny of merger review that consumer-surplus-oriented competition authorities should adopt. I call the pair of technological synergies $(\hat{\Delta}^A, \hat{\Delta}^B)$ of strictly CS-neutral mergers as *CS-neutral technological synergies*.

Next, using the notion of CS-neutral technological synergies, I analyze the interaction between the firm's pre-merger network shares and the technological synergies required for strictly CS-neutral mergers. Unlike the mergers in one-sided markets, the relation between the types and the equilibrium market shares is not intuitively clear, and thus characterizing CS-neutral technological synergies in terms of the types of firms does not necessarily provide clear guidance. Thus, rather than directly characterizing CS-neutral technological synergies in terms of the types of merging parties, I use the pre-merger network shares of merging parties for characterizing the CS-neutral synergies. To this end, I take the following procedure:

1. First, for any fixed network shares n^A and n^B, compute the ι-markup implied by the first-order condition:
$$\mu^B = \frac{1}{1-n^B}\left(1-\beta\frac{n^A}{n^B}\right), \tag{2.36}$$
and
$$\mu^A = \frac{1}{1-n^A}. \tag{2.37}$$

2. Using these formulas for ι-markups, backup the types T^A and T^B using the formulas for network shares
$$n^A = \frac{T^A\left(T^B\right)^\beta}{H^A}\exp\left(-\mu^A-\beta\mu^B\right), \tag{2.38}$$
and
$$n^B = \frac{T^B}{H^B}\exp\left(-\mu^B\right). \tag{2.39}$$
Solving this system of equations with respect to T^A and T^B derives the type induced by network shares as $T^J = \tau^J(n^A, n^B, H^A, H^B)$ for $J = A, B$, where
$$\tau^A\left(n^A, n^B, H^A, H^B\right) := \frac{H^A}{\left(H^B\right)^\beta}\frac{n^A}{\left(n^B\right)^\beta}\exp\left(\frac{1}{1-n^A}\right), \tag{2.40}$$
and
$$\tau^B\left(n^A, n^B, H^A, H^B\right) := H^B n^B \exp\left(\frac{1}{1-n^B}\left(1-\beta\frac{n^A}{n^B}\right)\right). \tag{2.41}$$
Finally, with pre-merger network shares (n_f^A, n_f^B) and (n_g^A, n_g^B) and pre-merger aggregators (H^A, H^B), the type that achieves the strictly CS-neutral merger $\tau^J(n_f^A + n_g^A, n_f^B + n_g^B, H^A, H^B)$ for $J = A, B$ can be computed. Using this definition of the type, the CS-neutral technological synergies can be computed as functions
$$\begin{aligned}&\tilde{\Delta}^J\left(n_f^A, n_g^A, n_f^B, n_g^B, H^A, H^B\right)\\ :=&\tau^J\left(n_f^A+n_g^A, n_f^B+n_g^B, H^A, H^B\right) - \tau^J\left(n_f^A, n_f^B, H^A, H^B\right) - \tau^J\left(n_g^A, n_g^B, H^A, H^B\right).\end{aligned} \tag{2.42}$$

To understand the novel impact of two-sidedness on the competitive effects of mergers, I consider the mergers between similar firms where $n_f^J = n_g^J = n^J$ for $j = A, B$. Let $\theta := n^A/n^B$ be the ratio of network shares of side A to B. Then, the type for the strictly CS-neutral merger is given by
$$\tau^A(2n^A, 2n^B, H^A, H^B) = \frac{H^A}{H^B}2^{1-\beta}\frac{n^A}{\left(n^B\right)^\beta}\exp\left(\frac{1}{1-2n^A}\right) \tag{2.43}$$

and

$$\tau^B(2n^A, 2n^B, H^A, H^B) = H^B 2n^B \exp\left(\frac{1}{1-2n^B}(1-\beta\theta)\right) \tag{2.44}$$

The CS-neutral technological synergy on side J is positive if and only if

$$\frac{\tau^J(2n^A, 2n^B, H^A, H^B)}{2\tau^J(n^A, n^B, H^A, H^B)} > 1,$$

which is equivalent to the condition

$$\frac{1}{1-2n^A} - \frac{1}{1-n^A} - \beta \log 2 > 0 \tag{2.45}$$

for $J = A$ and

$$1 - \beta\theta > 0 \tag{2.46}$$

for $J = B$, respectively. This fact directly leads to the following proposition.

Proposition 7. *Consider a merger between firms with the same pre-merger network shares n^A and n^B. Let $\theta := n^A/n^B$. There exists a critical value $\hat{n}^A$ such that CS-neutral technological synergy on side A is positive if and only if n^A is smaller than $\hat{n}^A$. CS-neutral technological synergy on side B is positive if and only if* $1 - \beta\theta > 0$.

The fact that whether a strictly CS-neutral merger requires positive technological synergies only depends on the ratio of network shares θ is a striking feature of the two-sided markets. If the merging parties are sufficiently large on the subsidizing segment (θ being large), then the strictly CS-neutral merger does not involve any technological synergy on the subsidized segment, while it may still require technological synergy on the subsidizing segment, which is the segment where merging parties exercise market power. However, as in the case with direct network externalities, the benefit from the network expansion may offset the cost of the increase in markups accompanying mergers, which may renders even the technological synergies on side A unnecessary.

Note that in this setting, the condition $\beta\theta > 1$ is equivalent to the condition that merging parties set negative markups to consumers on the subsidized segment. In this sense, the CS-neutral technological synergy on subsidized segment is negative if and only if firms set negative markups before the merger. This can provide a practical test to evaluate the impact of two-sidedness on mergers involving platforms: consumers who are subsidized through negative markups are likely to benefit from mergers.

In summary, two-sidedness affects the welfare properties of mergers in two ways. First, for consumers on the subsidizing segment, a merger may be desirable when the benefit of network expansion outweighs the cost of the accompanying market power. Second, when the merging parties set negative markups to consumers on the subsidized segment, the merger increases the subsidization incentives and benefits consumers on the subsidized segment.

2.7 Discussion

The merger analysis in this study is based on a static framework of price competition with single-homing consumers. While it derives useful insights on the merger policy toward platforms, there are also several limitations due to the simplicity of the framework, such as the restriction on demand systems and the absence of consumer multi-homing, compatibility choices, dynamics of competition, and the endogenous mergers. This section discusses how these elements potentially alter the implication of the main analysis.

General demand system To adopt an aggregative-games approach, it is assumed that the demand system has an IIA property across firms' products. However, this restricts the substitution patterns among products and is not realistic assumption in several applications. To incorporate the rich substitution patterns into the merger analysis, one needs to give up exploiting aggregative properties and thus obtaining clear theoretical results. In this case, a better way to conduct merger analysis is resorting to the simulation with an estimated demand model, which is a standard procedure in the empirical industrial organization literature.

The main analysis in Section 2.3 also assumes that there is not outside option. This assumption can be relaxed in the following manner. Suppose that each consumer z has an outside option with value $\log H_0 + \varepsilon_{0z}$, where ε_{0z} follows an i.i.d. type-I extreme-value distribution, and chooses whether to participate in one of the networks. Then, the probability that a consumer participate in some network is characterized by the following equation:

$$n_f = \frac{\left(H_f(p_f)\right)^{\frac{1}{1-\alpha}}}{H_0 \frac{\left(H_f(p_f)\right)^{\frac{1}{1-\alpha}}}{(n_f)^{\alpha}} + H(p)}.$$

Then the network share can be written as the function $\tilde{n}_f(H_f, H)$. Since the demand of each product conditional on that consumers participate in some network is given by equation (2.8), the unconditional demand function is given by

$$-\tilde{n}_f(H_f(p_f), H(p)) \frac{h_i'(p_i)}{H_f(p_f)}.$$

A similar analysis can be made with this demand system, and the cases where $H_0 = 0$ corresponds with the model in Section 2.3. Also, as α approaches to 0, the demand system corresponds with that of Nocke and Schutz (2018b).

Finally, I discuss the specification of network externalities. In the model, the form of indirect utility is given by equation (2.52), and thus network externalities enter into the utility in a logarithm form. While this specification is not common in the theoretical studies on competition in two-sided markets or competition with network externalities, which mainly use linear network externalities (Katz and Shapiro, 1985; Armstrong, 2006), logarithm specification of network externalities are often adopted in the empirical studies that try to estimate the magnitude of network externalities or consumers' preference for variety (Ohashi, 2003; Rysman, 2004, 2007). Whether linear or logarithm specification is more plausible depends on the application. In this study, I adopt the logarithm specification to generate a closed-form demand functions, which greatly improves the tractability of analysis. Another reason to adopt logarithm form of network externalities is that, under linear network externalities, even consumers' choice probability of each products given a price profile may not be unique, which necessarily complicates the analysis of price competition and merger policy.[¶¶]

Multi-homing The model in the main analysis assumed that the consumers on both sides single-home, to describe the demand on each side as a function of single aggregator. However, in some environments, consumers on one side often multi-home, and consumers on both side multi-home in another environment. The framework of this study fails to capture the implication of such kind of multi-homing. Thus, when consumers are likely to multi-home, we need to modify the results of the analysis according to the environments under consideration.

Partly, the following type of ad-sponsored competitive-bottleneck model can be considered, which is a slight modification of Anderson and Peitz (2015):

$$v_i - A_i + \alpha \log n_f + \varepsilon_{iz}. \tag{2.47}$$

Then, the demand is given by

$$s_i = \frac{\exp(v_i - A_i) n_f^{\alpha}}{\sum_{f' \in \mathcal{F}} \sum_{j \in \mathcal{N}_{f'}^J} \exp(v_i - A_i) n_{f'}^{\alpha}} \tag{2.48}$$

On the advertisers' side, the inverse advertising demand is given by $r_i(A_i) = 1 + \frac{b_i}{A_i}$, which comes from a utility function $A_i + b_i \log A_i$ and generates the advertisers' surplus function as

$$S_i(A_i) = b_i + b_i \log A_i$$

Assuming that the marginal cost is zero, the profit of each firm is given by

$$\sum_{i \in \mathcal{N}_f} \frac{(H_f)^{\frac{1}{1-\alpha}}}{H} \frac{\exp(v_i - A_i)}{H_f} (A_i + b_i). \tag{2.49}$$

The first-order condition for A_i is given by

$$\begin{aligned} & D_i - D_i (A_i + b_i) \\ & - \exp(v_i - A_i) \sum_{j \in \mathcal{N}_f} \left\{ \frac{\alpha}{1-\alpha} \frac{D_j}{H_f} \left(A_j + b_j\right) - \frac{(H_f)^{\frac{1}{1-\alpha}}}{H} \frac{1}{H_f} \frac{1}{1-\alpha} D_j \left(A_j + b_j\right) \right\} = 0, \end{aligned} \tag{2.50}$$

which can be simplified to

$$A_i + b_i = 1 + \frac{1}{1-\alpha} \Pi_f - \frac{\alpha}{1-\alpha} \frac{\Pi_f}{n_f} =: \mu_f \tag{2.51}$$

[¶¶]This issue is highlighted in Chapter 7.8 of Anderson, De Palma and Thisse (1992).

As a result, we have $\Pi_f = n_f \mu_f$. Finally, the type-aggregation property is preserved on the consumers' side.

One crucial difference from the model of single-homing consumers is that, in the competitive bottleneck framework, advertisers' surplus cannot be expressed by aggregators, which makes the welfare analysis on advertisers' side complicated.

Using this framework, we can show that any CS-neutral merger improves advertisers' welfare, which is partly in line with Anderson and Peitz (2015)'s "media see-saw" argument that there are several situations where the welfare effects on consumers and advertisers conflict with each other.

Compatibility choices The main analysis has assumed that once firms merge, their products immediately become compatible. However, the choice of compatibility itself is firms' choice variables, as discussed in Katz and Shapiro (1985). In this regard, the products owned by different firms can be compatible. When the compatibility is a choice variable, some policy implication might change. For example, when the strong network effects are likely to harm the consumer surplus, it may be more appropriate to require the compatibility between a product of the merged entity and the products of other firms.

Dynamic competition The model of this study assumed that firms compete in prices in a static manner. However, the digital markets are typically characterized by rapid technological changes because of R&D, continuous entries of startups, and the dynamic evolution of consumer bases. These dynamic considerations may make the merger analysis based on a static market share less useful to evaluate the competitive effects. One potential effect of dynamic consideration is that a dominant firm is probably more dominant in the future time period, which requires a more stringent merger policy than that under a static framework.

Endogenous merger Merger is endogenous in various aspects (Nocke and Whinston, 2010, 2013; Mermelstein, Nocke, Satterthwaite and Whinston, 2014). Firms merge only when it is profitable, which may have some dynamic implication (Nocke and Whinston, 2010). Similarly, an acquirer can choose a target to acquire, which is biased toward an increase in profit rather than an improvement in consumer welfare (Nocke and Whinston, 2013). Further, firms can choose whether to invest in an asset by themselves or to acquire a firm (Mermelstein et al., 2014). All of these aspects may affect the conclusion of the main analysis. The results of this study can be interpreted as a benchmark to consider more complicated situations.

2.8 Conclusion

This study used a model of multiproduct-firm oligopoly to analyze the impact of network externalities on the consumer-surplus effects of mergers. The impact of direct network externalities on the welfare properties of mergers depends on the sizes of merging parties relative to the industry. When an incumbent tries to acquire an innovative entrant, the incumbent may have a greater incentive to innovate due to the demand-side scale economies. In two-sided markets, the pre-merger structure of market shares on two sides of markets as well as the sizes of merging parties can predict the post-merger consumer surplus. These implications give theoretical guidance to the competition authorities involved in the merger policy toward platforms.

This study abstracts several aspects of the merger policy, which leaves the avenue for future research. First, the analytical framework is static and does not consider dynamics such as R&D competition and sequential mergers. Incorporating such dynamics would enrich some policy prescriptions. Second, the framework of this study focuses on the case where consumers on each side single-home. However, various online services, consumer behavior is better characterized by multi-homing. The possibility of multi-homing can affect platform competition and the potential effects of mergers on competition in important ways, as discussed by Anderson et al. (2019). It would be constructive to analyze the competitive effects of mergers in general settings with consumer multi-homing, but the discrete-choice framework of this study does not fit well to tackle with this issue. Thus, I leave these issues for future research.

2.9 Section A: General Framework of Multiproduct-Firm Oligopoly with Network Externalities

In this section, I present a general framework which our specific analyses belong to. There is an industry with two sides of markets A, B with a set $\mathcal{N}^J$ of imperfectly substitutable products on side $J \in \{A, B\}$, produced by a set of firms $\mathcal{F}$, with its generic element f. Each firm f produces the set $\mathcal{N}_f^J$ of products on side J, where $\mathcal{N}_f^J \cap \mathcal{N}_{f'}^J = \emptyset$ for $f \neq f'$ and $\bigcup_{f \in \mathcal{F}} \mathcal{N}_f^J = \mathcal{N}^J$.

Consumer Demand

There is a mass of consumers on side $J \in \{A, B\}$ who derive firm-level network externalities from the purchase of each product $i \in \mathcal{N}_f^J$, which depends on the number of consumers who purchase the products of firm f. Specifically, each consumer $z \in [0, 1]$ on side $J \in \{A, B\}$ yields the indirect subutility from the purchase of product $i \in \mathcal{N}_f^J$ by

$$\log h_i^J(p_i) + \alpha_J \log n_f^J + \beta_J \log n_f^I + \varepsilon_{iz}^J, \tag{2.52}$$

where $\log h_i^J(p_i)$ is the stand-alone indirect subutility from product i at price p_i, and n_f^J and n_f^I are the numbers of consumers on side J and $I \neq J$ who purchase products provided by firm f. $\alpha_J \in [0, 1)$ represents the magnitude of direct network externalities, $\beta_J \in [0, 1)$ represents the magnitude of indirect network externalities, and ε_{iz}^J is an idiosyncratic taste shock that follows i.i.d. type-I extreme value distributions. The direct and indirect network externalities are based on the firm-level number of participants of each network. I assume that network effects are not too strong so that $1 - \alpha_J - \beta_I > 0$ holds for each $J \in \{A, B\}$ and $I \neq J$, which guarantees that firms do not set infinite negative prices. An intuitive interpretation of this assumption is that a marginal increase in the stand-alone indirect utility of consumers on one side, 1, is greater than the accompanying increase in the indirect utility of consumers on two sides of markets from the resulting network expansions, $\alpha_J + \beta_I$. In the analysis, I adopt two specific forms of functions h_i^J. One is *MNL-class* demand specification where

$$h_i^J(p_i) = \exp\left(\frac{a_i - p_i}{\lambda^J}\right),$$

and another is *CES-class* demand specification where

$$h_i^J(p_i) = \begin{cases} a_i p_i^{1-\sigma^J} & \text{if } p_i > 0 \\ +\infty & \text{if } p_i \leq 0 \end{cases}$$

with $\sigma^J > 1$. In both specifications, a_i represents the quality of each product. I mean by the words "MNL-class" and "CES-class" that, if $\alpha_J = \beta_J = 0$, $J \in \{A, B\}$, the demand system obtained from indirect subutilities $h_i^J(p_i) = \exp\left(\frac{a_i - p_i}{\lambda^J}\right)$ and $h_i^J(p_i) = a_i p_i^{1-\sigma^J}$ corresponds with that of multinomial-logit and CES demand functions, respectively.

Given the network sizes $(n_f^A, n_f^B)_{f\in\mathcal{F}}$ and prices $p := (p_i)_{i\in\mathcal{N}^A\cup\mathcal{N}^B}$, consumers choose one product to purchase and the amount of the purchase. I assume that there is no outside option so that all consumers on side J purchase some product in the set $\mathcal{N}^J$. I further assume that consumers on each side single-home, that is, each consumer chooses only one product to purchase. From the above utility specification, the corresponding demand system is derived as a rational-expectation equilibrium among consumers. That is, based on the expectation over the network sizes, consumers choose their own decision to maximize the utilities, and the realized network sizes are consistent with the original expectation.

First, define the firm-level and industry-level aggregators as

$$H_f^J(p_f^J) = \sum_{i\in\mathcal{N}_f^J} h_i^J(p_i), \quad \text{where } p_f^J := (p_i)_{i\in\mathcal{N}_f^J}.$$

$$H^J(p) = \sum_{f\in\mathcal{F}} \left(H_f^J(p_f^J)\right)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I(p_f^I)\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}.$$

Next, I derive demand for each product conditional on the purchase. Applying Roy's identity, the conditional demand function for product i conditional on the purchase is given by $-(h_i^J)'(p_i)/h_i^J(p_i)$. I assume that consumers form the correct expectation that all firm have positive network shares. I call the network choice of consumers based on such expectation as an *interior consumption equilibrium*. Applying Holman and Marley's Theorem, the consumer choice probability s_i^J of product $i \in \mathcal{N}_f^J$ given the expectation over network shares $(n_{f'}^J)_{f'\in\mathcal{F}}$, $J = A, B$ is given by

$$s_i^J = \frac{h_i^J(p_i)\left(n_f^J\right)^{\alpha_J}\left(n_f^I\right)^{\beta_J}}{\sum_{f'\in\mathcal{F}}\sum_{j\in\mathcal{N}_{f'}^J} h_j^J(p_j)\left(n_{f'}^J\right)^{\alpha_J}\left(n_{f'}^I\right)^{\beta_J}}. \tag{2.53}$$

I require the network share is consistent with the consumers' behaviors, that is, the network share n_f^J of firm f on side J is given by the sum of the choice probability of products produced by firm f on side J:

$$n_f^J = \sum_{i\in\mathcal{N}_f^J} s_i^J. \tag{2.54}$$

From equations (2.53) and (2.54), the share of product $i \in \mathcal{N}_f^J$ in the set of products sold by firm f is given by

$$\frac{s_i^J}{n_f^J} = \frac{h_i^J(p_i)}{H_f^J(p_f^J)}. \tag{2.55}$$

As derived in the Section 2.9, the network share n_f^J of firm f on side J in the interior consumption equilibrium is given by

$$n_f^J(p) = \frac{1}{H^J(p)} \left(\left((H_f^J(p_f^J) \right)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I(p_f^I) \right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}} \right). \tag{2.56}$$

Combining equations (2.55) and (2.56) the probability that product $i \in \mathcal{N}_f^J$ is purchased by a consumer is given by the equation

$$s_i^J(p) = n_f^J(p) \frac{h_i^J(p_i)}{H_f^J(p_f^J)}. \tag{2.57}$$

Finally, the demand for the product $i \in \mathcal{N}_f^J$ given the profile of prices p has the following form.

$$\begin{aligned} D_i^J(p) &= \hat{D}_i^J \left(p_i, H_f^J(p_f^J), H_f^I(p_f^I), H^J(p) \right) \\ &= s_i^J(p) \times \frac{-(h_i^J)'(p_i)}{h_i^J(p_i)} \\ &= -\left(H_f^J(p_f^J) \right)^{\frac{(1-\alpha_I)\alpha_J+\beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I(p_f^I) \right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}} \frac{(h_i^J)'(p_i)}{H^J(p)} \end{aligned} \tag{2.58}$$

With CES-class demand and negative price, we cannot use Roy's identity to derive demand. To allow for the demand for negative prices, we assume that $D_i^J(p) = \lim_{p_i \to 0} D_i^J(p) = +\infty$ for $p_i < 0$.

Finally, the consumer surplus CS^J on side J is given by the expected indirect utility of consumers, and the aggregate consumer surplus CS is given by the some of consumer surplus on both sides :

$$\begin{aligned} CS^J &= \log \left(\sum_{f \in \mathcal{F}} \left(H_f^J \right)^{\frac{1-\alpha_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I \right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)\beta_J\beta_I}} \frac{1}{(H^J)^{\alpha_J}(H^I)^{\beta_J}} \right) \\ &= (1-\alpha_J) \log H^J - \beta_J \log H^I, \end{aligned} \tag{2.59}$$

and

$$\begin{aligned} CS &= CS^A + CS^B \\ &= (1-\alpha_A-\beta_B) \log H^A + (1-\alpha_B-\beta_A) \log H^B. \end{aligned} \tag{2.60}$$

Note that in the presence of indirect network externalities $\beta_J > 0$, the consumer surplus on side J is decreasing in the value of aggregator H^I on side $I \neq J$ while it increases with the value of the aggregator on the same side H^J. This is because the large value of aggregator on the other side shrinks the market share and thereby resulting in the fragmentation of networks.

Firm pricing

Each product $i \in \mathcal{N}^J$ has a constant marginal cost $c_i > 0$ of production. Given the demand system, the profit function of each firm $f \in \mathcal{F}$ is written as a function of the profile of firm's own prices $p_f = (p_i)_{\mathcal{N}_f^A \cup \mathcal{N}_f^B}$ and aggregators H^A and H^B:

$$\Pi_f\left(p_f, H^A(p), H^B(p)\right) = \Pi_f^A + \Pi_f^B, \tag{2.61}$$

where

$$\Pi_f^J = \sum_{i \in \mathcal{N}_f^J} \hat{D}_i^J\left(p_i, H_f^J(p_f^J), H_f^I(p_f^I), H^J(p)\right)(p_i - c_i). \tag{2.62}$$

The pricing game consists of a demand system $\{(D_i^J)_{i \in \mathcal{N}^J}\}_{J \in \{A,B\}}$, the set of firms $\mathcal{F}$, and a profile of marginal costs $(c_i)_{i \in \mathcal{N}^J}$, $J \in \{A, B\}$. In a pricing game, firms simultaneously set the prices $p_f := (p_i)_{i \in \mathcal{N}_f^A \cup \mathcal{N}_f^B}$ of their products, with the payoff function Π_f defined by equation (2.61). I call a Nash equilibrium of this pricing game as a *pricing equilibrium*. In the following analysis, I often suppress the arguments of functions for the sake of readability.

As formally derived in Section 2.9, the first-order condition for the profit-maximization of each firm f is given by

$$\begin{aligned}
&-\frac{(h_i^J)''}{(h_i^J)'}(p_i - c_i) \\
&= 1 - \underbrace{\frac{1}{n_f^J}\frac{\{(1-\alpha_I)\alpha_J + \beta_J\beta_I\}\Pi_f^J + \beta_I\Pi_f^I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}}_{\text{network-externality terms}} + \underbrace{\frac{(1-\alpha_I)}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\left(\Pi_f^J + \frac{\beta_I}{1-\alpha_I}\frac{n_f^I}{n_f^J}\Pi_f^I\right)}_{\text{cannibalization terms}} \\
&=: \mu_f^J
\end{aligned} \tag{2.63}$$

As Nocke and Schutz (2018b) do, I call μ_f^J as the ι-markup of firm f on side J. This ι-markup summarizes the pricing incentive of each firm.

Let me explain how the ι-markup of each firm is determined in the equation (2.63). the first term, 1, in the second line of the equation (2.63) is the baseline ι-markup, which would be set under the monopolistic competition. The second term is the downward-pricing pressure due to the direct and indirect network externalities. The third term is the upward-pricing pressure due to the cannibalization effects under oligopoly. The relative magnitudes of the second and the last terms on each side determine the price level and the price structure of each firm.

We have $-(h_i^J)''/(h_i^J)' = 1/\lambda^J$ and thus $p_i = c_i + \lambda^J\mu_f^J$ in the case of multinomial logit demand, and $-(h_i^J)''/(h_i^J)' = \sigma^J/p_i$ and thus $p_i = c_i/(1 - \mu_f^J/\sigma^J)$ in the case of CES-class demand. Using these functional forms, the formula for the subaggregators and profit functions are given by

$$H_f^J = \begin{cases} T_f^J \exp(-\mu_f^J) & \text{in the case of MNL-class demand,} \\ T_f^J\left(1 - \frac{\mu_f^J}{\sigma^J}\right)^{\sigma^J - 1} & \text{in the case of CES-class demand,} \end{cases} \tag{2.64}$$

and

$$\Pi_f^J = \begin{cases} n_f^J \mu_f^J & \text{in the case of MNL-class demand,} \\ \frac{\sigma^J - 1}{\sigma^J} n_f^J \mu_f^J & \text{in the case of CES-class demand,} \end{cases} \tag{2.65}$$

where $T_f^J := \sum_{i \in \mathcal{N}_f^J} \exp\left(\frac{a_i - c_i}{\lambda^J}\right)$ for MNL-class and $T_f^J := \sum_{i \in \mathcal{N}_f^J} a_i c_i^{1-\sigma^J}$ for CES-class demand. T_f^J is the "type" of firm f that corresponds with the value of the subaggregator of firm f when it engages in the marginal cost pricing. The property that all the pricing information is summarized by unidimensional type T_f^J is called as the "type-aggregation property" (Nocke and Schutz, 2018b). This property greatly simplifies the analysis of merger policy. As a result, the ι-markup μ_f^J and the network share n_f^J depends only on T_f^A, T_f^B, H^A, and H^B. Whenever they are unique, I write the ι-markups μ_f^J and market shares n_f^J under each firm's optimal pricing as functions

$$\mu_f^J = m^J\left(T_f^J, T_f^I, H^J, H^I\right), \tag{2.66}$$

and

$$n_f^J = N^J\left(T_f^J, T_f^I, H^J, H^I\right) \tag{2.67}$$

for $J \in \{A, B\}$.

Pricing Equilibrium

Given the above market share functions, the equilibrium condition for the aggregators (H^A, H^B) is

$$\sum_{f \in \mathcal{N}_f^J} N^J\left(T_f^J, T_f^I, H^J, H^I\right) = 1, \quad J \in \{A, B\}, \quad I \neq J. \tag{2.68}$$

The general characterization of the equilibrium and the analysis of merger policy are not easy to conduct. Thus, to obtain the clear-cut insights, I divide the analysis of merger policy into two special cases where there is only direct network externalities and where there is only indirect network externalities. For each special case, I separately prove the existence and the uniqueness of the equilibrium.

Proposition 8. *(Uniqueness of equilibrium) The following statements hold:*

1. *For any pair of aggregators (H^A, H^B), each firm's optimal pricing is characterized by the first-order condition (2.63).*

2. *If $\beta_A = \beta_B = 0$, then there is unique pricing equilibrium.*

3. *If the demand system is given by MNL-class demand, $\alpha_A = \alpha_B = 0$, and either β_A or β_B is close to 0, then there is unique pricing equilibrium.*

I provide the proof of this proposition in Section 2.9.

The first part of Proposition 8 shows that in any equilibrium, each firm's pricing is characterized by the first-order condition (2.63). The second and the third parts of Proposition 8 guarantee the existence and the uniqueness of equilibrium in special cases analyzed in Section 2.4. and Section 2.6.

Proof of Proposition 8

Proof of Proposition 8.1

I prove Proposition 1-1 for CES-class demand and MNL-class demand separately.

Before proceeding to individual proofs, I introduce several notations that are used in both proofs. First, let

$$\Omega_f^{JI}(p_f) := \left(H_f^J(p_f^J)\right)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(H_f^I(p_f^I)\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}}$$

and

$$H_{-f}^J = \sum_{f'\in\mathcal{F}\setminus\{f\}} \Omega_{f'}^{JI}(p_{f'}).$$

Then, the profit-maximization problem of firm f the problem can be rewritten as

$$\max_{p_f\in\mathbb{R}^{\mathcal{N}_f^A\cup\mathcal{N}_f^B}} G_f(p_f) := \Pi_f\left(p_f, \Omega_f^{AB}(p_f) + H_{-f}^A, \Omega_f^{BA}(p_f) + H_{-f}^B\right) \tag{2.69}$$

I show that the solution to (2.69) takes unique finite value and given by the first-order condition (2.63). Here, I list the steps of the proof. For CES-class demand,

1. To whatever extent a subaggregator H_f^J of firm f on one side grows, the profit on the other side Π_f^I is bounded above.
2. Setting zero price for some good is never optimal.
3. Setting infinite price for some good is never optimal.
4. The optimal prices should satisfy the first-order condition (2.63).

For MNL-class demand,

1. Fixing p_f^A, p_f^B that maximizes $G_f(p_f^A, p_f^B)$ is finite and unique. Let $\tilde{p}_f^B(p_f^A)$ denote such p_f^B.
2. p_f^A that maximizes $G_f(p_f^A, \tilde{p}_f^B(p_f^A))$ is finite and unique.
3. Setting infinite price for some good is never optimal.
4. The optimal prices should satisfy the first-order condition (2.63).

CES-class demand I first show that whatever the value of subaggregator H_f^J is, the value Π_f^I is bounded. To see this, note that

$$\Pi_f^I = \frac{\Omega^{IJ}}{H^I}\sum_{j\in\mathcal{N}_f^I}\frac{-(h_j^I)'(p_j)}{H_f^I(p_f^I)}(p_j - c_j) \le \sum_{j\in\mathcal{N}_f^I}\frac{-(h_j^I)'(p_j)}{H_f^I(p_f^I)}(p_j - c_j) = \frac{\sum_{j\in\mathcal{N}_f^I}(\sigma_J-1)(a_j p_j^{-\sigma_I})(p_j-c_j)}{\sum_{k\in\mathcal{N}_f^I} a_k p_k^{\sigma_I-1}}$$
$$\le \sigma_J - 1, \tag{2.70}$$

where the last inequality follows from computing $\max_{p_f^I} \left\{ \sum_{j \in \mathcal{N}_f^I (\sigma_J - 1)(a_j p_j^{-\sigma_I})(p_j - c_j)} \right\} / \left\{ \sum_{k \in \mathcal{N}_f^I a_k p_k^{\sigma_I - 1}} \right\}$

Next, I show that the price of each product should be in the set $(0, \infty)$, and thus the optimal pricing should be characterized by the equation (2.63). To see this, suppose that $p_i \leq 0$ for some $i \in \mathcal{N}^J$. Then, $h_i^J(p_i) = \infty$ and thus the $D_j^J(p) = 0$ for all $j \in \mathcal{N}_f^J \setminus \{i\}$. Therefore the profit of firm f is given by

$$D_i^J(p)(p_i - c_i) + \Pi_f^I. \tag{2.71}$$

Since Π_f^I is finite as shown above, $D_i^J(p) = \infty$, and $p_i - c_i < 0$, the profit is negative. Thus, setting non-positive for some product is never optimal for firm f.

The fact that firm never sets infinite price is shown in the same manner as Nocke and Schutz (2018a).

Thus, the optimal profile of prices is interior, which implies that the optimal prices should satisfy the first-order condition (2.63).

MNL-class demand I first show that all firms' prices are bounded below. Next, I show that all firm' prices are bounded above.

Fix $p_f^I := (p_i^I)_{i \in \mathcal{N}_f^I}$. Then, I show that the value of $(p_f^J) := (p_i^J)_{i \in \mathcal{N}_f^J}$ that maximize the profit of firm f has finite absolute values. To see this, note that

$$\begin{aligned} &\operatorname{sign}\left(\frac{\partial G(p_f)}{\partial p_i}\right) \\ =&\operatorname{sign}\left(1 - \frac{p_i - c_i}{\lambda_J} + \frac{(1-\alpha_I) - \{(1-\alpha_I)\alpha_J + \beta_I\beta_J\}\frac{1}{n_f^J}}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\Pi_f^J - \frac{\beta_I}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\frac{(1-n_f^I)}{n_f^J}\Pi_f^I\right). \end{aligned} \tag{2.72}$$

the last term converges to 0 as $p_i \to -\infty$ because $n_f^I \to 1$ as $p_i \to -\infty$. The sum of the second and the third terms is nonnegative as $p_i \to -\infty$ because

$$\begin{aligned} &-\frac{p_i - c_i}{\lambda_J} + \frac{(1-\alpha_I) - \{(1-\alpha_I)\alpha_J + \beta_I\beta_J\}\frac{1}{n_f^J}}{(1-\alpha_I)(1-\alpha_J) - \beta_I\beta_J}\Pi_f^J \\ &\geq -\frac{p_i - c_i}{\lambda_J} + \Pi_f^J \\ &\geq -\frac{p_i - c_i}{\lambda_J} + n_f^J \frac{p_i - c_i}{\lambda_J} \end{aligned} \tag{2.73}$$

as $p_i \to -\infty$. Thus we have $\partial G / \partial p_i > 0$ for sufficiently small p_i.

The fact that firm never sets infinite price is shown in the same manner as Nocke and Schutz (2018a).

As a result, fixed the values of p_f^I, the optimal pricing for p_f^J is given by the common ι-markup pricing, which is given by (2.63). Let $\mu_f^J(p_f^I)$ be the optimal ι-markup on side J given the profile of prices on the other side p_f^I. Then, under MNL-class demand, we have $p_i = c_i + \lambda_J \mu_f^J(p_f^I)$ for $i \in \mathcal{N}_f^J$.

Next, I show that the optimal value of p_f^I that maximizes $G(p)$ where $p_f^J = (c_i + \lambda_J \mu_f^J(p_f^I))_{i\in\mathcal{N}_f^J}$ is finite. To see this, it is sufficient to show that

$$\begin{aligned}
&\text{sign}\left(\frac{\partial G((p_f^I,(c_i+\lambda_J\mu_f^J(p_f^I))_{i\in\mathcal{N}_f^J})}{\partial p_j}\right)\\
=&\text{sign}\left(1-\frac{p_j-c_j}{\lambda_I}+\frac{(1-\alpha_J)-\{(1-\alpha_I)\alpha_J+\beta_I\beta_J\}\frac{1}{n_f^I}}{(1-\alpha_I)(1-\alpha_J)-\beta_I\beta_J}\Pi_f^I-\frac{\beta_J}{(1-\alpha_I)(1-\alpha_J)-\beta_I\beta_J}\frac{(1-n_f^J)}{n_f^I}\Pi_f^J\right).
\end{aligned} \tag{2.74}$$

Using the first-order condition for μ_f^J, 2.63, we have

$$\begin{aligned}
&\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}(1-n_f^J)\mu_f^J\\
=&1-\frac{\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}\frac{n_f^I}{n_f^J}(1-n_f^I)\sum_{j'\in\mathcal{N}_f^I}\frac{\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)}{H_f^I}\frac{p_{j'}-c_{j'}}{\lambda^I}
\end{aligned} \tag{2.75}$$

By l'Hopital's rule, we have

$$\begin{aligned}
&\lim_{p_i\to-\infty}\frac{\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}\frac{n_f^I}{n_f^J}(1-n_f^I)\sum_{j'\in\mathcal{N}_f^I}\frac{\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)}{H_f^I}\frac{p_{j'}-c_{j'}}{\lambda^I}\\
=&\lim_{p_j\to-\infty}\frac{\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}\frac{\sum_{j'\in\mathcal{N}_f^I\setminus\{i\}}\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)}{\sum_{j'\in\mathcal{N}_f^I\setminus\{i\}}\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)+\exp\left(\frac{a_i-p_i}{\lambda^I}\right)}\frac{p_i-c_i}{\lambda^I}\\
=&\lim_{p_j\to-\infty}\frac{\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}\frac{\sum_{j'\in\mathcal{N}_f^I\setminus\{i\}}\exp\left(\frac{a_{j'}-p_{j'}}{\lambda^I}\right)}{-\lim_{p_i\to\infty}\lambda^I\exp\left(\frac{a_i-p_i}{\lambda^I}\right)}\\
=&0.
\end{aligned}$$

Thus, we have

$$\lim_{p_j\to-\infty}\frac{(1-n_f^J)}{n_f^I}\Pi_f^J=\frac{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}{1-\alpha_I},$$

and

$$\begin{aligned}
&\text{sign}\lim_{p_j\to-\infty}\left(\frac{\partial G((p_f^I,(c_i+\lambda_J\mu_f^J(p_f^I))_{i\in\mathcal{N}_f^J})}{\partial p_j}\right)\\
=&\text{sign}\lim_{p_j\to-\infty}\left(1-\frac{\beta_J}{1-\alpha_I}-\frac{p_j-c_j}{\lambda_I}+\frac{(1-\alpha_J)-\{(1-\alpha_I)\alpha_J+\beta_I\beta_J\}\frac{1}{n_f^I}}{(1-\alpha_I)(1-\alpha_J)-\beta_I\beta_J}\Pi_f^I\right)>0.
\end{aligned} \tag{2.76}$$

as $p_j \to -\infty$ for some $\mathcal{N}_f^I$.

Again, the fact that firm never sets infinite price is shown in the same manner as Nocke and Schutz (2018a).

Thus, the optimal profile of prices is interior, which implies that the optimal prices should satisfy the first-order condition (2.63).

Note that in when $\beta_A = \beta_B = 0$ or $\alpha_A = \alpha_B = 0$ and either $\beta_A \simeq 0$ or $\beta_B \simeq 0$, equation (2.63) is also sufficient for the profit maximization. In the subsequence sections (Section 2.9- 2.9), I prove that the solution to (2.63) is unique when $\beta_A = \beta_B = 0$ or $\alpha_A = \alpha_B = 0$ and either $\beta_A \simeq 0$ or $\beta_B \simeq 0$. Then, by the facts that (i) there is finite price profile that maximizes the profit, (ii) any optimal price profile should satisfy equation (2.63), and (iii) when $\beta_A = \beta_B = 0$ or $\alpha_A = \alpha_B = 0$ and either $\beta_A \simeq 0$ or $\beta_B \simeq 0$, the pair of ι-markups (μ_f^A, μ_f^B) that satisfies equation (2.63) is unique jointly show that there is a unique solution to (2.63) that maximizes the firm's profit. If the solution to (2.63) does not maximize the firm's profit, then there must be some finite price profile that maximizes the firm's profit but does not satisfy equation (2.63), which contradicts the necessity of (2.63).

Proof of Proposition 8.2

Suppose that $\beta_A = \beta_B = 0$. Then, two sides of markets are independent, and thus it suffice to focus on one side of the market. The ι-markup is uniquely given by the equation (FOC-MNL) for MNL-class demand and (FOC-CES) for CES-class demand. Finally, since the market share equation $N_0(\gamma(T)/H)$ defined by is monotonically decreasing in H, $\lim_{x\to 0} N_0(x) = 0$, and $\lim_{x\to\infty} N_0(x) = 1$, the intermediate value theorem implies that the value of aggregator H that satisfies $\sum_f N_0(\gamma(T_f)/H) = 1$ is unique.

Proof of Proposition 8.3

I show that there is the unique pair of ι-markup that satisfies the system of equations (2.63), and the unique pair of aggregators that satisfies the system of equation (2.63) when the demand system is given by MNL-class demand, $\alpha_A = \alpha_B = 0$, and at at least one of β_A or β_B is sufficiently close to 0.

Because the solution to equation (2.63) is continuous in β_J at $\beta_J = 0$, without loss of generality, suppose that $\beta_B = 0$. After several manipulations, the system of first-order conditions (2.63) can be rewritten as

$$g_A(\mu_f^A, \mu_f^B) = (1 - n_f^A)\mu_f^A - 1 = 0, \tag{2.77}$$

$$g_B(\mu_f^A, \mu_f^B) = (1 - n_f^B)\mu_f^B - 1 + \beta_A \frac{n_f^A}{n_f^B} = 0. \tag{2.78}$$

Let $g(\mu_f^A, \mu_f^B) := \{g_A(\mu_f^A, \mu_f^B, g_B(\mu_f^A, \mu_f^B)\}$. To show that this system of equations has unique solution, I show that the determinant of $D(g(\mu_f^A, \mu_f^B)$ is positive.*** A calculation leads to

$$\begin{aligned}\det G_f =& \left(\frac{n_f^A}{1-n_f^A} + 1 - n_f^A\right)\left(\frac{n_f^B}{1-n_f^B} + 1 - n_f^B + \beta_A\left(\frac{1}{n_f^B} - \frac{1}{1-n_f^B}\right)\right) \\ &+ \beta_A^3 \frac{(n_f^A)^2}{n_f^B(1-n_f^A)} > 0,\end{aligned} \tag{2.79}$$

where

$$G_f := \begin{pmatrix} \frac{\partial g_A}{\partial \mu_f^A} & \frac{\partial g_A}{\partial \mu_f^B} \\ \frac{\partial g_B}{\partial \mu_f^A} & \frac{\partial g_B}{\partial \mu_f^B} \end{pmatrix}, \tag{2.80}$$

and the last inequality follows from the fact that

$$\begin{aligned}&\frac{n_f^B}{1-n_f^B} + 1 - n_f^B + \beta_A\left(\frac{1}{n_f^B} - \frac{1}{1-n_f^B}\right) \\ =&\frac{1}{n_f^B(1-n_f^B)}\left((n_f^B)^3 - (n_f^B)^2 + n_f^B + \beta_A n_f^A(1-2n_f^B)\right)\end{aligned} \tag{2.81}$$

is positive. To see this, all the terms in expression (2.81) is positive when $n_f^B < 1/2$. When $n_f^B \geq 1/2$, the last term in expression (2.81) is nonpositive. In that case, by the fact that $\beta_A n_f^A \in (0,1)$ the following inequality holds:

$$\nu(n_f^B) := (n_f^B)^3 - (n_f^B)^2 + n_f^B + \beta_A n_f^A(1-2n_f^B) \geq (n_f^B)^3 - (n_f^B)^2 - n_f^B + 1, \tag{2.82}$$

which is positive for all $n_f^B \in (0,1)$. This is because $\nu(1) = 0$, and $\nu'(n_f^B) < 0$ for all $n_f^B \in (0,1)$.

Thus, the pair of ι-markups that satisfies the first-order condition (2.63) is unique.

Finally, I show that there is unique equilibrium. To do this, I present several comparative statics of m^J, $J \in \{A, B\}$, with respect to several parameters x. This is given by the Implicit Function Theorem

$$G_f \begin{pmatrix} \frac{\partial m^A}{\partial x} \\ \frac{\partial m^B}{\partial x} \end{pmatrix} = -\begin{pmatrix} \frac{\partial g_A}{\partial x} \\ \frac{\partial g_B}{\partial x} \end{pmatrix}. \tag{2.83}$$

by Cramer's Rule, I obtain

$$\frac{\partial \mu^A}{\partial x} = \frac{\det\begin{pmatrix} -\frac{\partial g_A}{\partial x} & \frac{\partial g_A}{\partial \mu_B} \\ -\frac{\partial g_B}{\partial x} & \frac{\partial g_B}{\partial \mu_B} \end{pmatrix}}{\det G_f}, \quad \frac{\partial \mu^B}{\partial x} = \frac{\det\begin{pmatrix} \frac{\partial g_A}{\partial \mu_A} & -\frac{\partial g_A}{\partial x} \\ \frac{\partial g_B}{\partial \mu_A} & -\frac{\partial g_B}{\partial x} \end{pmatrix}}{\det G_f}. \tag{2.84}$$

Using this comparative statics in ι-markups, I conduct a comparative statics in market shares N^A and N^B:

$$\frac{\partial N^A}{\partial x} = \frac{\partial n^A}{\partial x} - \frac{\partial m^A}{\partial x} n^A - \beta_A \frac{\partial m^B}{\partial x} n^A \tag{2.85}$$

***See chapter 2 of Vives (2001).

$$\frac{\partial N^B}{\partial x} = \frac{\partial n^B}{\partial x} - \frac{\partial m^B}{\partial x} n^B \tag{2.86}$$

Based on this observation, I derive the effects of H^A and H^B on N^A and N^B. Fist, for H^A, we have

$$\begin{aligned}
\frac{\partial m^A}{\partial H^A} &= -\frac{n_f^A}{H^A}\frac{1}{\det(G_f)}\left[\mu_f^A\left(n_f^B\mu_f^B + 1 - n_f^B + \beta_A\frac{n_f^A}{n_f^B}\right) + \mu_f^A\beta_A^2\frac{n_f^A}{n_f^B}\right] < 0 \\
\frac{\partial m^B}{\partial H^A} &= \frac{n_f^A}{H^A}\frac{1}{\det(G_f)}\left(\beta_A\frac{n_f^A}{n_f^B}\mu_f^A(1-\beta_A) + \beta_A\frac{1-n_f^A}{n_f^B}\right) > 0,
\end{aligned}$$

and thus

$$\frac{\partial N^A}{\partial H^A} = -\frac{n_f^A}{H^A}\frac{1}{\det(G_f)}(1-n_f^A)\left(n_f^B\mu_f^B + 1 - n_f^B + \beta_A(1+\beta_A)\frac{n_f^A}{n_f^B}\right) < 0 \tag{2.87}$$

$$\frac{\partial N^B}{\partial H^A} = -\frac{n_f^A}{H^A}\frac{1}{\det(G_f)}\left(\beta_A\frac{n_f^A}{n_f^B}\mu_f^A(1-\beta_A) + \beta_A\frac{1-n_f^A}{n_f^B}\right)n_f^B < 0. \tag{2.88}$$

For H^B, we have

$$\begin{aligned}
\frac{\partial m^A}{\partial H^B} &= \frac{n_f^B}{H^B}\frac{1}{\det(G_f)}\left(\mu_f^B + \beta_A\frac{n_f^A}{(n_f^B)^2}\right)\beta_A n_f^A\mu_f^A > 0 \\
\frac{\partial m^B}{\partial H^B} &= -\frac{n_f^B}{H^B}\frac{1}{\det(G_f)}\left(\mu_f^B + \beta_A\frac{n_f^A}{(n_f^B)^2}\right)(n_f^A\mu_f^A + 1 - n_f^A) < 0,
\end{aligned}$$

and thus

$$\frac{\partial N^A}{\partial H^B} = \beta_A\frac{n_f^A}{n_f^B}\frac{n_f^B}{H^B}\frac{1}{\det(G_f)}\left(n_f^B\mu_f^B + \beta_A\frac{n_f^A}{n_f^B}\right)\left(1-n_f^A\right) > 0, \tag{2.89}$$

$$\frac{\partial N^B}{\partial H^B} = -\frac{n_f^B}{H_f^B}\frac{1}{\det(G_f)}\left((n_f^A\mu_f^A + 1 - n_f^A)(1-n_f^B) + \beta_A^3\frac{n_f^A}{n_f^B}n_f^A\mu_f^A\right) < 0. \tag{2.90}$$

As a result, we have

$$\det\begin{pmatrix} \sum\frac{\partial N^A}{\partial H^A} & \sum\frac{\partial N^A}{\partial H^B} \\ \sum\frac{\partial N^B}{\partial H^A} & \sum\frac{\partial N^B}{\partial H^B}\end{pmatrix} > 0, \tag{2.91}$$

which implies that the pair (H^A, H^B) that satisfies the condition (2.68) is unique.

Omitted derivation of demand system and firm pricing in Section 2.9 and 2.9

Consumer demand I first present a general framework to which our specific analyses belong to.

$$\log h_i^J(p_i) + \alpha_J \log n_f^J + \beta_J \log n_f^I + \varepsilon_i, \tag{2.92}$$

where $J, I \in \{A, B\}$, $J \neq I$, $\alpha^J \in [0, 1)$, and ε follows the Type-I extreme-value distribution. As a result, the consumer choice probability of product $i \in \mathcal{N}_f^J$ is given by

$$s_i^J = \frac{h_i^J(p_i)(n_f^J)^{\alpha_J}(n_f^I)^{\beta_J}}{\sum_{f' \in \mathcal{F}} \sum_{j \in \mathcal{N}_f^J} h_j^J(p_j)(n_{f'}^J)^{\alpha_J}(n_{f'}^I)^{\beta_J}} \tag{2.93}$$

network share n_f^J is given by

$$n_f^J = \frac{H_f^J(n_f^J)^{\alpha_J}(n_f^I)^{\beta_J}}{\sum_{f' \in \mathcal{F}} H_{f'}^J(n_{f'}^J)^{\alpha_J}(n_{f'}^I)^{\beta_J}}. \tag{2.94}$$

Then, we have

$$\frac{n_f^J}{n_{f'}^J} = \frac{H_f^J}{H_{f'}^J} \frac{(n_f^J)^{\alpha_J}}{(n_{f'}^J)^{\alpha_J}} \frac{(n_f^I)^{\beta_J}}{(n_{f'}^I)^{\beta_J}} \tag{2.95}$$

which can be rewritten as

$$\frac{n_f^J}{n_{f'}^J} = \left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{1}{1-\alpha_J}} \left(\frac{n_f^I}{n_{f'}^I}\right)^{\frac{\beta_J}{1-\alpha_J}} = \left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{1}{1-\alpha_J}} \left(\left(\frac{H_f^I}{H_{f'}^I}\right)^{\frac{1}{1-\alpha_I}} \left(\frac{n_f^J}{n_{f'}^J}\right)^{\frac{\beta_I}{1-\alpha_I}}\right)^{\frac{\beta_J}{1-\alpha_J}}. \tag{2.96}$$

Finally, we obtain

$$\frac{n_f^J}{n_{f'}^J} = \left(\left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{1}{1-\alpha_J}} \left(\frac{H_f^I}{H_{f'}^I}\right)^{\frac{\beta_J}{(1-\alpha_I)(1-\alpha_J)}}\right)^{\frac{1}{1-\frac{\beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I)}}} = \left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(\frac{H_f^I}{H_{f'}^I}\right)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \tag{2.97}$$

Plugging this equation into the definition of market share function, I obtain the closed-form expression for the network share

$$\begin{aligned} n_f^J &= \frac{H_f^J}{\sum_{f' \in \mathcal{F}} H_{f'}^J \left(\frac{H_f^J}{H_{f'}^J}\right)^{\frac{(1-\alpha_I)\alpha_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(\frac{H_f^I}{H_{f'}^I}\right)^{\frac{\alpha_J\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(\frac{H_{f'}^I}{H_f^I}\right)^{\frac{(1-\alpha_J)\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} \left(\frac{H_{f'}^J}{H_f^J}\right)^{\frac{\beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}}} \\ &= \frac{(H_f^J)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} (H_f^I)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}}{\sum_{f' \in \mathcal{F}} (H_{f'}^J)^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} (H_{f'}^I)^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}} \end{aligned} \tag{2.98}$$

and the demand for product $i \in \mathcal{N}_f^J$

$$
\begin{aligned}
D_i^J(p) &= \hat{D}_i^J(p_i, H_f^J(p_f^J), H_f^I(p_f^I), H^J(p)) \\
&= n_f^J(p) \times \frac{s_i^J(p)}{n_f^J(p)} \times \left(-\frac{(h_i^J)'(p_i)}{h_i^J(p_i)}\right) \\
&= -\frac{(H_f^J(p_f^J))^{\frac{1-\alpha_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} (H_f^I(p_f^I))^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}}{H^J(p)} \frac{(h_i^J)'(p_i)}{H_f^J(p_f^J)} \\
&= -\frac{(H_f^J(p_f^J))^{\frac{(1-\alpha_I)\alpha_J+\beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I)-\beta_J\beta_I}} (H_f^I(p_f^I))^{\frac{\beta_J}{(1-\alpha_J)(1-\alpha_I)-\beta_I\beta_J}}}{H^J(p)} (h_i^J)'(p_i)
\end{aligned}
\tag{2.99}
$$

Firm pricing Given this demand system derived above, the profit function of each firm f is given by

$$
\begin{aligned}
\Pi_f(p_f, H^A(p), H^B(p)) =&\Pi_f^A(p_f^A, H_f^A(p_f^A), H_f^B(p_f^B), H^A(p), H^B(p)) \\
&+ \Pi_f^B(p_f^B, H_f^B(p_f^B), H_f^A(p_f^A), H^B(p), H^A(p))
\end{aligned}
\tag{2.100}
$$

where

$$
\Pi_f^J(p_f^J, H_f^J(p_f^J), H_f^I(p_f^I)) = \sum_{i \in \mathcal{N}_f^J} \hat{D}_i^J(p_i, H_f^J(p_f^J), H_f^I(p_f^I), H^J(p))(p_i - c_i). \tag{2.101}
$$

The first-order condition for profit-maximization of each firm f is given by

$$
\begin{aligned}
0 &= \frac{\partial \Pi_f^J}{\partial p_i} + (h_i^J)' \left(\frac{\partial \Pi_f^J}{\partial H_f^J} + \frac{\partial \Pi_f^I}{\partial H_f^J} + \frac{\partial H^J}{\partial H_f^J}\frac{\partial \Pi_f^J}{\partial H^J} + \frac{\partial H^I}{\partial H_f^J}\frac{\partial \Pi_f^I}{\partial H^I}\right) \\
&= \hat{D}_i^J - \hat{D}_i^J \frac{(h_i^J)''}{(h_i^J)'}(p_i - c_i) + (h_i^J)' \left(\frac{(1-\alpha_I)\alpha_J + \beta_J\beta_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\frac{1}{H_f^J}\Pi_f^J + \frac{\beta_I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\frac{1}{H_f^J}\Pi_f^I\right) \\
&\quad - (h_i^J)' \left(\frac{\partial H^J}{\partial H_f^J}\frac{1}{H^J}\Pi_f^J + \frac{\partial H^I}{\partial H_f^J}\frac{1}{H^I}\Pi_f^I\right) \\
&= \hat{D}_i^J \left(1 + \frac{(h_i^J)''}{(h_i^J)'}(p_i - c_i) - \frac{1}{N_f^J}\frac{\{(1-\alpha_I)\alpha_J + \beta_J\beta_I\}\Pi_f^J + \beta_I\Pi_f^I}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\right. \\
&\quad \left. + \frac{(1-\alpha_I)}{(1-\alpha_J)(1-\alpha_I) - \beta_J\beta_I}\left(\Pi_f^J + \frac{\beta_I}{1-\alpha_I}\frac{N_f^I}{N_f^J}\Pi_f^I\right)\right)
\end{aligned}
\tag{2.102}
$$

From this equation, we observe that there exists μ_f^J such that

$$
-\frac{(h_i^J)''(p_i)}{(h_i^J)'(p_i)}(p_i - c_i) = \mu_f^J \tag{2.103}
$$

for all $i \in \mathcal{N}_f^J$.

2.10 Section B: Proofs of Propositions

Proof of Proposition 1

Suppose that T_g is sufficiently close to 0. In both cases with MNL-class and CES-class demand systems, we have $N(0,\alpha) = 0$, the left-hand-side minus the right-hand-side in (2.15) can be approximated around $\tilde{T}_g = 0$ by

$$\frac{d}{d} N\left(\frac{\gamma(T_f)}{H^*}, \alpha\right) T_g - \frac{d}{d} N(0,\alpha) T_g \tag{2.104}$$

For each cases with MNL-class and CES-class demand systems, I show that expression 2.104 is positive.

MNL-class demand In the proof of Proposition 3, I show that in the case with MNL-class demand system, the expression (2.104) is positive.

CES-class demand From equation FOC-CES, we have

$$\mu_f = \frac{\sigma(1-\alpha)}{\sigma - \alpha - (\sigma-1)N_f}$$

Next, by equation Share-CES, we can write the value of T that leads to the market share N as

$$T = H^{1-\alpha} N^{1-\alpha} \left(1 - \frac{\mu}{\sigma}\right)^{1-\sigma}.$$

Finally, using The Implicit Function Theorem and rearranging, we have

$$\frac{dN}{dT} = \frac{\left(1-\frac{\mu}{\sigma}\right)^{\sigma-1}}{H^{1-\alpha}} N^{\alpha} \left(1 - \frac{\mu_f N_f \frac{\sigma-1}{\sigma(1-\alpha)} \left(1 - \frac{\mu_f}{\sigma}\right)^{-1}}{\sigma - \alpha - (\sigma-1) N \left(1 - \frac{\mu_f}{\sigma}\right)^{-1}}\right) \to 0 \text{ as } N \to 0.$$

Thus, the condition (2.15) holds for sufficiently small value of $\tilde{T}_g$.

Proof of Proposition 2

First, prove the proposition in the MNL case. By the equation $N((\gamma(T)/H^*, \alpha) = 1/|\mathcal{F}|$, we obtain

$$H^* = |\mathcal{F}|\gamma(T) \exp\left(-\frac{|\mathcal{F}|}{|\mathcal{F}|-1}\right) \tag{2.105}$$

Thus, we have

$$N\left(\frac{\gamma(2T)}{H^*},\alpha\right) = N\left(2^{\frac{1}{1-\alpha}}\frac{\exp\left(\frac{|\mathcal{F}|}{|\mathcal{F}|-1}\right)}{|\mathcal{F}|},\alpha\right), \tag{2.106}$$

which is increasing in α. Further, we have $N(\gamma(2T)/H^*,\alpha) = 2/|\mathcal{F}|$ at $\hat{\alpha}$ such that

$$2^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} = \exp\left(\frac{|\mathcal{F}|}{(|\mathcal{F}|-2)(|\mathcal{F}|-1)}\right). \tag{2.107}$$

Thus, if $\alpha > \hat{\alpha}$, the merger between two firms improves the consumer surplus. Since $\frac{|\mathcal{F}|}{(|\mathcal{F}|-2)(|\mathcal{F}|-1)}$ decreases with $|\mathcal{F}|$, $\hat{\alpha}$ decreases with $|\mathcal{F}|$.

Next, I show the proposition in the case of CES-class demand. First, the equilibrium level of the aggregator is given by

$$H^* = |\mathcal{F}|\gamma(T)\left(\frac{(\sigma-1)\frac{|\mathcal{F}|-1}{|\mathcal{F}|}}{\sigma-\alpha-\frac{(\sigma-1)}{|\mathcal{F}|}}\right)^{\frac{\sigma-1}{1-\alpha}} \tag{2.108}$$

Thus, we have

$$N\left(\frac{\gamma(2T)}{H^*},\alpha\right) = N\left(2^{\frac{1}{1-\alpha}}\frac{\left(\frac{(\sigma-1)\frac{|\mathcal{F}|-1}{|\mathcal{F}|}}{\sigma-\alpha-\frac{(\sigma-1)}{|\mathcal{F}|}}\right)^{\frac{1-\sigma}{1-\alpha}}}{|\mathcal{F}|},\alpha\right), \tag{2.109}$$

Let $\hat{\alpha}$ such that

$$\Lambda(\hat{\alpha},|\mathcal{F}|) := 2^{\hat{\alpha}} - \left(\frac{(\sigma-1)\frac{|\mathcal{F}|-1}{|\mathcal{F}|}}{\sigma-\hat{\alpha}-\frac{(\sigma-1)}{|\mathcal{F}|}}\right)^{\sigma-1} = 0. \tag{2.110}$$

Then, a merger between two firms is CS-neutral at $\alpha = \hat{\alpha}$. Since $\Lambda(0,|\mathcal{F}|) < 0$ and

$$\frac{\partial\Lambda}{\partial\alpha} = 2^{\hat{\alpha}}\log 2 + (\sigma-1)2^{-\hat{\alpha}\frac{\sigma}{\sigma-1}}\frac{|\mathcal{F}|-2}{|\mathcal{F}|-1}\frac{\sigma-1}{\mathcal{F}(\sigma-\hat{\alpha})-2(\sigma-1)} > 0.$$

at $\alpha = \hat{\alpha}$, the merger between two firms is CS-increases if and only if $\alpha > \hat{\alpha}$. To see that $\hat{\alpha}$ decreases with $|\mathcal{F}|$, I apply the implicit function theorem to obtain

$$\frac{d\hat{\alpha}}{d|\mathcal{F}|} = -\frac{\frac{\partial\Lambda}{\partial|\mathcal{F}|}}{\frac{\partial\Lambda}{\partial\alpha}},$$

where

$$\frac{\partial\Lambda}{\partial|\mathcal{F}|} = (\sigma-1)2^{\alpha\frac{-\sigma}{\sigma-1}}\frac{(1-\hat{\alpha})\left(\sigma-\hat{\alpha}-\frac{2(\sigma-1)}{|\mathcal{F}|^2}\right)}{\left[\left(\sigma-\hat{\alpha}-\frac{2(\sigma-1)}{|\mathcal{F}|}\right)(|\mathcal{F}|-1)\right]^2} > 0. \tag{2.111}$$

Proof of Proposition 3

First, note that

$$\frac{d}{dT}N_0\left(\frac{\gamma(T)}{H}\right) = \frac{1}{1-\alpha}\frac{1}{T}\frac{\gamma(T)}{H}N_0'\left(\frac{\gamma(T)}{H}\right). \tag{2.112}$$

Next, suppose that $N_0(\gamma(T)/H) = N$. Then, by the equations FOC-MNL and Share-MNL, we must have

$$N = \frac{\gamma(T)}{H}\exp\left(-\frac{1}{1-N}\right), \tag{2.113}$$

which implies that

$$T = H^{1-\alpha}N^{1-\alpha}\exp\left(\frac{1-\alpha}{1-N}\right). \tag{2.114}$$

Thus, we can rewrite $dN_0(\gamma(T)/H)/dT$ as

$$\frac{d}{dT}N_0\left(\frac{\gamma(T)}{H}\right) = \chi(N) := \frac{1}{1-\alpha}H^{\alpha-1}\frac{N^{\alpha}(1-N)^2}{1-N+N^2}\exp\left(-\frac{1-\alpha}{1-N}\right), \tag{2.115}$$

which approaches to 0 as $N \to 0$ or as $N \to 0$. Further, we have

$$\chi'(N) = \frac{\exp\left(-\frac{1-\alpha}{1-N}\right)N^{\alpha-1}}{(1-N+N^2)^2}\zeta(N) \tag{2.116}$$

where

$$\zeta(N) := \left[(1-N+N^2)\left\{\alpha(1-N)^2 - 2N(1-N) - (1-\alpha)N\right\} + (1-2N)(1-N)^2N\right]. \tag{2.117}$$

Note that

$$\zeta(0) = \alpha > 0, \text{ and } \zeta(1) = -(1-\alpha) < 0,$$

which implies that $\xi(N)$ is positive around $N = 0$ and positive around $N = 1$. and

$$\zeta'(0) = -2 - 2\alpha > 0, \text{ and } \zeta'(1) = 2\alpha > 0.$$

By the fact that $\zeta(N)$ is a fourth polynomial of N with negative coefficient of N^4, have only one $\bar{N} \in (0,1)$ such that $\zeta'(\bar{N}) = 0$. Further, $\zeta'(N) < 0$ for $N \in [0,\bar{N})$ and $\zeta'(N) > 0$ for $N \in (\bar{N},1]$. Putting these observations together, we have N^* such that $\zeta(N) \geq$ if and only if $N \leq N^*$. Thus, $\xi(N)$ increases with N if and only if $N \leq N^*$.

The fact that $\xi(0) = \xi(1)$ and that $\xi(N)$ increases with N if and only if $N \leq N^*$ together imply that (i) for any $N, N' \leq N^*$, $\xi(N) > \xi(N')$ if and only if $N > N'$, (ii) for any $N, N' \geq N^*$, $\xi(N) > \xi(N')$ if and only if $N < N'$, and (iii) for any $N < N^*$, there exists $N' > N^*$ such that $\xi(N) = \xi(N')$. These observations leads to the statements of Proposition 3.

Proof of Proposition 4

Proof of Proposition 4.1 and Proposition 4.2

I first analyze the concavity and the convexity of the function $\tilde{N}'(x)x$ with respect to the network share, and then use this analysis to prove Proposition 4.1 and Proposition 4.2.

In MNL-class demand system, we have

$$N_0'(x)x\bigg|_{N_0(x)=N} = \frac{N^3 - 2N^2 + N}{N^2 - N + 1} =: \lambda(N).$$

Then, we have

$$\lambda'(N) = \frac{(1-N)(N^3 - N^2 + 3N - 1)}{(N^2 - N + 1)^2}, \tag{2.118}$$

which is nonnegative in $N \in [0, \hat{N}]$ and negative in $(\hat{N}, 1]$ for some critical value $\hat{N} \in (0,1)$ that satisfies

$$\hat{N}^3 - \hat{N}^2 + 3\hat{N} - 1 = 0.$$

Further, Nocke and Schutz (2018a) show that $\lambda(N)$ is concave in N. In summary, $\lambda(N)$ is increasing in $N \in [0, \hat{N})$, nonincreasing in $N \in [\hat{N}, 1]$, and concave in $N \in [0,1]$

Using this result, I prove Proposition 4.1 and Proposition 4.2.

The required synergies $\hat{\Delta}$ decreases with α if and only if

$$A(T_f + T_g + \hat{\Delta})B(T_f + T_g + \hat{\Delta}) \geq A(T_f)B(T_f) + A(T_g)B(T_g) \tag{2.119}$$

where

$$A(T_f) = N_0'\left(\frac{\gamma(T)}{H^*}\right)\frac{\gamma(T)}{H^*} \tag{2.120}$$

and

$$B(T) = \frac{H^*}{\gamma(T)}\frac{d}{d\alpha}\left(\frac{\gamma(T)}{H^*}\right) = \frac{d}{d\alpha}\left(\frac{\gamma(T)}{H^*}\right) = \frac{1}{(1-\alpha)^2}\frac{\sum_{f'\in\mathcal{F}}(\log T - \log T_{f'})N_0'\left(\frac{\gamma(T_{f'})}{H^*}\right)}{\sum_{f'\in\mathcal{F}} N_0'\left(\frac{\gamma(T_{f'})}{H^*}\right)}. \tag{2.121}$$

Note that $B(T)$ is increasing in T.

Suppose that two merging firms f and g are weak. If the merged entity is strong, the LHS of (2.119) is positive while the RHS of (2.119) is negative. Next, suppose that the merged entity is weak. We have $A(T_f + T_g + \hat{\Delta}) \leq A(T_f) + A(T_g)$ by the concavity of $N_0'(x)x$ in $N \in [0,1]$. Finally, we have the following inequality

$$\begin{aligned} A(T_f + T_g + \hat{\Delta})B(T_f + T_g + \hat{\Delta}) &\geq (A(T_f) + A(T_g))B(T_f + T_g + \hat{\Delta}) \\ &\geq A(T_f)B(T_f) + A(T_g)B(T_g), \end{aligned} \tag{2.122}$$

where the last inequality follows from the fact that $B(T)$ is increasing in T and $T_f + T_g + \hat{\Delta} \geq \max\{T_f, T_g\}$. Thus, $\hat{\Delta}$ decreases with α.

Next, I show that $\hat{\Delta}$ for the merger between firm f and g decreases with α if firm f is weak and the firm g is strong and if $N_f + N_g < \hat{N}$. This can be observed by

$$A(T_f + T_g + \hat{\Delta})B(T_f + T_g + \hat{\Delta}) \geq A(T_f)B(T_f) \tag{2.123}$$

and

$$A(T_g)B(T_g) \leq 0. \tag{2.124}$$

Proof of Proposition 4.3

When $N_f + N_g \simeq 1$, then $A(T_f + T_g + \hat{\Delta}) \simeq 0$.

$$T_f + T_g + \hat{\Delta} = \left[H^*(N_f + N_g)\exp\left(\frac{1}{1 - N_f - N_g}\right)\right]^{1-\alpha}. \tag{2.125}$$

Thus, we have

$$\log(T_f + T_g + \hat{\Delta}) = (1-\alpha)\left[\log H^* + \log(N_f + N_g) + \frac{1}{1 - N_f - N_g}\right]$$

As a result,

$$N'(x)x\log(T_f + T_g + \hat{\Delta}) \longrightarrow 0 \text{ as } N_f + N_g \longrightarrow 1. \tag{2.126}$$

Thus, $A(T_f + T_g + \hat{\Delta})B(T_f + T_g + \hat{\Delta}) \longrightarrow 0$ as $N_f + N_g \longrightarrow 1$. Thus, if two merging firms are strong, the LHS of the equation (2.119) is zero, while the RHS of the equation (2.119) is positive. Thus, $\hat{\Delta}$ increases with α.

Proof of Proposition 5

Let $\Pi(\gamma(T)/H)$ be the profit of a firm with is type $\gamma(T)$ at the value of aggregator H. The profit of a firm with the market share N is given by

$$\pi(N) = \Pi\left(\frac{\gamma(T)}{H}\right)\Big|_{N_0(\gamma(T)/H)=N} = (1-\alpha)\frac{N}{1-N}.$$

Thus, we have

$$\frac{d\Pi}{dT} = \frac{dN}{dT}\frac{d\pi}{dN} = \frac{dN}{dT}\frac{1}{(1-N)^2}. \tag{2.127}$$

Next, I consider the change in the equilibrium value of the aggregator H^* with respect to the change in the type T_f of firm f. Using The Implicit Function Theorem to the equation 2.14, we obtain

$$\frac{dH^*}{d\gamma(T_f)} = \frac{N_0'(x_f)}{\sum_g x_g N_0'(x_g)},$$

where $x_g = \gamma(T_f)/H^*$ for each $g \in \mathcal{F}$. Thus, we have

$$\frac{d}{d\gamma(T_f)}\left(\frac{\gamma(T_f)}{H^*}\right) = \frac{1}{H}\left(1 - \frac{x_f N_0'(x_f)}{\sum_g x_g N_0'(x_g)}\right).$$

Finally, we obtain

$$\begin{aligned}\frac{d\Pi_f}{dT_f} &= \frac{1}{1-\alpha}\frac{1}{T_f}\frac{\gamma(T_f)}{H}N_0'(x_f)\frac{\sum_{g\neq f} x_g N_0'(x_g)}{\sum_g x_g N_0'(x_g)} \\ &= \frac{\chi(N_0(x_f))}{(1-N_0(x_f))^2}\frac{\sum_{g\neq f} x_g N_0'(x_g)}{\sum_g x_g N_0'(x_g)} = \frac{1}{1-\alpha}H^{\alpha-1}\frac{N^\alpha}{1-N+N^2}\exp\left(-\frac{1-\alpha}{1-N}\right)\frac{\sum_{g\neq f} x_g N_0'(x_g)}{\sum_g x_g N_0'(x_g)}.\end{aligned} \tag{2.128}$$

Drawing on these preliminaries, I show that if T_E is sufficiently small, $\Delta\pi^I > \Delta\Pi^E$ holds. When T_E is sufficiently small, $\Delta\pi^I$ can be approximated as

$$\Delta\pi^I \simeq \frac{d\Pi(\gamma(T_I)/H_0)}{dT_I}T_E > 0$$

and $\Delta\pi^E$ can be approximated as

$$\Delta\pi^E \simeq \frac{d\Pi(0)}{dT_E}T_E = 0,$$

which implies that $\Delta\pi^I > \Delta\pi^E$ for sufficiently small values of T_E.

Next, I show that when T_E is sufficiently large, $\Delta\pi^I < \Delta\pi^E$ holds. Since $N(\gamma(T)/H) \to 1$ as $T \to \infty$, $H \to \infty$ as $T \to \infty$ and

$$\frac{\sum_{g\neq f} x_g N_0'(x_g)}{\sum_g x_g N_0'(x_g)} \to 1 \quad \text{as } T_f \to \infty,$$

we obtain $\lim_{T_f\to\infty} d\Pi_f/dT_f = 0$. As a result, we have $\Pi^E \simeq \Pi^I$ when $T_E \simeq \infty$, and thus

$$\Delta\pi^I = \Delta\pi^E - \Pi_0^I < \Delta\pi^E.$$

Proof of Lemma 3

To see the effects of T_f^A and T_f^B on n_f^A and n_f^B note that

$$\begin{aligned}\frac{\partial m^A}{\partial T_f^A} &= \frac{n_f^A}{T_f^A}\frac{1}{\det(G_f)}\left[\mu_f^A\left(n_f^B\mu_f^B + 1 - n_f^B + \beta_A\frac{n_f^A}{n_f^B}\right) + \mu_f^A\beta_A^2\frac{n_f^A}{n_f^B}\right] > 0 \\ \frac{\partial m^B}{\partial T_f^A} &= -\frac{n_f^A}{T_f^A}\frac{1}{\det(G_f)}\left(\beta_A\frac{n_f^A}{n_f^B}\mu_f^A(1-\beta_A) + \beta_A\frac{1-n_f^A}{n_f^B}\right) < 0,\end{aligned}$$

and thus

$$\frac{\partial N^A}{\partial T_f^A} = \frac{n_f^A}{T_f^A}\frac{1}{\det(G_f)}(1-n_f^A)\left(n_f^B\mu_f^B + 1 - n_f^B + \beta_A(1+\beta_A)\frac{n_f^A}{n_f^B}\right) < 0 \tag{2.129}$$

$$\frac{\partial N^B}{\partial T_f^A} = \frac{n_f^A}{T_f^A}\frac{1}{\det(G_f)}\left(\beta_A\frac{n_f^A}{n_f^B}\mu_f^A(1-\beta_A) + \beta_A\frac{1-n_f^A}{n_f^B}\right)n_f^B < 0. \tag{2.130}$$

Further, note that

$$\frac{\partial m^A}{\partial T_f^B} = \frac{1}{T_f^B}\frac{1}{\det(G_f)}\left(\beta_A n_f^A \mu_f^A (1-n_f^B) + \beta_A^3 \frac{n_f^A}{n_f^B} n_f^A \mu_f^A\right) > 0$$
$$\frac{\partial m^B}{\partial T_f^B} = \frac{1}{T_f^B}\frac{1}{\det(G_f)}\left[(n_f^A \mu_f^A + 1 - n_f^A)\left(n_f^B \mu_f^B + \beta_A(1-\beta_A)\frac{n_f^A}{n_f^B}\right) + \beta_A^3 n_f^A \mu_f^A \frac{n_f^A}{n_f^B}\right] > 0,$$

and thus

$$\frac{\partial N^A}{\partial T_f^B} = \beta_A \frac{n_f^A}{T_f^B}\frac{1}{\det(G_f)}(1-n_f^A)\left(1 - n_f^B + \beta_A^2 \frac{n_f^A}{n_f^B}\right) > 0 \tag{2.131}$$
$$\frac{\partial N^B}{\partial T_f^B} = \frac{n_f^B}{T_f^B}\frac{1}{\det(G_f)}(n_f^A \mu_f^A + 1 - n_f^A)\left(1 - n_f^B + \beta_A^2 \frac{n_f^A}{n_f^B}\right) > 0 \tag{2.132}$$

Proof of Proposition 6

First, consider the effect of $x_f \in \{T_f^A, T_f^B\}$ on H^A. By the Implicit Function Theorem, we have

$$\begin{pmatrix} \sum \frac{\partial N_f^A}{\partial H^A} & \sum \frac{\partial N_f^A}{\partial H^B} \\ \sum \frac{\partial N_f^B}{\partial H^A} & \sum \frac{\partial N_f^B}{\partial H^B} \end{pmatrix} \begin{pmatrix} \frac{dH^A}{dx_f} \\ \frac{dH^B}{dx_f} \end{pmatrix} = -\begin{pmatrix} \frac{\partial N_f^A}{\partial x_f} \\ \frac{\partial N_f^B}{\partial x_f} \end{pmatrix}, \tag{2.133}$$

for $x_f \in \{T_f^A, T_f^B\}$. Using Cramer's rule, we obtain

$$\operatorname{sign}\left(\frac{\partial H^A}{\partial x_f}\right) = \operatorname{sign}\left[-\frac{\partial N_f^A}{\partial x_f}\left(\sum_{f' \in \mathcal{F}} \frac{\partial N_f^B}{\partial H^B}\right) + \frac{\partial N_f^B}{\partial x_f}\left(\sum_{f' \in \mathcal{F}} \frac{\partial N_f^A}{\partial H^B}\right)\right] \tag{2.134}$$

Since $\partial N_f^A / \partial H^B > 0$ for all $f \in \mathcal{F}$ and $\partial N_f^J / \partial x_f > 0$ for any $J \in \{A, B\}$ and $x_f \in \{T_f^A, T_f^B\}$, we have $\partial H^A / \partial T_f^A > 0$ and $\partial H^B / \partial T_f^B > 0$.

Next, consider the effects of T_f^A and T_f^B on H^B. Using the Cramer's rule, we have

$$\operatorname{sign}\left(\frac{\partial H^B}{\partial x_f}\right) = \operatorname{sign}\left[-\frac{\partial N_f^B}{\partial x_f}\left(\sum_{f' \in \mathcal{F}} \frac{\partial N_f^A}{\partial H^A}\right) + \frac{\partial N_f^A}{\partial x_f}\left(\sum_{f' \in \mathcal{F}} \frac{\partial N_f^B}{\partial H^A}\right)\right] \tag{2.135}$$

for $x_f \in \{T_f^A, T_f^B\}$. A calculation shows that $\partial N_f^B / \partial T_f^B > \partial N_f^A / \partial T_f^B$, and that $|\partial N_f^A / \partial H^A| >$

$|\partial N_f^B/\partial H^A|$. These jointly imply that $dH^B/dT_f^B > 0$. Finally, a further calculation shows that

$$\begin{aligned}&\text{sign}\left(\frac{dH^B}{dT_f^A}\right)\\ =&\text{sign}\left[\frac{n_f^A\mu_f^A(1-\beta)+1-n_f^A}{(1-n_f^A)\left(n_f^B\mu_f^B+1-n_f^B+\beta(1+\beta)\frac{n_f^A}{n_f^B}\right)}-\frac{\sum_{f'\in\mathcal{F}}\left(n_{f'}^A\mu_{f'}^A(1-\beta)+1-n_{f'}^A\right)}{\sum_{f'\in\mathcal{F}}\left[(1-n_{f'}^A)\left(n_{f'}^B\mu_{f'}^B+1-n_{f'}^B+\beta(1+\beta)\frac{n_{f'}^A}{n_{f'}^B}\right)\right]}\right]\end{aligned} \tag{2.136}$$

When $n_f^A \simeq 1$, the first item approaches to ∞, while the second item remains to be finite. Thus, if $n_f^A \simeq 1$, $dH^B/dT_f^A > 0$.

Finally, I provide an example where H^B decreases with T_f^A. For simplicity, suppose that $\beta \simeq 0$. Then, we have

$$\begin{aligned}&\text{sign}\left(\frac{dH^B}{dT_f^A}\right)\\ \simeq&\text{sign}\left[\frac{n_f^A\mu_f^A+1-n_f^A}{(1-n_f^A)\left(n_f^B\mu_f^B+1-n_f^B\right)}-\frac{\sum_{f'\in\mathcal{F}}\left(n_{f'}^A\mu_{f'}^A+1-n_{f'}^A\right)}{\sum_{f'\in\mathcal{F}}\left[(1-n_{f'}^A)\left(n_{f'}^B\mu_{f'}^B+1-n_{f'}^B\right)\right]}\right]\end{aligned} \tag{2.137}$$

Consider further the case where all firm but firm f are symmetric, and firm f's shares are given by $n_f^A \simeq 0$ and $n_f^B \simeq 0$. Then, the terms in the brackets in the second line of equation (2.137) can be rewritten as

$$1-\frac{1+\frac{|\mathcal{F}|^2-|\mathcal{F}|+1}{|\mathcal{F}|-1}}{1+\frac{|\mathcal{F}|^2-|\mathcal{F}|+1}{|\mathcal{F}|}}<0.$$

Proof of Lemma 4

Let N_k^J be the equilibrium market share of firm $k \in \{f, g\}$ on side $J \in \{A, B\}$. Let $N_M^J = N_f^J + N_g^J$ for each $J \in \{A, B\}$. Consider the merging entity's type (T_M^A, T_M^B) such that

$$\begin{aligned}N^A(T_M^A, T_M^B, H^A, H^B) &= N_M^A\\ N^B(T_M^A, T_M^B, H^A, H^B) &= N_M^B\end{aligned} \tag{2.138}$$

By the first-order condition, we obtain

$$\mu_M^A = \frac{1}{1-N_M^A} \tag{2.139}$$

$$1-(1-N_M^B)\mu_M^B-\beta\frac{N_M^A}{N_M^B}=0. \tag{2.140}$$

Thus, we obtain the unique markup such that the market shares given the aggregators H^A and H^B is given by the unique pair of μ_M^A and μ_M^B respectively. Finally, the pair (T_M^A, T_M^B) must satisfy the following system of equations:

$$\begin{aligned} N_M^A &= \frac{T_M^A (T_M^B)^\beta \exp(-\mu_M^A - \beta \mu_M^B)}{H^A} \\ N_M^B &= \frac{T_M^B \exp(-\mu_M^B)}{H^B}. \end{aligned} \tag{2.141}$$

By the latter equation, T_M^B is uniquely determined. Further, once T_M^B is given, T_M^A is also uniquely determined. Let $(\hat{T}_M^A, \hat{T}_M^B)$ be such pair of types. As a result, there is a unique pair of technological synergies $\hat{\Delta}_M^J := \hat{T}_M^J - T_f^J - T_g^J$.

Chapter 3

Monopoly Regulation in the Presence of Consumer Demand-Reduction*

3.1 Introduction

In utility sectors, consumers often save the demand by engaging in some investments. For example, households can use rooftop solar energies or introduce electricity-efficient consumer electronics to reduce the demand for electricity or purchase bicycles or cars to reduce the dependence on public transportation. Business enterprises also engage in energy-saving investments in the face of various environmental regulations.[1] Such demand-saving activities often have important effects on the rate-setting of the regulated firms. As reviewed by Costello and Hemphill (2014), the consequence of such an interaction between rate-setting and demand-reduction is sometimes called as “death-spiral", the situation where a high rate leads to demand-reducing investments, which damages the utility’s financial viability and requires even higher rates to break even. The aim of this chapter is to study the design of optimal regulatory mechanisms in the face of such demand-reducing activities.

Specifically, I study the model of optimal monopoly regulation á la Laffont and Tirole (1993), in the setting where consumers can engage in demand-reducing investments.

I first consider the case where the regulatory mechanism is designed ignoring the consumers’ investments. In this case, the resulting level of investments is too high in terms of aggregate welfare. Thus, the optimal mechanism should be designed so as to limit the consumers’ investments. This result is consistent with the view that to deal with the problem of death-spiral, the rate should be set at a lower level. Next, I proceed to the analysis of optimal regulation policy explicitly taking the consumers’ investments into account and study the effects of asymmetric information between regulator and monopolist on the optimal policy. I show that the presence of asymmetric information results in the higher average price than the first best, which leads to the higher level of buyers’ investments. Thus, the presence of asymmetric information exacerbates the problem of excess investments. Finally, I show that the regulated prices for the most efficient monopolists under asymmetric information are set below the first best levels. This result contrasts with the standard

*This chapter is based on Sato (2018).

[1] See Matsumura and Yamagishi (2017) for the examples of such investments.

“no distortion at the top" principle that the regulated price for the most efficient types corresponds with the first best. These results would provide some theoretical guidance for policy design in utility sectors subject to a demand-reducing investments, such as gas and electricity.

3.2 Model

I consider the model of monopoly regulation á la Laffont and Tirole (1993), where a continuum of consumers can engage in demand-reducing investments. I set the primitives of the model below.

Consumers Consumers derive the utility $S(q,x) - pq$, where q is the amount of purchase, p is the unit price of the good, $x \in \mathbb{R}_+$ is the level of demand-reducing investment. I assume that S is concave, $S_q > 0$, $S_{qq} < 0$, $S_{qx} < 0$, $S_{xx} < 0$. I also assume that $S_x(q,0) > 0$ for any q and that for any q, there exists $\bar{x}_q$ such that $S_x(q,\bar{x}_q) = 0$. These assumptions guarantee that given any level of x, there is a demand function $D(p,x)$ derived from the condition

$$S_q(D(p,x),x) - p = 0 \tag{3.1}$$

that is decreasing in p and x. Let $V(p,x) = S(D(p,x),x) - pD(p,x)$ be the corresponding indirect utility function. Note that $V_p(p,x) = -D(p,x)$.

The first component in the consumer utility, $S(q,x)$, can be specified depending on the contexts. For example, the cost of investment can be explicitly incorporated by specifying $S(q,x) = u(q,x) - c(x)$, where $c(x)$ is the monetary or opportunity cost of investments that may reflect the prices of substitute goods or subsidies for investments. In this case, the required assumptions on u and c are that $u(q,x) - c(x)$ is concave, $u_q > 0$, $u_{qq} < 0$, $u_{qx} < 0$, $u_{xx} - c_{xx} < 0$, $u_x(q,0) - c_x(0) > 0$, and that there exists $\bar{x}_q$ such that $u_x(q,\bar{x}_q) - c(\bar{x}_q) = 0$ for any q.

Monopolist The monopolist incurs a constant marginal cost $\beta \in [\beta_L, \beta_H]$ and a fixed cost K of production. Thus, the profit of the monopolist at a price level p and the sales q is given by

$$(p-\beta)q - K. \tag{3.2}$$

β is privately known by the monopolist and distributed according to the strictly increasing smooth distribution function F with the density function f. I assume that the function F/f is increasing.

Regulator The regulator can offer a menu $(p(\beta), s(\beta))_{\beta\in[\beta_L,\beta_H]}$ of contracts that specifies the price p and the amount of subsidy s. I assume that the subsidy is costly due to shadow cost of public funds λ.

Then the aggregate welfare with the price p, the subsidy s, investment level x, and the marginal cost β is given by the sum of consumer surplus, producer surplus, and the welfare loss from subsidy:

$$S(D(p,x),x) - \beta D(p,x) - K - \lambda s. \tag{3.3}$$

Timing The timing of this game is as follows:

1. The regulator offers a menu $(p(\beta), s(\beta))_{\beta \in [\beta_L, \beta_H]}$ of contracts.

2. Consumers choose the level of demand-reducing investments x. At the same time, the monopolist observes β and chooses the contract $(p(\beta'), s(\beta'))$ that maximizes his profit.

3. Given the price $p(\beta')$ consumers choose the amount of purchase.[2]

3.3 Optimal Regulation

In this section, I study the optimal regulation under two scenarios: (i) complete information with exogenous investments, (ii) asymmetric information with endogenous investments. In the course of analysis, I study how the presence of demand-reducing investments affects the aggregate welfare and interacts with the asymmetric information.

To this end, I first consider how the consumers make investment decisions. Suppose that the monopolist with type β is regulated to set the price $p(\beta)$. Then the expected surplus is $E_\beta[V(p(\beta), x)]$. Thus, the first-order condition for consumer investment is given by

$$\mathbb{E}_\beta[S_x(D(p(\beta), x), x)] = 0. \tag{3.4}$$

Benchmark: Complete Information with Exogenous Investments

As a first benchmark, consider the setting where there is no asymmetric information between the monopolist and the regulator, and the regulator takes the consumers' investments as given. In this setting, a standard derivation yields

$$s(\beta) = K - (p(\beta) - \beta)D(p(\beta), x)$$

and

$$\frac{p(\beta) - \beta}{p(\beta)} = \frac{\lambda}{1 + \lambda} \frac{1}{\eta(p(\beta), x)}, \tag{3.5}$$

where

$$\eta(p, x) := -\frac{D_p(p, x)p}{D(p, x)} > 0 \tag{3.6}$$

is the price elasticity of demand. This is the standard Lerner formula obtained in the models of monopoly regulation with a cost of public funds.

[2]Here I assume that the regulator chooses the policy before the consumers engage in investments. This might no be realistic in several situations where the investment decision takes a longer time than the regulatory decision. Even if a fraction of consumer investments are allowed to take place before the investment decision, the qualitative results are unchanged if at least there is some fraction of investment decisions which take place after the regulatory decision. Our analysis can also be seen as a normative one on how the regulator should design the policy when she can commit to a long-run policy.

Now consider the welfare consequence of buyer investments. Let x be determined by the condition (3.4) and slightly increase from the equilibrium level. After some algebra, its effect on the welfare is written as

$$(1+\lambda)\mathbb{E}_\beta\left[D_x(p(\beta),x)(p(\beta)-\beta)\right] < 0. \tag{3.7}$$

This immediately implies the following proposition.

Proposition 1. *Under the complete information, if the regulator sets the policy taking the consumers' investments as given, the amount of the investments is too high in terms of social welfare.*

The intuition is as follows. While the government needs to guarantee some profit of the monopolist to reduce the subsidy, consumers choose their investment level to maximize their surplus. As a result, the welfare loss due to an increase in subsidy is ignored by consumers, resulting in the excess investments.

This proposition gives an implication that the optimal regulation should be designed so as to keep consumers from engaging in too many investments. With this proposition in mind, let me proceed to the analysis of optimal regulation in the presence of consumers' investments.

Regulation under Asymmetric Information

In this subsection, I solve the regulator's problem taking the information asymmetry and consumers' investments into account.[3] The regulator needs to set the menu of contracts so as to satisfy the incentive compatibility constraints:

$$\beta = \arg\max_{\beta'}(p(\beta')-\beta)D(p(\beta'),x) - K + s(\beta'). \tag{3.8}$$

Thus, the envelope theorem yields the following:

$$s(\beta) = (p(\beta_H)-\beta_H)D(p(\beta_H),x) - K + s(\beta_H) - (p(\beta)-\beta)D(p(\beta),x) + K + \int_\beta^{\beta_H} D(p(y),x)dy. \tag{3.9}$$

As standard in the literature, $s(\beta_H) = K - (p(\beta_H)-\beta_H)q(\beta_H,\bar{\psi})$ so that the participation constraint for the most inefficient monopolist binds.

A standard integral-by-parts manipulation yields the following expression:

$$\mathbb{E}_\beta[s(\beta)] = \mathbb{E}_\beta\left[-\left(p(\beta)-\beta-\frac{F(\beta)}{f(\beta)}\right)D(p(\beta),x) + K\right]. \tag{3.10}$$

Thus, the expected welfare can finally be written as

$$\mathbb{E}_\beta\left[\left[S(D(p(\beta),x)) + \left(\lambda p(\beta) - (1+\lambda)\beta - \lambda\frac{F(\beta)}{f(\beta)}\right)D(p(\beta),x)\right] - (1+\lambda)K\right]. \tag{3.11}$$

[3] For the detail of each derivation step, see Laffont and Tirole (1993) Chapter 2.

The regulator's maximization problem is thus to maximize the above welfare over $p(\cdot)$ and x subject to the equation (3.4). Let μ^{**} be the Lagrangian multiplier for the constraint (3.4). Then the first-order condition is obtained by

$$\begin{aligned}&\mathbb{E}_\beta\left[D_x(p(\beta),x)\left[(1+\lambda)(p(\beta)-\beta)-\lambda\frac{F(\beta)}{f(\beta)}\right]\right]\\&+\mu^{**}\mathbb{E}_\beta[D_x(p(\beta),x)S_{xq}(D(p(\beta),x),x)+S_{xx}(D(p(\beta),x),x)]=0\end{aligned} \tag{3.12}$$

and

$$\left[D_p\left((1+\lambda)p(\beta)-(1+\lambda)\beta-\lambda\frac{F(\beta)}{f(\beta)}\right)+\lambda D(p(\beta),x)\right]+\mu^{**}D_pS_{qx}=0 \tag{3.13}$$

Thus, a similar condition for the equilibrium price schedule $p(\cdot)$ is obtained as follows:

$$\frac{p(\beta)-\beta-\frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)}}{p(\beta)}=\underbrace{\frac{\lambda}{1+\lambda}\frac{1}{\eta(p(\beta),x)}}_{\text{Standard inverse elasticity}}+\underbrace{\left(\frac{S_{qx}}{p(\beta)E_\beta[D_xS_{qx}+S_{xx}]}\right)\mathbb{E}_\beta\left[\left(p(\beta)-\beta-\frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)}\right)D_x\right]}_{\text{Investment reduction term}}. \tag{3.14}$$

Note that the investment reduction term in the right-hand side is negative and thus generates a downward-pricing pressure to the regulator. Let $p^{**}(\cdot)$ and x^{**} be the price schedule and the investment level under the optimal regulation policy.

Comparison

I have derived the conditions for the optimal regulation policies under asymmetric information. I analyze the property of optimal policy by comparing it with the first-best policy that could be achieved under complete information. Under complete information, the first-best policy correspond with the optimal policy with $F(\beta)/f(\beta)=0$, that is:

$$\frac{p(\beta)-\beta}{p(\beta)}=\underbrace{\frac{\lambda}{1+\lambda}\frac{1}{\eta(p(\beta),x)}}_{\text{Standard inverse elasticity}}+\underbrace{\left(\frac{S_{qx}}{p(\beta)E_\beta[D_xS_{qx}+S_{xx}]}\right)\mathbb{E}_\beta\left[(p(\beta)-\beta)D_x\right]}_{\text{Investment reduction term}}. \tag{3.15}$$

Let $p^*(\cdot)$ and x^* be the price schedule and the investment level under the first-best policy.

To obtain a clear comparison and good intuition, I adopt the following linear-quadratic specification: $S(q,x)=aq-\frac{b}{2}(\theta x+q)^2+Ax-\frac{B}{2}x^2$. Then, $D(p,x)=(a-p)/b-\theta x$, $x^*=(A+\theta\mathbb{E}_\beta[p^*(\beta)]-a\theta)/B$, and $x^{**}=(A+\theta\mathbb{E}_\beta[p^{**}(\beta)]-a\theta)/B$. Using this specification, the following proposition is obtained.

Proposition 2. *The average regulated price is higher under the asymmetric information than the first-best level. That is, $\mathbb{E}_\beta[p^{**}(\beta)]>\mathbb{E}_\beta[p^*(\beta)]$. The equilibrium consumer investment is also higher under asymmetric information. That is, $x^{**}>x^*$.*

Proof. In Section 3.5. □

This result is straightforward. Since the asymmetric information generates information rent, the regulator has incentive to increase prices so as to reduce the rent given up to the monopolist. This incentive directly leads to the first statement of the proposition. As a result, the amount of consumers' demand-reducing investments increases, which is the second statement of the proposition.

As expected, the demand reduction due to the asymmetric information generates a downward pricing pressure on the regulator. The next proposition shows that for efficient firms, this downward effect dominates the upward effects of asymmetric information.

Proposition 3. *Under the asymmetric information, the optimal pricing for the most efficient type is lower than the first-best, i.e.,* $p^{**}(\beta_L) < p^*(\beta_L)$.

Proof. In Section 3.5. □

This result can be understood by combining Proposition 2 and the famous "no distortion at the top" principle. As shown in Proposition 2, the presence of asymmetric information generates an upward-pricing distortions on average. Expecting this distortion, more buyers engage in demand-reducing investments, which results in a weaker demand and higher investment cost. These changes generate a downward pricing pressure to the regulator. Since there is no informational distortion for the most efficient types (no distortion at the top), the downward effect dominates for the efficient firms. This kind of global distortion generated through the distortion due to informational asymmetry is similar to the overprovision of quality in dynamic regulation observed by Auray, Mariotti and Moizeau (2011).

3.4 Discussion

I have assumed that the surplus from investment is given by an exogenous function. Introducing public policies regarding the investment decisions (e.g., subsidy on energy-saving investments) and studying its interaction with monopoly regulation would be an interesting avenue for future research.

3.5 Section

Proof of Proposition 2 The equations (3.14) and (3.15) can be written as

$$\frac{p^{**}(\beta)-\beta-\frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)}}{b}-\frac{\lambda}{1+\lambda}\left(\frac{a-p^{**}(\beta)}{b}-\theta x^{**}\right)-\frac{b\theta^2}{B}\mathbb{E}_\beta\left[p^{**}(\beta)-\beta-\frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)}\right]=0 \quad (3.16)$$

$$\frac{p^{*}(\beta)-\beta}{b}-\frac{\lambda}{1+\lambda}\left(\frac{a-p^{*}(\beta)}{b}-\theta x^{*}\right)-\frac{b\theta^2}{B}\mathbb{E}_\beta\left[p^{*}(\beta)-\beta\right]=0 \quad (3.17)$$

Let $m^i = \mathbb{E}_\beta[p^i(\beta)]$ for $i \in \{*, **\}$ and suppose to the contrary that $m^* > m^{**}$. Then, we have $x^* > x^{**}$. Putting equations (3.16) and (3.17) together, we have

$$\frac{1+\frac{\lambda}{1+\lambda}}{b}\left(p^{**}(\beta) - p^*(\beta)\right) - \frac{1}{b}\frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)} - \frac{\lambda}{1+\lambda}\theta\left(x^* - x^{**}\right) - \frac{b\theta^2}{B}\left(m^* - m^{**} + \frac{\lambda}{1+\lambda}\mathbb{E}\left[\frac{F(\beta)}{f(\beta)}\right]\right) = 0 \tag{3.18}$$

This implies that $p^{**}(\beta) > p^*(\beta)$ for all β, which implies that $m^{**} > m^*$, a contradiction. Thus, we must have $m^{**} > m^*$ and $x^{**} > x^*$.

Proof of Proposition 3 Since $F(\beta_L)/f(\beta_L) = 0$, we have

$$\frac{1+\frac{\lambda}{1+\lambda}}{b}\left(p^{**}(\beta_L) - p^*(\beta_L)\right) - \frac{\lambda}{1+\lambda}\theta\left(x^* - x^{**}\right) - \frac{b\theta^2}{B}\left(m^* - m^{**}\right) = 0. \tag{3.19}$$

Since $m^{**} > m^*$ and $x^{**} > x^*$ as shown above, we have $p^{**}(\beta_L) - p^*(\beta_L) < 0$.

Chapter 4

Dynamic Privatization Policy*

4.1 Introduction

Although the privatization of state-owned public enterprises has been a global phenomenon for more than 50 years, many public and semipublic enterprises owned fully or partially by the public sector remain active. Further, while some public enterprises are traditional monopolists in natural monopoly markets, a considerable number of public and semipublic enterprises compete with private enterprises in a wide range of industries.[1] The optimal privatization policies in these industries have attracted extensive attention from economics researchers in such fields as industrial organization, public economics, financial economics, and development economics.

Specifically, the literature on mixed oligopolies has investigated the optimal privatization policy in different situations. Matsumura (1998) showed that the optimal degree of privatization is never zero unless full nationalization yields a public monopoly. Lin and Matsumura (2012) and Matsumura and Okamura (2015) found that the optimal degree of privatization increases with the number of private firms and decreases with the foreign ownership share in private firms. In free entry markets, Matsumura and Kanda (2005) showed that the optimal degree of privatization is zero when private competitors are domestic, while Cato and Matsumura (2012) found that it is strictly positive when they are foreign and increases with the foreign ownership share in private firms. In addition, Chen (2017) showed that the optimal degree of privatization is positive even in free entry markets if privatization improves production efficiency. Fujiwara (2007) showed a non-monotonic relationship between the degree of product differentiation and optimal degree of privatization. Cato and Matsumura (2015) discussed the relationship between the optimal trade and privatization policies, showing that a higher tariff rate reduces the optimal degree of privatization in free entry markets. Xu, Lee and Matsumura (2017) and Lee, Matsumura and Sato (2018) showed that the optimal degree of privatization depends on the timing of privatization.

*This chapter is based on Sato and Matsumura (2019).

[1] Examples include the United States Postal Service, Deutsche Post AG, Areva, Nippon Telecom and Telecommunication (NTT), Japan Tobacco (JT), Volkswagen, Renault, Electricite de France, the Japan Postal Bank, Kampo, the Korea Development Bank, and the Korea Investment Corporation. For other examples of mixed oligopolies and recent developments in this field, see Ishibashi and Matsumura (2006), Ishida and Matsushima (2009), Dong and Bàrcena-Ruiz (2017), Chen (2017), and the works cited therein.

While these studies all assumed that a government privatizes a public firm as a one-time event, public enterprises are often privatized gradually over time. For example, the Japanese government continued to sell shares in NTT (JT), which was a state-owned public monopolist until 1985, from 1986 to 2016 (to 2013). Moreover, it still holds a one-third share in both NTT and JT. In 2015, the Japanese government sold a minor share in the Japan Postal Bank, the largest bank in Japan, and Kampo, a major life insurance company, and it announced its plans to sell more shares in these enterprises in the near future again. Although these examples are cases in which the government gradually reduces the public ownership shares, examples of the opposite also exist. The French government increased its ownership of Renault from 15% to 19.4% in 2015. Therefore, it is reasonable to assume that governments change the degree of privatization of public enterprises over time, and existing studies fail to capture this dynamic aspect of privatization policies.

In this chapter, we formulate a simple model to analyze the dynamics of privatization policies. We present a two-period model in which the government can change the degree of privatization twice and has an incentive to raise revenue from selling shares in the public firm due to the shadow cost of public funding (i.e., the excess burden of taxation).[2]

First, we consider the case in which the government can commit to the entire plan of privatization policies at the beginning of the game as a benchmark. Because we assume the demand and cost structure does not change over periods, the optimal policy also does not change over periods.

Next, we consider the case wherein the government chooses the degree of privatization at the beginning of each period, period 1 and period 2. We find that the government has an incentive to change the degree of privatization in period 2. In period 2, the government has a distorted incentive to raise the profit of the public firm because it sells some share of the public firm in period 1 and obtains a higher profit from this firm only partially. Therefore, the government has a weaker incentive to raise the public firm's profit in period 2, which yields a change in the degree of privatization over time. Moreover, the government chooses a degree of privatization in period 1 that is lower than the optimal level to reduce future distortion in the privatization policy.

Whether it increases or decreases the degree of privatization depends on the number of private competitors (i.e., the competitiveness of the market) and their nationality (i.e., foreign penetration in the domestic market). If the private competitors are domestic and the number of firms is large, the government sells additional shares in the public firm over time. As De Fraja and Delbono (1989) showed, when the private competitors are domestic and the number of them is larger, the optimal degree of privatization is higher without excess burden of taxation. Therefore, the government chooses a degree of privatization that is higher than the profit-maximizing one. This accelerates the distortion mentioned above, and thus the government chooses a further higher degree of privatization in period 2.

[2]See Meade (1944) and Laffont and Tirole (1986) for more detail on this concept. Without the shadow cost of public funding, a welfare-maximizing government has no incentive to raise the stock price for the public firm because a lower stock price reduces the government's surplus, but increases investors' surplus. By contrast, with the shadow cost of public funding, the government has an incentive to maintain a high stock price for the public firm because it reduces the excess burden of taxation in other markets. Thus, we introduce the shadow cost of public funding in our analysis. Governments often try to sell shares of public firms at higher prices. For example, the Japanese government recently postponed the additional stock sale of Japan Post because it determined that the stock price of Japan Post was too low. For discussions of the shadow cost of public funding in mixed oligopolies, see Capuano and De Feo (2010) and Matsumura and Tomaru (2013, 2015)

On the contrary, if the private competitors are foreign, the government is likely to have an incentive to renationalize the public firm. We explain the intuition. When the competitors are foreign firms, the government has a stronger incentive to reduce the price because a lower price restricts surplus outflow to foreign investors. because of this expected aggressive behavior of the public firm, the share price of the public firm falls. To raise the share price, the government sells larger shares of the public firm in period 1 and ensures less aggressive behavior of the public firm. In period 2, the government has a distorted incentive as mentioned above, and thus it increases the public ownership share in period 2 in order to restrict the surplus outflow to foreign investors.

Finally, we show that when the optimal degree of privatization is full privatization or full nationalization, the government can implement that policy. When the optimal degree of privatization is full nationalization, there is no distortion in period 2 and thus the government can choose full nationalization in both periods. The optimal degree of privatization is full privatization only when the welfare gain of privatization is sufficiently large, and thus, the government chooses full privatization in both periods.

These results have some policy implications. First, an intertemporal adjustment in the degree of privatization may not be the optimal policy for the government, and thus committing to not changing the degree of privatization may improve welfare. Second, if the optimal policy is either full privatization or full nationalization, there is no need to restrict the government's behavior. Third, if it is impossible to commit to a future privatization policy and the static optimal privatization is neither full privatization nor full nationalization, the government should choose the lower degree of privatization in the early stage of privatization to mitigate a future distortion of the privatization policy.

4.2 The Model

Consider a two-period model in which one domestic state-owned public firm, firm 0, competes against n private firms. Each period is indexed by $t(=1,2)$. We assume that every agent has the same discount factor $\delta \in (0,1)$.

At the beginning of the game, the government owns all the shares in firm 0 and sells them over two periods. The government sells α_1 shares at the beginning of period 1 and $\alpha_2 - \alpha_1$ shares at the beginning of period 2. We assume that the investors of firm 0 are domestic.[3] α_t is a measure of the degree of privatization in period t. If $\alpha_2 - \alpha_1 < 0$, this implies that the government buys back the shares in firm 0 and renationalizes it.

Following the standard formulation in the literature on mixed oligopolies formulated by Matsumura (1998), we assume that firm 0 maximizes the weighted average of social welfare (discounted sum of social surplus over two periods) and its own profit (discounted sum of profits over two periods) and that the weight depends on α_t, whereas private firms maximize their own profits (discounted

[3] The assumption that the investors for privatized firms are domestic is standard in the literature (Cato and Matsumura (2012); Lee et al. (2018); Xu et al. (2017)), and might be realistic. For example, the foreign ownership share in Postal Bank among private ownership is about one-fifth of the Mitsubishi UFJ Financial Group. If the investors of firm 0 are foreign, the time-inconsistency problem discussed below becomes more serious, and thus the equilibrium degree of privatization more likely departs from the optimal degree of privatization. For discussions on foreign investors for privatized firms in a static model, see Lin and Matsumura (2012)

sum of profits over two periods). Let W_t denote domestic social surplus and $\pi_{i,t}$ denote firm i's profit in period t.

In each period, firms produce perfectly substitutable commodities for which the stationary inverse demand function is denoted by $p_t = p(Q_t)$, where p_t is the price and Q_t is the total output in period t. We assume that the function p is twice continuously differentiable and $p' < 0$ as long as $p > 0$. Firm 0's cost function is $c_0(q_{0,t})$, where $q_{0,t}$ is the output of firm 0 in period t. Each private firm i $(= 1, \ldots, n)$ has an identical cost function, $c(q_{i,t})$, where $q_{i,t}$ is the output of private firm i in period t and $c(q_{i,t})$ is the cost.[4] We assume that the functions c_0 and c are twice continuously differentiable as well as the interior solution in the output competition stages.

The profit of firm 0 in period t is given by $\pi_{0,t} = p(Q_t)q_{0,t} - c_0(q_{0,t})$ and that of firm i $(= 1, \ldots, n)$ in period t is given by $\pi_{i,t} = p(Q_t)q_{i,t} - c(q_{i,t})$. Domestic social surplus in period t is defined as

$$W_t = \int_0^{Q_t} p(q)dq - p(Q_t)Q_t + \pi_{0,t} + (1-\theta)\sum_{i=1}^{n} \pi_{i,t} + \lambda(D_t + R_t), \tag{4.1}$$

where $\lambda > 0$ is the additional social cost of public funding,[5] D_t is the revenue from firm 0's dividends, R_t is the revenue from privatization, and θ is the foreign ownership share in private firms. Private firms are foreign (domestic) when $\theta = 1$ $(\theta = 0)$.[6] The social cost of public funding is the deadweight loss from collecting a unit of tax (i.e., the excess burden of taxation). Thus, the government's revenue from firm 0 yields a λ welfare gain because it saves the excess burden of taxation in other markets.[7] We assume that $\lambda < 1$ for the tractability of our analysis.[8]

We assume that the financial market is perfect. That is, the government sells its shares in firm 0 at the fair value of the firm. The fair value of firm 0 in period 1, V_1, is equal to $\pi_{0,1} + \delta\pi_{0,2}$ and that in period 2, V_2, is equal to $\pi_{0,2}$. Therefore, at the beginning of period 1 (2), the government obtains $R_1 = \alpha_1 V_1$ $(R_2 = (\alpha_2 - \alpha_1)V_2)$. In addition, at the end of period t, the government obtains $D_t = (1-\alpha_t)\pi_{0,t}$.

The game is the following four-stage complete information game over two periods. Period 1 contains the first and second stages, and period 2 contains the third and fourth stages. In the first stage, the government chooses $\alpha_1 \in [0, 1]$. Domestic investors who are price takers with a rational expectation purchase α_1 share at the fair value of firm 0, and thus, R_1 is determined. In the second stage, each firm i chooses $q_{i,1} \in [0, \infty)$ independently. In the third stage, the government chooses

[4]In this study, we allow a cost difference between public and private firms, although we do not allow a cost difference among private firms. While some readers might think that the public firm must be less efficient than the private firm, not all empirical studies support this view. See Megginson and Netter (2001) and Stiglitz (1988). In addition, Martin and Parker (1997) suggested that corporate performance can either increase or decrease after privatization, based on their study in the United Kingdom. See Matsumura and Matsushima (2004) for a theoretical discussion of the endogenous cost differences between public and private enterprises.

[5]$(1+\lambda)$ is the so called marginal cost of public funding (MCF).

[6]For discussions on the nationality of private enterprises in mixed oligopolies, see the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). See also Pal and White (1998) and Bárcena-Ruiz and Garzón (2005a,b), Lee, Xu and Chen (2013), and Xu, Lee and Wang (2016)

[7]See Matsumura and Tomaru (2013). Introducing the shadow cost of public funding λ is popular in many contexts, as used by the studies listed in footnote 2.

[8]According to Laffont (2005), λ is estimated to be around 0.3 in developed countries and thus this assumption is realistic in a developed country setting.

$\alpha_2 \in [0,1]$. Domestic investors who are price takers with a rational expectation purchase $\alpha_2 - \alpha_1$ share at the fair value of firm 0, and thus, R_2 is determined. In the final stage, each firm i chooses $q_{i,2} \in [0,\infty)$ independently.

4.3 Equilibrium Analysis and Results

We solve the game by backward induction. In the second stage in each period, each firm chooses its output simultaneously. Note that at the beginning of period t, the government has already sold firm 0's shares. Therefore, when firm 0 chooses $q_{0,t}$, R_t is given exogenously. In period 1, firm 0 maximizes $(1-\alpha_1)(W_1 + \delta W_2) + \alpha_1(\pi_{0,1} + \delta\pi_{0,2})$. However, because neither W_2 nor $\pi_{0,2}$ is affected by $q_{0,1}$, it chooses $q_{0,1}$ to maximize $(1-\alpha_1)W_1 + \alpha_1\pi_{0,1}$. In period 2, firm 0 chooses $q_{0,2}$ to maximize $(1-\alpha_2)W_2 + \alpha_2\pi_{0,2}$. By substituting $D_t = (1-\alpha_t)\pi_{0,t}$ into (4.1), we obtain the payoff of firm 0. The first-order condition of firm 0 in period t is

$$(1+(1-\alpha_t)^2\lambda)p+(1-(1-\alpha_t)(1-\theta)+(1-\alpha_t)^2\lambda)p'q_{0,t}-(1+(1-\alpha_t)^2\lambda)c_0'-(1-\alpha_t)\theta p'Q_t = 0. \quad (4.2)$$

The first-order condition of private firm i ($i = 1,\dots,n$) is

$$p + p'q_{i,t} - c' = 0. \quad (4.3)$$

We assume that the second-order conditions,

$$(1+(1-\alpha_t)^2\lambda)(p''q_{0,t} + 2p' - c_0'') + (1-\alpha_t)(-p' - \theta p''Q_t - (1-\theta)p''q_{0,t}) < 0 \quad (4.4)$$

and

$$2p' + p''q_{i,t} - c'' < 0 \quad (4.5)$$

are satisfied. A sufficient but not necessary condition is that c_0'' and c'' are sufficiently large. We also assume

$$p' + p''q < 0. \quad (4.6)$$

This assumption implies that the strategies of private firms in the quantity competition stage are strategic substitutes.[9] A sufficient but not necessary condition is $p'' \le 0$. These are standard assumptions in the literature.

Henceforth, we focus on the symmetric equilibrium wherein all private firms produce the same output level q (i.e., $q_{i,t} = q_{j,t} = q_t$ for all $i, j = 1,\dots,n$). Solving equations (4.2), (4.3), and the following equation (4.7) leads to the equilibrium outputs in the second and fourth stage, given α_t and n:

$$Q_t = q_{0,t} + nq_t. \quad (4.7)$$

Let $q_0(\alpha_t)$, $q(\alpha_t)$, and $Q(\alpha_t) := q_0(\alpha_t) + nq(\alpha_t)$ be the equilibrium output of firm 0, that of each private firm, and the equilibrium total output given α_t. Note that these functions are not affected by time index t except for the effect through α_t.

[9] We do not assume that the strategy of the public firm is a strategic substitute because it may be a strategic complement under plausible assumptions when private firms are foreign. See Matsumura (2003).

Lemma 1. *$q_0(\alpha)$ and $Q(\alpha)$ are decreasing in α, and $q(\alpha)$ is increasing in α.*

Proof. In Section 4.5. □

Lemma 1 is intuitive and indicates the standard results in the literature. Thus, we omit the formal proof. A decrease in α makes the public firm, firm 0, more aggressive because it is more concerned about the consumer surplus. Although the objective of each private firm is not related to α, a decrease in α reduces the output of each private firm through the strategic interaction. Note that private firms' strategies are strategic substitutes. Then, the first direct effect dominates the second indirect strategic effect and thus a decrease in α increases the total output.

Before solving the two-period game formulated above, we consider the game in which the government chooses the degree of privatization in both periods, α_1 and α_2, in period 1 as a benchmark. In other words, in this benchmark game, the government can commit to α_2 in period 1. The government's problem is

$$\max_{\alpha_1,\alpha_2} \sum_{t=1}^{2} \delta^{t-1} \left(\int_0^{Q_t} p(q)dq - p(Q_t)Q_t + (1+\lambda)\pi_{0,t} + (1-\theta)\sum_{i=1}^{n} \pi_{i,t} \right).$$

Let α_1^{**} and α_2^{**} be the solutions to this problem. Because of the time invariance property of our model formulation, the first-order conditions for α_1 and α_2 are common and the common first-order condition is

$$\left(\frac{dq_0}{d\alpha}\right)(-p'Q+(1+\lambda)(p+p'q_0-c_0')+(1-\theta)np'q)+n\left(\frac{dq}{d\alpha}\right)p'(\lambda q_0-(\theta(Q-q_0)+(1-\theta)q)) = 0. \quad (4.8)$$

We assume that the second-order condition is satisfied.

We present a result on this optimal privatization policy.

Lemma 2. *(i) $\alpha_1^{**} = \alpha_2^{**}$. (ii) $\alpha_1^{**} = \alpha_2^{**} := \alpha^{**} = 0$ if and only if $\theta(Q(0) - q_0(0)) + (1-\theta)q(0) - \lambda q_0(0) \le 0$. (iii) $\alpha^{**} < 1$ if $\theta = 1$ or $c_0(q) = c(q)$ for all q.*

Proof. In Section 4.5. □

Lemma 2(i) implies that the ex-post change in the degree of privatization in period 2 is undesirable from the welfare viewpoint. The discounting factor does not affect the benchmark case in (8), which implies that the committed privatization policy is optimal in both periods. Thus, committing not to change the degree of privatization improves welfare. However, this commitment may be difficult. The government may commit to not reducing public ownership in the future by enacting a law with a minimal public ownership share obligation. For example, the Japanese government must hold more than one-third of the shares in NTT and JT by law. However, there was a rule forcing the Japanese government to hold a two-thirds share in JT until 2012, which was subsequently reduced to one-third. Because the government can change the law, committing to not changing the public ownership share in the future is hard to implement.

We now solve the original privatization game over two periods. In the third stage (the first stage in period 1), the government chooses α_2 to maximize W_2. By substituting $R_2 = (\alpha_2 - \alpha_1)\pi_{0,2}$ and $D_2 = (1 - \alpha_2)\pi_{0,2}$, we obtain the following first-order-condition for the interior solution:

$$\begin{aligned}\frac{dW_2}{d\alpha_2} \quad = \quad & \Big(\frac{dq_0}{d\alpha}\Big)(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q) \\ & +n\Big(\frac{dq}{d\alpha}\Big)(-\theta(p'Q - p'q_0) - (1-\theta)p'q + \lambda p'q_0) \\ & -\lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0\right) = 0. \qquad (4.9)\end{aligned}$$

From (4.9), we see that the equilibrium α_2 of this subgame depends on α_1. Let $\alpha_2(\alpha_1)$ be the equilibrium degree of privatization in period 2 in this subgame. Substituting $\alpha_1 = 0$ into (4.9) and comparing it with (4.8) yield the following Lemma 3.

Lemma 3. $\alpha_2^*(0) = \alpha^{**}$.

Lemma 3 implies that if the government holds all the shares of firm 0 at the beginning of period 2, there is no distortion. However, a positive degree of privatization in period 1 distorts the privatization policy in period 2. At the beginning of period 2, the government holds $(1 - \alpha_1)$ shares in firm 0. A one-unit increase in the profit of firm 0 in period 2 increases welfare by $(1 - \alpha_1)(1 + \lambda)$ units in period 2. Therefore, an increase in α_1 reduces the government's incentive to raise the profit in firm 0 at the cost of consumer surplus (i.e., the government puts a larger weight on the consumer surplus and puts a smaller weight on firm 0's profit). This yields the deviation of α_2 from α^{**} in period 2.

In the first stage, the government maximizes $W_1 + \delta W_2$ with respect to α_1. By substituting $R_1 = \alpha_1(\pi_{0,1} + \delta\pi_{0,2})$, $R_2 = (\alpha_2 - \alpha_1)\pi_{0,2}$, and $D_t = (1 - \alpha_t)\pi_{0,t}$ into it, we obtain the following first-order-condition for the interior solution:

$$\begin{aligned}\frac{d(W_1 + \delta W_2)}{d\alpha_1} \quad = \quad & \Bigg[\Big(\frac{dq_0}{d\alpha}\Big)(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q) \\ & +n\Big(\frac{dq}{d\alpha}\Big)p'(\lambda q_0 - (\theta(Q - q_0) + (1-\theta)q))\Bigg]_{\alpha=\alpha_1} \\ & +\delta\left[\frac{d\alpha_2}{d\alpha_1}\lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0\right)\right]_{\alpha=\alpha_2(\alpha_1)} = 0. \qquad (4.10)\end{aligned}$$

We assume that the second-order condition is satisfied. Let α_1^* be the equilibrium degree of privatization in period 1. Define $\alpha_2^* := \alpha_2(\alpha_1^*)$.

In the static model (the model without period 2), the government chooses $\alpha_1 = \alpha^{**}$. However, a positive α_1 distorts its privatization policy in period 2. An increase in α_1 makes the government place less emphasis on the profit of firm 0 in period 2, and thus the profit of firm 0 decreases because of the distorted choice of α_2. Investors expect this privatization policy in period 2 and the stock price is then lower than that when the government can commit to the privatization policy in the future, thereby reducing the government's revenue from the privatization. To reduce the welfare loss from a future deviation from the optimal privatization policy, the government chooses a lower degree of privatization than the optimal one (α^{**}). This yields our main result, Proposition 1(i).

Proposition 1. *(i) $\alpha_1^* \leq \alpha^{**}$. (ii) $\alpha_1^* = 0$ if and only if $\alpha^{**} = 0$. (iii) $\alpha_1^* = 1$ if and only if $\alpha^{**} = 1$. (iv) $\alpha_1^* = \alpha^{**} = 0$ if and only if $\theta(Q(0) - q_0(0)) + (1-\theta)q(0) - \lambda q_0(0) \leq 0$. (v) $\alpha_1^* < 1$ if $\theta = 1$ or $c_0(q) = c(q)$ for all q. (vi) α_1^* is decreasing in δ if $(\alpha_1^*, \alpha_2^*) \in (0,1)^2$. (vii) $|\alpha_2^* - \alpha^{**}|$ is decreasing in δ if $(\alpha_1^*, \alpha_2^*) \in (0,1)^2$.*

Proof. In Section 4.5. □

Proposition 1(i) states that the government, when it has the opportunity to adjust the degree of privatization in the future, may choose a lower degree of privatization but never a degree larger than the optimal one. For example, the Japanese government sold small stakes in the Japan Postal Bank, Kampo, NTT, and Japan Rail (JR) East, West, and Central during the first stage of privatization, and Proposition 1(i) may thus explain these actions well.

We now briefly discuss the other results in Proposition 1. Proposition 1(ii) states that the equilibrium degree of privatization is strictly positive if the optimal degree of privatization is positive. On one hand, a marginal increase in α_1 from zero yields the distortion in period 2; however, this is by the second order because of the envelope theorem. When $\alpha_1 = 0$, the government chooses the optimal α_2 at the equilibrium and there is no distortion. On the other hand, the welfare-improving effect of an increase in α_1 from zero has a first-order effect as long as $\alpha^{**} > 0$. Therefore, $\alpha_1^* > 0$ as long as $\alpha^{**} > 0$.

Matsumura (1998) investigated the case with $\theta = \lambda = 0$ and showed that the full nationalization of the public firm is never optimal unless full nationalization yields a public monopoly. Proposition 1(iv) states that full nationalization can be optimal if λ is large, even when $\theta = 0$. An increase in α reduces the output of the public firm and increases that of private firms, while it may also decrease the profit of the public firm. When λ is large, it may harm welfare. This is why full nationalization can be optimal.

Proposition 1(iii) states that if $\alpha^{**} = 1$, the government sells all of its shares during the first stage of privatization. In period 2, the government's choice of α_2 is more biased toward lowering the public firm's profit than under the optimal policy. However, a marginal decrease in the degree of privatization from one always increases profit through the strategic effect (Fershtman and Judd, 1987). Thus, when the optimal degree of the privatization is one, the government chooses $\alpha_2 = 1$. Since there is no distortion in period 2, the government optimally chooses $\alpha_1 = 1$ in period 1. Indeed, the Japanese government sold all of its shares in the state-owned J-Power in 2004, which may support our result.

Proposition 1(v) states that if the private competitors are foreign, or the public firm is as efficient as private firms, it is never optimal to fully privatize the public firm. Note that $\theta = 1$ or $c_0(q) = c(q)$ is a sufficient, but very far from necessary condition for $\alpha_1^* < 1$. Even if $\theta < 1$ and/or the public firm is less efficient than private firms, α_1^* can be strictly lower than one.

Proposition 1(vi) states that the equilibrium degree of privatization in period 1 is decreasing in δ as long as the solution is interior. The larger δ is, the more important is social welfare in period 2. The larger α_1 is, the more distorted is the government's choice in period 2. Therefore, when α is larger, the government chooses smaller α_1 to reduce this distortion.

Proposition 1(vii) states that α_2^* is closer to α^{**} when δ is larger. As mentioned above, the larger δ is, the government chooses smaller α_1^* to avoid distortion in period 2. As a result, the equilibrium degree of privatization in period 2 is closer to the optimal degree of privatization when δ is larger.

From Lemma 3 and Proposition 1(ii), we observe that if $\alpha^{**} = 0$, then $\alpha_1^* = 0$ and $\alpha_2^* = \alpha_2(0) = \alpha^{**} = 0$. This means that if the optimal degree of privatization is zero in the static model, there is no additional distortion in the dynamic model. However, when $\alpha^{**} > 0$, the privatization policy in period 2 is distorted. As discussed above, the government places less weight on the profit of firm 0 in period 2, and the resulting profit of firm 2 drops below the optimal level. Under these conditions, if an increase in α decreases (increases) the profit of the public firm, the government chooses too high (low) a degree of privatization in period 2.

We speculate that whether an increase in α decreases the profit of the public firm depends on the foreign ownership share in private firms and number of private firms. When the foreign ownership share in private firms is high, the aggressive behavior of firm 0 is more beneficial. Therefore, in the static model, the government chooses a lower degree of privatization (Lin and Matsumura, 2012), and the public firm is more aggressive than profit-maximizing at the equilibrium. Under these conditions, firm 0's profit may be increasing in α, and the government has an incentive to reduce α in period 2 to raise the consumer surplus at the cost of firm 0's profit. However, the profit motive for the government is weaker in period 2.

Suppose that private firms are domestic. The government is concerned about both the profits of these private firms and the consumer surplus as well as firm 0's profit. An increase in α decreases q_0 and increases q_i for $i = 1, 2, ..., n$. Because firm 0's price-cost margin is lower than each private firm's as long as $\alpha < 1$, the above production substitution from the public to private firms improves welfare.[10] When this welfare-improving production substitution effect is strong, the government chooses α above the profit-maximizing level.[11] The welfare effect is stronger when n is larger. Thus, we naturally expect that the government reduces the public ownership share in firm 0 in period 2 when n is larger.

Unfortunately, we fail to derive clear results or prove the above speculation under general demand and cost functions. Hence, we adopt linear demand and common quadratic cost functions ($p(Q) = a - Q$ and $c_0(q) - c(q) = q^2/2$), which are popular in the literature on mixed oligopolies to present a clear-cut result.[12]

[10] For a discussion of welfare-improving production substitution, see Lahiri and Ono (1988). See Matsumura (1998) in the context of mixed oligopolies.

[11] The profit-maximizing level of α is not one because of the strategic effect discussed in the literature on delegation games. See Vickers (1985).

[12] The pioneering work of De Fraja and Delbono (1989) adopted this setting. See also Matsumura and Shimizu (2010) and the works cited therein.

Proposition 2. *Suppose that $p(Q) = a - Q$ and $c_0(q) = c(q) = q^2/2$. (i) $\alpha_2^* > \alpha^{**}$ if and only if*

$$\theta < \theta(n) := \frac{n^2 - 8}{3n(n+4)},$$

and $\theta(n)$ is increasing in n. (ii) $\alpha_1^* = \alpha_2^* = \alpha^{**} = 0$ if and only if $g(n, \lambda, \theta) := (n-1)\theta(2+\lambda) + 2(1-\lambda^2) - \lambda\theta - n\theta^2 \leq 0$. (iii) $g(n, \lambda, \theta) \leq 0$ only if $n < 2$, and $g(n, \lambda, \theta)$ is decreasing in both λ and θ for $n < 2$.

Proof. In Section 4.5. □

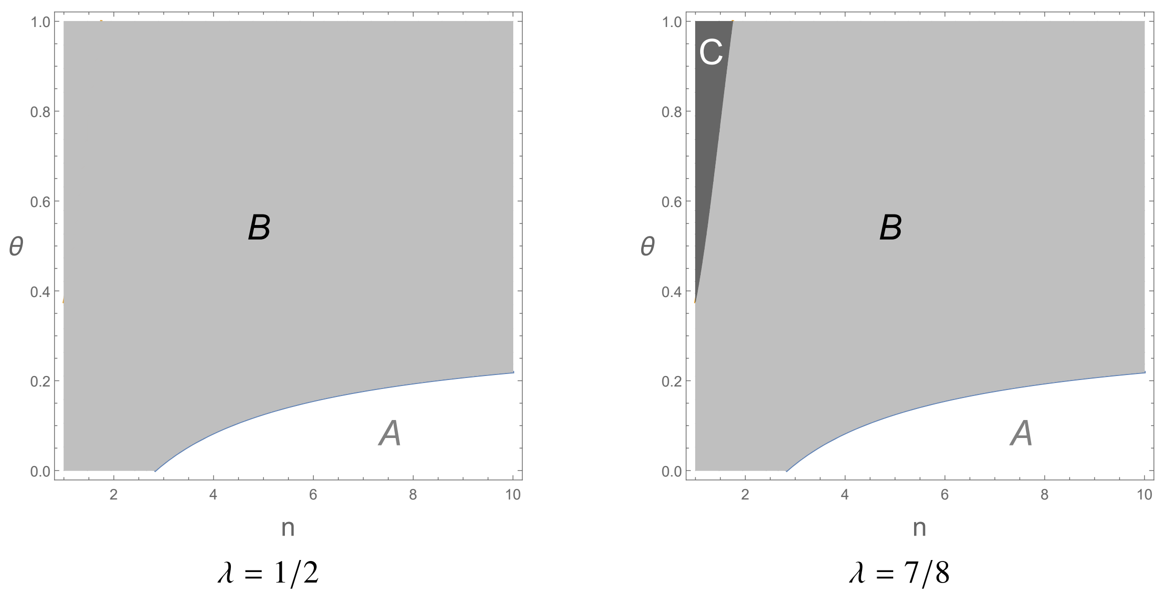


Figure 4.1: The comparison between α_2^* and α^{**}. The left figure is the case where $\lambda = 1/2$, and the right figure is the case where $\lambda = 7/8$. The region A shows the area where $\alpha_2^* > \alpha^{**}$, the region B shows the area where $\alpha_2^* < \alpha^{**}$, and the region C shows the area where $\alpha_2^* = \alpha^{**} = 0$.

Figure 1 indicates that over-privatization ($\alpha_2^* > \alpha^{**}$) is more likely to take place when the number of private firms is high and the foreign ownership share in private firms is low. Furthermore, as the right-hand figure in Figure 1 shows, full nationalization may be realized under duopoly when λ and θ are large.

In regions A and B, $\alpha_1^* < \alpha^{**}$. In region C as well as on the border between region A and region B, $\alpha_1^* = \alpha^{**}$. Thus, this example illustrates that for a fairly wide range of parameters, the government chooses a degree of privatization below the optimal one during the early stage of privatization to reduce distortion in the later stage.

Proposition 2 also suggests the possible danger of using a duopoly model in which one public firm competes against one private firm. If we consider a duopoly model, we may conclude that over-privatization never takes place. However, Proposition 2(i) suggests that this is possible if the number of private firms exceeds three. Moreover, if we consider only a duopoly model, we may conclude that full nationalization takes place for a reasonable value of λ. However, Proposition 2(ii) states that full nationalization does not take place if the number of private firms exceeds two. Proposition 2 thus suggests that we should carefully check the robustness of the results in mixed oligopolies.

We now discuss another implication of Proposition 2. Suppose that $\alpha^{**} > 0$. Because $\alpha_1^* < \alpha^{**}$ (Proposition 1(i)), $\alpha_2^* > \alpha^{**}$ is a sufficient (but not necessary) condition for $\alpha_1^* < \alpha_2^*$. Thus, Proposition 2 implies the following corollary.

Corollary 1. *Under the linear demand and quadratic cost specified in Proposition 2, $\alpha_2^* > \alpha_1^*$ if $\theta < \theta(n)$.*

As discussed in the Introduction, the Japanese government has often increased the degree of privatization over time, such as in the cases of the Japan Postal Bank, Kampo, NTT, and JT. Our result may explain such a gradual privatization process over time.

We present a result including the welfare implications of full privatization. In the model discussed in Proposition 2, $\alpha^{**} < 1$ (Proposition 1(v)). However, under more general cost conditions, $\alpha^{**} = 1$ can hold. We find that once the government chooses full privatization in period 1, it never renationalizes in period 2. However, the government chooses full privatization in period 2 even when $\alpha^{**} < 1$.

Proposition 3. *(i) If $\alpha^{**} = 1$, then $\alpha_1^* = \alpha_2^* = 1$. (ii) Even if $\alpha^{**} < 1$, α_2^* can be one.*

Proof. In Section 4.5. □

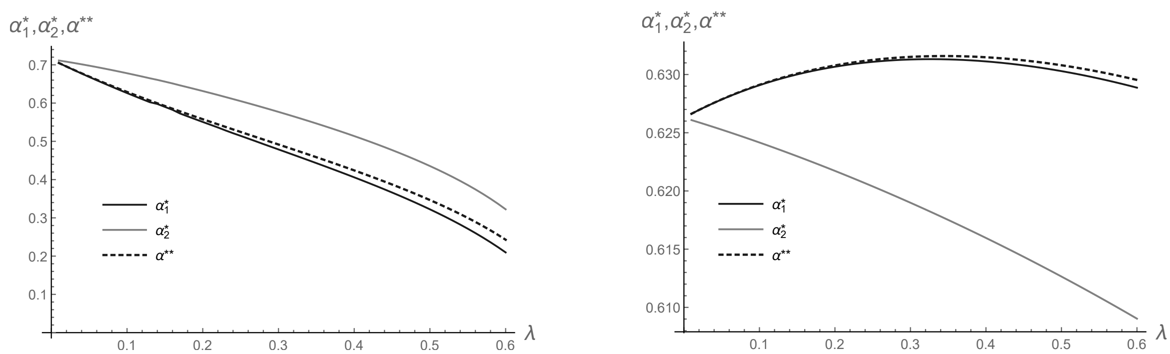
Proposition 3(i) states that if full privatization is optimal, the government fully privatizes the public firm during the early stage (in period 1) and never renationalizes it in the later stage (in period 2). When the public firm is fully privatized in period 1, a marginal reduction in α in period 2 makes the public firm more aggressive and this increases firm 0's profit.[13] Thus, if $\alpha_1 = 1$ is optimal, then $\alpha_2 = 1$ must be optimal because the government's profit motive is weaker in period 2.

Indeed, the renationalization of firms that are fully privatized is exceptional. For example, the Japanese government fully privatized Japan Airlines (JAL) in 1987, KDDI in 1998, J-Power and JR East in 2002, JR West in 2004, JR Central in 2006, and JR Kyushu in 2016. Similarly, the Korean government fully privatized Korean Air in 1969 and POSCO in 2000. Except for JAL, these firms were not renationalized, even partially. Although JAL was renationalized in 2010 because it faced bankruptcy, the government fully privatized it again in 2012.

[13] A decrease in α increases q_0 and decreases q_i for $(i = 1, 2, ..., n)$. The former reduces firm 0's profit but $\alpha = 1$ by the second order (from the envelope theorem). The latter increases firm 0's profit by the first order. Therefore, the latter effect dominates the former.

Our result, however, depends on the stationarity assumption. In other words, the inverse demand function, cost functions, and number of private firms do not change over time. If these conditions do change over time, the optimal policy also changes over time. Further, political reasons explain why the degree of privatization changes over time. For example, if the ruling party changes from right to left, it is plausible that fully privatized firms might be renationalized.[14] Our result suggests that without such changes in circumstances, the renationalization of fully privatized firms is exceptional, although the public ownership share in partially privatized firms may often be adjusted.

Proposition 3(ii) states that even if the government fully privatizes a firm, this does not imply that full privatization is optimal. As discussed above, the government has a distorted incentive to sell its remaining shares in a partially privatized firm but this may harm welfare.



$\theta = 0$ $\theta = 1/4$

Figure 4.2: The comparison among α_1^*, α_2^*, and α^{**} with varying λ when $n = 10$. The left figure is the case where $\theta = 0$, and the right figure is the case where $\theta = 1/4$.

Finally, we discuss how λ affects α_1^*, α_2^*, and α^{**}. Even under the linear demand and quadratic cost specified in Proposition 2, it is difficult to derive α_1^*, α_2^*, and α^{**} explicitly. Thus, we present numerical results.

Figure 2 plots the loci of α_1^*, α_2^*, and α^{**} with varying λ. As Lemma 4 states, $\alpha_2^* > \alpha_1^*$ if θ is small. In the case where $\theta = 0$, $\alpha_2^* > \alpha^{**} > \alpha_1^*$ holds, and all of these variables are decreasing in λ. We explain the intuition. Given α, an increase in λ reduces firm 0's output because firm 0 has stronger profit-motivation. This lesser aggressive behavior of firm 0 reduces its resulting profit. In order to keep firm 0 aggressive, the second-best solution α^{**} becomes smaller, and naturally, both α_1^* and α_2^* also become smaller.

By contrast, in the case where $\theta = 1/4$, a different ranking appears (i.e., $\alpha^{**} > \alpha_1^* > \alpha_2^*$ holds). More interestingly, α^{**} and α_1^* increases with λ when λ is small, while α_2^* decreases with λ, regardless of λ. We explain why α^{**} and α_1^* can increase with λ. Given $\alpha > 0$, firm 0's output is increasing in θ because firm 0 has a stronger incentive to restrict outflow of private firms' profits

[14]From 1945 to 1977, the Labour government in the United Kingdom repeatedly renationalized several major enterprises that had been privatized by the Conservative government. Similar fluctuations in privatization policies have been observed in France, too.

to foreign investors. As we stated, firm 0's output is decreasing in α and λ. The resulting price is lower when θ is larger and α and λ are smaller, and thus the difference between profit-maximizing and equilibrium outputs for firm 0 is also larger. An increase in λ strengthens the profit-motivation of the government. This increases α^{**} to reduce the difference between profit-maximizing and equilibrium outputs for firm 0, which naturally increases α_1^*. This effect increases α_2^*, too (common effect). However, an increase in α_1^* reduces the profit-motive for the government and thus reduces α_2^* (time-inconsistency effect). This time-inconsistency effect dominates the above common effect, and thus, an increase in λ reduces α_2^*.

When λ is large, the equilibrium price is high, and thus the difference between profit-maximizing and equilibrium outputs for firm 0 is small. Therefore, a further reduction of firm 0's output is less likely to increase its profit (the larger is firm 0's output, the more likely will it too to be increase its profits). Under these conditions, a further increase in λ reduces α^{**}, similar to the case in which $\theta = 0$, and it naturally reduces α_1^* and α_2^*.

4.4 Concluding Remarks

In this study, we formulate a two-period model of privatization and investigate the welfare implications of privatization policies across two periods. We find that the government changes its privatization policy over time even when external circumstances (e.g., demand and cost conditions) remain unchanged. In the later stage, the government is less concerned about the public firm's profit as it raises the stock price and increases the revenue from the firm because it has already sold some of its shares in the public firm to the private sector, and this yields the distortion.

We show that whether the government increases or decreases the public ownership share in the public firm depends on the competitiveness of the product market and nationality of private competitors. When private firms are domestic, the government has an ex-post incentive to increase the degree of privatization when the number of private firms is above some threshold. When private firms are foreign, the government has an ex-post incentive to decrease the degree of privatization. These changes in the degree of privatization are undesirable in terms of social welfare. Hence, the desirable policy depends on the number and nationality of private firms. When private firms are domestic and their number is above some threshold, it is important to commit not to further privatize semipublic firms. When private firms are foreign, on the contrary, it is important to commit not to renationalize the public firm.

Let us add one caveat at this point. According to our analysis, an ex-post change in the degree of privatization harms social welfare. This observation depends on the assumption that the environment is fixed. If demand or supply conditions change, an ex-post change in the degree of privatization becomes desirable. Thus, we must also consider whether further privatization is an adaptation to the changing environment or simply opportunistic behavior by the government.

In this study, we consider a two-period model. We assume that similar results hold because the distorted incentive for the government must exist in a general multi-period model. However, even in a two-period model, the problem is quite complicated, and it is quite tough to solve a general n-period model. This remains a promising research topic for future research.[15]

[15] If we consider an infinitely repeated game, further efficient outcome might be achieved by cooperation between

In this study, we assume that firms face quantity competition. As Matsumura and Ogawa (2012) showed, if firms can choose whether they compete on price or quantity, they choose price competition in contrast to the private market (Singh and Vives, 1984).[16] Thus, an analysis of price competition remains for future research.

Moreover, we assume that the number of private firms is given exogenously. Owing to recent deregulation and liberalization, entry restrictions in mixed oligopolies have significantly weakened. As a result, private enterprises have newly entered many mixed oligopolies such as the banking, insurance, telecommunications, and transportation industries and the literature on mixed oligopolies has intensively investigated the optimal privatization policy in free entry markets.[17] However, it is commonly assumed that governments privatize public firms only once. Dynamic privatization policies are important in this context, too. Extending our analysis to free entry markets remains another promising avenue for future research.

4.5 Section

In the following proofs, we suppress the arguments of the functions.

Proof of Lemma 2 (i) This is immediately derived from the time invariance property of our model formulation.
(ii) Define $\mathcal{W}$ by

$$\mathcal{W} := \sum_{t=1}^{2} \delta^{t-1} \left(\int_0^{Q_t} p(q)dq - p(Q_t)Q_t + (1+\lambda)\pi_{0,t} + (1-\theta)\sum_{i=1}^{n} \pi_{i,t} \right) \text{ s.t. } \alpha_1 = \alpha_2 = \alpha$$

We obtain

$$\left.\frac{\partial \mathcal{W}}{\partial \alpha_t}\right|_{\alpha=0} = (1+\delta)\left[n\left(\frac{dq}{d\alpha}\right)(\lambda p' q_0 - (\theta p'(Q-q_0) + (1-\theta)p'q)) \right]_{\alpha=0}. \tag{4.11}$$

$\alpha^{**} = 0$ if (4.11) is nonpositive, and it is nonpositive only if $\theta(Q(0) - q_0(0)) + (1-\theta)q(0) - \lambda q_0(0)$ is nonpositive.
(iii) By substituting $\theta = 1$ into (4.10) and using (4.2), we obtain

$$\left.\frac{\partial \mathcal{W}}{\partial \alpha_t}\right|_{\alpha_t=1} = \left[\left(\frac{dQ}{d\alpha}\right)(-p'Q) + \left(n\frac{dq}{d\alpha}\right)p'(1+\lambda)q_0 \right]_{\alpha=1}. \tag{4.12}$$

Because $dQ/d\alpha < 0$, $dq/d\alpha > 0$ (Lemma 1), $p' < 0$, and $q_0 > 0$, (4.12) is negative and thus $\alpha^{**} < 1$.

public and private firms. However, as Haraguchi and Matsumura (2018) showed, the first best is never achieved in such a game.

[16] For the oligopoly version in mixed oligopolies, see Haraguchi and Matsumura (2016).

[17] See Matsumura and Kanda (2005). For recent discussions on this topic, see Chen (2017) and the works cited therein.

Suppose that $c_0 = c$. Then, $q_0 = q$ at $\alpha = 1$, and thus

$$\left.\frac{\partial \mathcal{W}}{\partial \alpha_t}\right|_{\alpha_t=1} = \left[-\frac{dQ}{d\alpha}(1+n\theta)p'q + n\frac{dq}{d\alpha}(\lambda+\theta)p'q \right]_{\alpha=1} < 0. \tag{4.13}$$

Therefore, $\alpha^{**} < 1$. Q.E.D.

To prove Proposition 1, we present a supplementary lemma.

Lemma 4. *For $\alpha_2(\alpha_1) \in (0,1)$, $\alpha_2'(\alpha_1)$ is positive (negative, zero) if and only if*

$$\left[\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0 \right]_{\alpha=\alpha_2(\alpha_1)}$$

is negative (positive, zero).

Proof of Lemma 5 By applying the implicit function theorem to (4.9), we obtain

$$\frac{d\alpha_2}{d\alpha_1} = -\frac{\frac{\partial^2 W_2}{\partial\alpha_1\partial\alpha_2}}{\frac{\partial^2 W_2}{\partial\alpha_2^2}}. \tag{4.14}$$

Because the denominator in (4.14) is negative, (4.14) is positive (negative, zero) if and only if

$$\frac{\partial^2 W_2}{\partial\alpha_1\partial\alpha_2} = -\lambda\left(\left[\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0 \right]_{\alpha=\alpha_2} \right)$$

is positive (negative, zero). Q.E.D.

Proof of Proposition 1 (i) First, we show that if $\alpha^{**} < 1$, then $d(W_1 + \delta W_2)/d\alpha_1$ is nonpositive at $\alpha_1 = \alpha^{**}$, and thus, $\alpha_1^* \leq \alpha^{**}$.

$$\left.\frac{d(W_1+\delta W_2)}{d\alpha_1}\right|_{\alpha_1=\alpha^{**}} = \Bigg[\left(\frac{dq_0}{d\alpha}\right)(-p'Q + (1+\lambda)(p+p'q_0-c_0') + (1-\theta)np'q) \tag{4.15}$$

$$+n\left(\frac{dq}{d\alpha}\right)p'(\lambda q_0 - (\theta(Q-q_0)+(1-\theta)q))\Bigg]_{\alpha=\alpha^{**}} \tag{4.16}$$

$$+\delta\left[\frac{d\alpha_2}{d\alpha_1}\lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p+p'q_0-c_0'\right) + n\frac{dq}{d\alpha}p'q_0 \right) \right]_{\alpha=\alpha_2(\alpha^{**})} \tag{4.17}$$

Suppose that $\alpha^{**} \in (0,1)$. From (4.8),(4.15)+(4.16) is zero. Lemma 5 implies that (4.17) is nonpositive. Therefore, $d(W_1 + \delta W_2)/d\alpha_1$ is nonpositive at $\alpha_1 = \alpha^{**} \in (0,1)$.

Suppose that $\alpha^{**} = 0$. (4.15)+(4.16) is nonpositive. (4.17) is zero when $\alpha_1 = \alpha^{**} = 0$. Thus, $d(W_1 + \delta W_2)/d\alpha_1$ is nonpositive at $\alpha_1 = \alpha^{**} = 0$.

Finally, we show that if $\alpha^{**} = 1$, then $\alpha_1^* = 1$. First, we show that if $\alpha^{**} = 1$, $\alpha_2(\alpha_1) = 1$ for any $\alpha_1 \in [0,1]$.

$$\frac{dW_2}{d\alpha_2}\Big|_{\alpha_2=1} = \Bigg[\Big(\frac{dq_0}{d\alpha}\Big)(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q) \tag{4.18}$$

$$+ \quad n\Big(\frac{dq}{d\alpha}\Big)(-\theta(p'Q - p'q_0) - (1-\theta)p'q + \lambda p'q_0) \tag{4.19}$$

$$- \quad \lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0\right)\Bigg]_{\alpha=1} \tag{4.20}$$

When $\alpha^{**} = 1$, (4.18)+(4.19) is nonnegative. Furthermore, (4.20) is also nonnegative for any $\alpha \in [0,1]$. Under these conditions, $dW_2/d\alpha_2$ is nonnegative at $\alpha_2 = 1$ for any $\alpha_1 \in [0,1]$, and thus $\alpha_2(\alpha_1) = 1$ for any $\alpha_1 \in [0,1]$.

From the discussion above, we find that $\alpha_2'(\alpha_1) = 0$ for all $\alpha_1 \in [0,1]$ when $\alpha^{**} = 1$. By substituting $d\alpha_2/d\alpha_1 = 0$ into (4.10) and comparing it with (4.8), we obtain $\alpha_1^* = 1$.

(ii) Proposition 1(i) implies that if $\alpha^{**} = 0$, then $\alpha_1^* = 0$. We show that $\alpha_1^* = 0$ only if $\alpha^{**} = 0$.

$$\frac{dW_1 + \delta W_2}{d\alpha}\Big|_{\alpha=0} = \Bigg[n\Big(\frac{dq}{d\alpha}\Big)(\lambda p'q_0 - (\theta p'(Q - q_0) + (1-\theta)p'q))\Bigg]_{\alpha=0}. \tag{4.21}$$

$\alpha_1^* = 0$ if (4.21) is nonpositive and it is nonpositive only if $\theta(Q(0) - q_0(0)) + (1-\theta)q(0) - \lambda q_0(0)$ is nonpositive. From Lemma 2(ii) we find that $\alpha_1^* = 0$ only if $\alpha^{**} = 0$.

(iii) Proposition 1(i) implies that $\alpha_1^* = 1$ only if $\alpha^{**} = 1$ since $\alpha_1^* \le \alpha^{**}$. In the Proof of Proposition 1(i), we showed that if $\alpha^{**} = 1$, then $\alpha_1^* = \alpha_2^* = 1$.

(iv) Lemma 2(ii) and Proposition 1(i) imply Proposition 1(iv).

(v) Lemma 2(iii) and Proposition 1(i) imply Proposition 1(v).

(vi) We show that $d^2(W_1 + \delta W_2)/d\alpha_1 d\delta < 0$, which implies Proposition 1(vi). Since

$$\frac{d^2(W_1 + \delta W_2)}{d\alpha_1 d\delta} = \left[\frac{d\alpha_2}{d\alpha_1}\lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0\right)\right]_{\alpha=\alpha_2(\alpha_1)} < 0,$$

from Lemma 5, we obtain $d^2(W_1 + \delta W_2)/d\alpha_1 d\delta < 0$.

(vii) Suppose that $\alpha_2^* > \alpha^{**}$. Lemma 5 and the equation (4.9) imply that $d\alpha_2/d\alpha_1 < 0$. Proposition 1(vi) states that α_1 is decreasing in δ. Therefore, α_2^* (and thus $\alpha_2^* - \alpha^{**}$) is decreasing in δ.

Suppose that $\alpha_2^* < \alpha^{**}$. Lemma 5 and the equation (4.9) imply that $d\alpha_2/d\alpha_1 \le 0$ and strict inequalities hold if $\alpha_2^* \in (0,1)$. Proposition 1(vi) states that α_1 is nonincreasing in δ and decreasing in δ if $\alpha_2^* \in (0,1)$. Therefore α_2^* (and thus $\alpha_2^* - \alpha^{**}$) is increasing in δ.

These imply Proposition 1(vii). Q.E.D.

Proof of Proposition 2 From the first-order condition of each private firm in the quantity competition stage, we obtain $q = (a - q_0)/(n+2)$ and $Q = (na + 2q_0)/(n+2)$. Thus, we obtain

$$\frac{dq}{d\alpha} = -\frac{1}{n+2}\frac{dq_0}{d\alpha}.$$

From Lemma 2(iii) we obtain $\alpha^{**} \neq 1$. First, we consider the case where $\alpha^{**} \in (0,1)$. Note that $\alpha_1^* > 0$ when $\alpha^{**} > 0$ (Proposition 1(ii)). The first-order condition for the interior solution for α^{**} is

$$\begin{aligned}
\left.\frac{\partial \mathcal{W}}{\partial \alpha_t}\right|_{\alpha_t=\alpha^{**}} &= \left(\frac{dq_0}{d\alpha}\right)(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q) + n\left(\frac{dq}{d\alpha}\right)p'(\lambda q_0 - (\theta(Q - q_0) + (1-\theta)q)) \\
&= \frac{dq_0}{d\alpha}\left(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q - \frac{n}{n+2}p'(\lambda q_0 - (\theta(Q-q_0) + (1-\theta)q))\right) \\
&= \frac{dq_0}{d\alpha}p'\left(-Q - (1+\lambda)(a - Q - 2q_0) + (1-\theta)nq - \frac{n}{n+2}(\lambda q_0 - \theta(Q - q_0) - (1-\theta)q)\right) \\
&= -\frac{dq_0}{d\alpha}\Bigg(-\frac{na + 2q_0}{n+2} - (1+\lambda)\left(\frac{2a - (2n+6)q_0}{n+2}\right) + (1-\theta)\frac{n}{n+2}(a - q_0) \\
&\qquad -\frac{n}{n+2}\left(\lambda q_0 - \theta\frac{n(a-q_0)}{n+2} - (1-\theta)\frac{a-q_0}{n+2}\right)\Bigg) = 0.
\end{aligned}$$

From this, we obtain

$$q_0(\alpha^{**}) = \frac{2(1+\lambda) + \frac{2n}{n+2}\theta - \frac{n}{n+2}(1-\theta)}{(1+\lambda)(n+6) - 2 + \frac{2n}{n+2}\theta - \frac{n}{n+2}(1-\theta)}a.$$

By substituting it into W_2, we obtain

$$\begin{aligned}
\left.\frac{dW_2}{d\alpha_2}\right|_{\alpha_2=\alpha^{**}} &= -\lambda\alpha_1^*\left(\frac{dq_0}{d\alpha}(p + p'q_0 - c_0') + n\frac{dq}{d\alpha}p'q_0\right) \\
&= -\lambda\alpha_1^*\frac{dq_0}{d\alpha}\left(a - Q - 2q_0 + \frac{n}{n+2}q_0\right) \\
&= -\lambda\alpha_1^*\frac{dq_0}{d\alpha}\frac{a}{n+2}\left(2 - (n+6)\frac{2(1+\lambda) + \frac{2n}{n+2}\theta - \frac{n}{n+2}(1-\theta)}{(1+\lambda)(n+6) - 2 + \frac{2n}{n+2}\theta - \frac{n}{n+2}(1-\theta)}\right) \\
&= -\lambda\alpha_1^*\frac{dq_0}{d\alpha}\frac{a}{n+2}\frac{n^2 - 8 - 3n(n+4)\theta}{\left((1+\lambda)(n+6) - 2 + \frac{2n}{n+2}\theta - \frac{n}{n+2}(1-\theta)\right)(n+2)}.
\end{aligned}$$

This is positive if and only if $n^2 - 8 - 3n(n+4)\theta > 0$ (or equivalently $\theta < \theta(n)$). Thus, $\alpha_2^* > \alpha^{**}$ if $\theta < \theta(n)$. $\theta'(n) = 4(5n^2 + 12n + 8)/(3n(n+4))^2 > 0$.

Next, we consider the case in which $\alpha^{**} = 0$. In this case $\alpha_1^{**} = \alpha_2^{**} = 0$. From Lemma 2, we find that $\alpha^{**} = 0$ if and only if $\theta(Q(0) - q_0(0)) + (1-\theta)q(0) - \lambda q_0(0) \leq 0$. This holds if and only if $g(n,\lambda,\theta) := (n-1)\theta(2+\lambda) + 2(1-\lambda^2) - \lambda\theta - n\theta^2 \leq 0$.

Because $\partial g(n,\lambda,\theta)/\partial n = \theta(2+\lambda-\theta) \geq 0$ and $g(2,\lambda,\theta) = 2\theta(1-\theta)+2(1-\lambda^2) > 0$, $g(n,\lambda,\theta) < 0$ only if $n < 2$.

For $n < 2$, we find that $\partial g(n,\lambda,\theta)/\partial\lambda = (n-1)\theta - 4\lambda - \theta < 0$ and that $\partial g(n,\lambda,\theta)/\theta = (n-2)\lambda - 2 < 0$. Q.E.D.

Proof of Proposition 3 (i) We have already shown this in the proof of Proposition 1(i).
(ii)We present an example in which $\alpha_2^* = 1$ and $\alpha^{**} < 1$. Suppose that $\theta = 0$, $p(Q) = a - Q$, $c_0(q) = k_0 q$, $c(q) = kq$ (constant marginal costs), and $k_0 > k$. We normalize $k = 0$. Suppose that $n = 1$ and $\lambda = 1/2$.

In this specification, $q = (a - q_0)/2, Q = q + q_0$, and q_0 satisfies the first-order condition

$$\left(1 + \frac{1}{2}(1-\alpha)^2\right)\frac{a - q_0}{2} - \left(\alpha + \frac{1}{2}(1-\alpha)^2\right) q_0 - \left(1 + \frac{1}{2}(1-\alpha)^2\right) k_0 = 0,$$

which yields

$$q_0(\alpha) = \frac{\left(1 + \frac{1}{2}(1-\alpha)^2\right)\left(\frac{a}{2} - k_0\right)}{\frac{1}{2} + \alpha + \frac{3}{4}(1-\alpha)^2}.$$

By substituting this into W_2, we obtain

$$\begin{aligned}
\left.\frac{dW_2}{d\alpha_2}\right|_{\alpha_2=1} &= \left(\frac{dq_0}{d\alpha}\right)(-p'q_0) + \left(\frac{dq}{d\alpha}\right)(-p'q + \lambda p'q_0) - \lambda\alpha_1\left(\frac{dq}{d\alpha}p'q_0\right) \\
&= \frac{dq_0}{d\alpha}p'\left(-q_0 + \frac{1}{2}\left(q - \lambda q_0 + \lambda\alpha_1 q_0\right)\right) \\
&= \frac{dq_0}{d\alpha}p'\left(\frac{a}{4} - \frac{1}{4}q_0 - q_0 + \frac{1}{2}\left(-\lambda q_0 + \lambda\alpha_1 q_0\right)\right) \\
&= \frac{dq_0}{d\alpha}\frac{p'}{4}\left(a - (6-\alpha_1)\frac{a - 2k_0}{3}\right).
\end{aligned} \tag{4.22}$$

From Lemma 3, we find that $\alpha^{**} = 1$ if and only if (4.22) at $\alpha_1 = 0$ is nonnegative. Thus, $\alpha_2^* = 1$ if and only if $k_0 \geq a/4$. $\alpha_2^* = 1$ if and only if (4.22) at $\alpha_1 = \alpha_1^*$ is nonnegative. Therefore, $\alpha_2^* = 1$ if and only if $k_0 \geq y := a(3 - \alpha_1^*)/(12 - 2\alpha_1^*)$.

Suppose that $k_0 = a/4 - \varepsilon$, where ε is positive and sufficiently small. Then $\alpha^{**} < 1$. From Proposition 1, we obtain $\alpha_1^* > 0$. Because y is decreasing in α_1^* and $y = a/4$ when $\alpha_1^* = 0$, $y < a/4$. Therefore, $a/4 - \varepsilon > y$, and thus, $\alpha_2^* = 1$. This example yields $\alpha^{**} < 1$ and $\alpha_2^* = 1$. Q.E.D.

Bibliography

Affeldt, Pauline, Lapo Filistrucchi, and Tobias J. Klein, "Upward Pricing Pressure in Two-sided Markets," *The Economic Journal*, 2013, *123* (572), 505–523.

Anderson, Simon P and Martin Peitz, "Media See-saws: Winners and Losers on Media Platforms," Working Paper 2015.

_ , Andre De Palma, and Jacques-Francois Thisse, *Discrete Choice Theory of Product Differentiation*, MIT press, 1992.

Anderson, Simon P., Øystein Foros, and Hans Jarle Kind, "The Importance of Consumer Multihoming (Joint Purchases) for Market Performance: Mergers and Entry in Media Markets," *Journal of Economics & Management Strategy*, 2019, *28* (1), 125–137.

Armstrong, Mark, "Competition in Two-Sided Markets," *The RAND Journal of Economics*, 2006, *37* (3), 668–691.

Auray, Stéphane, Thomas Mariotti, and Fabien Moizeau, "Dynamic Regulation of Quality," *The RAND Journal of Economics*, 2011, *42* (2), 246–265.

Baranes, Edmond, Thomas Cortade, and Andreea Cosnita-Langlais, "Merger Control on Two-Sided Markets: Is There Need for an Efficiency Defense?," Working Paper 2014.

Bárcena-Ruiz, Juan Carlos and Marıa Begona Garzón, "Economic Integration and Privatisation under Diseconomies of Scale," *European Journal of Political Economy*, 2005, *21* (1), 247–267.

_ and María Begoña Garzón, "International Trade and Strategic Privatization," *Review of Development Economics*, 2005, *9* (4), 502–513.

Cabral, Luís, "Dynamic Price Competition with Network Effects," *The Review of Economic Studies*, 01 2011, *78* (1), 83–111.

Capuano, Carlo and Giuseppe De Feo, "Privatization in Oligopoly: The Impact of the Shadow Cost of Public Funds," *Rivista italiana degli economisti*, 2010, *15* (2), 175–208.

Cato, Susumu and Toshihiro Matsumura, "Long-run Effects of Foreign Penetration on Privatization Policies," *Journal of Institutional and Theoretical Economics JITE*, 2012, *168* (3), 444–454.

— **and** — , “Optimal Privatisation and Trade Policies with Endogenous Market Structure,” *Economic Record*, 2015, *91* (294), 309–323.

Chen, Tai-Liang, “Privatization and Efficiency: A Mixed Oligopoly Approach,” *Journal of Economics*, 2017, *120* (3), 251–268.

Corneo, Giacomo and Olivier Jeanne, “Oligopole Mixte dans un Marche Commun,” *Annales d’Economie et de Statistique*, 1994, pp. 73–90.

Correia-da-Silva, Joao, Bruno Jullien, Yassine Lefouili, and Joana Pinho, “Horizontal Mergers between Multisided Platforms: Insights from Cournot Competition,” *Journal of Economics & Management Strategy*, 2019, *28* (1), 109–124.

Costello, Kenneth W and Ross C Hemphill, “Electric Utilities’ ‘Death Spiral’: Hyperbole or Reality?,” *The Electricity Journal*, 2014, *27* (10), 7–26.

Cunningham, Colleen, Florian Ederer, and Song Ma, “Killer Acquisitions,” Working Paper 2018.

De Fraja, Giovanni and Flavio Delbono, “Alternative Strategies of a Public Enterprise in Oligopoly,” *Oxford Economic Papers*, 1989, *41* (2), 302–311.

Dong, Quan and Juan Carlos Bàrcena-Ruiz, “Privatization and Entry with Switching Costs,” *The Manchester School*, 2017, *85* (4), 491–510.

Evans, David S. and Michael D. Noel, “The Analysis of Mergers that Involve Multisided Platform Businesses,” *Journal of Competition Law & Economics*, 05 2008, *4* (3), 663–695.

Farrel, Joseph and Garth Saloner, “Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation.,” *American Economic Review*, 1986, *76* (5), 940–56.

Farrell, Joseph and Carl Shapiro, “Horizontal Mergers: An Equilibrium Analysis,” *American Economic Review*, 1990, *80* (1), 107–26.

Federico, Giulio, Gregor Langus, and Tommaso Valletti, “Reprint of: Horizontal Mergers and Product Innovation,” *International Journal of Industrial Organization*, 2018, *61*, 590–612.

Fershtman, Chaim and Kenneth L Judd, “Equilibrium Incentives in Oligopoly,” *The American Economic Review*, 1987, pp. 927–940.

Fjell, Kenneth and Debashis Pal, “A Mixed Oligopoly in the Presence of Foreign Private Firms,” *Canadian Journal of economics*, 1996, pp. 737–743.

Fujiwara, Kenji, “Partial Privatization in a Differentiated Mixed Oligopoly,” *Journal of Economics*, 2007, *92* (1), 51–65.

Haraguchi, Junichi and Toshihiro Matsumura, “Cournot–Bertrand Comparison in a Mixed Oligopoly,” *Journal of Economics*, 2016, *117* (2), 117–136.

__ **and** __ , “Government-Leading Welfare-Improving Collusion,” *International Review of Economics & Finance*, 2018, *56*, 363–370.

Ishibashi, Ikuo and Toshihiro Matsumura, “R&D Competition Between Public and Private Sectors,” *European Economic Review*, 2006, *50* (6), 1347–1366.

Ishida, Junichiro and Noriaki Matsushima, “Should Civil Servants be Restricted in Wage Bargaining? A Mixed-Duopoly Approach,” *Journal of Public Economics*, 2009, *93* (3-4), 634–646.

Jeziorski, PrzemysŁaw, “Effects of Mergers in Two-Sided Markets: The US Radio Industry,” *American Economic Journal: Microeconomics*, November 2014, *6* (4), 35–73.

Jullien, Bruno and Yassine Lefouili, “Horizontal Mergers and Innovation,” *Journal of Competition Law & Economics*, 2018, *14* (3), 364–392.

Katz, Michael L and Carl Shapiro, “Network Externalities, Competition, and Compatibility,” *American Economic Review*, 1985, *75* (3), 424–440.

Laffont, Jean-Jacques, *Regulation and Development*, Cambridge University Press, 2005.

__ **and Jean Tirole**, “Using Cost Observation to Regulate Firms,” *Journal of Political Economy*, 1986, *94* (3, Part 1), 614–641.

__ **and** __ , *A Theory of Incentives in Procurement and Regulation*, MIT press, 1993.

Lahiri, Sajal and Yoshiyasu Ono, “Helping Minor Firms Reduces Welfare,” *The Economic Journal*, 1988, *98* (393), 1199–1202.

Lee, Sand-Ho, Lili Xu, and Zhao Chen, “Competitive Privatization and Tariff Policies in an International Mixed Duopoly,” *The Manchester School*, 2013, *81* (5), 763–779.

Lee, Sang-Ho, Toshihiro Matsumura, and Susumu Sato, “An Analysis of Entry-then-Privatization Model: Welfare and Policy Implications,” *Journal of Economics*, 2018, *123* (1), 71–88.

Lin, Ming Hsin and Toshihiro Matsumura, “Presence of Foreign Investors in Privatized Firms and Privatization Policy,” *Journal of Economics*, 2012, *107* (1), 71–80.

Martin, Stephen and David Parker, *The Impact of Privatization: Ownership and Corporate Performance in the United Kingdom*, Routledge, 1997.

Matsumura, Toshihiro, “Partial Privatization in Mixed Duopoly,” *Journal of Public Economics*, 1998, *70* (3), 473–483.

__ , “Stackelberg Mixed Duopoly with a Foreign Competitor,” *Bulletin of Economic Research*, 2003, *55* (3), 275–287.

— **and Akira Ogawa**, "Price versus Quantity in a Mixed Duopoly," *Economics Letters*, 2012, *116* (2), 174–177.

— **and Atsushi Yamagishi**, "Long-run Welfare Effect of Energy Conservation Regulation," *Economics Letters*, 2017, *154*, 64–68.

— **and Daisuke Shimizu**, "Privatization Waves," *The Manchester School*, 2010, *78* (6), 609–625.

— **and Makoto Okamura**, "Competition and Privatization Policies Revisited: the Payoff Interdependence Approach," *Journal of Economics*, 2015, *116* (2), 137–150.

— **and Noriaki Matsushima**, "Endogenous Cost Differentials between Public and Private Enterprises: a Mixed Duopoly Approach," *Economica*, 2004, *71* (284), 671–688.

— **and Osamu Kanda**, "Mixed Oligopoly at Free Entry Markets," *Journal of Economics*, 2005, *84* (1), 27–48.

— **and Yoshihiro Tomaru**, "Mixed Duopoly, Privatization, and Subsidization with Excess Burden of Taxation," *Canadian Journal of Economics*, 2013, *46* (2), 526–554.

— **and** — , "Mixed Duopoly, Location Choice, and Shadow Cost of Public Funds," *Southern Economic Journal*, 2015, *82* (2), 416–429.

Meade, James E, "Price and Output Policy of State Enterprise," *The Economic Journal*, 1944, *54* (215/216), 321–339.

Megginson, William L and Jeffry M Netter, "From State to Market: A Survey of Empirical Studies on Privatization," *Journal of Economic Literature*, 2001, *39* (2), 321–389.

Mermelstein, Ben, Volker Nocke, Mark A Satterthwaite, and Michael D Whinston, "Internal versus external growth in industries with scale economies: A computational model of optimal merger policy," Technical Report, National Bureau of Economic Research 2014.

Nocke, Volker and Michael D. Whinston, "Dynamic Merger Review," *Journal of Political Economy*, 2010, *118* (6), 1200–1251.

— **and** — , "Merger Policy with Merger Choice," *American Economic Review*, 2013, *103* (2), 1006–33.

— **and Nicolas Schutz**, "An Aggregative Games Approach to Merger Analysis in Multiproduct-Firm Oligopoly," Working Paper 24578, National Bureau of Economic Research 2018.

— **and** — , "Multiproduct-Firm Oligopoly: An Aggregative Games Approach," *Econometrica*, 2018, *86* (2), 523–557.

Ocello, Eleonora and Cristina Sjödin, "Digital Markets in EU Merger Control: Key Features and Implications," *Competition Policy International*, 2018.

Ohashi, Hiroshi, "The Role of Network Effects in the US VCR Market, 1978–1986," *Journal of Economics & Management Strategy*, 2003, *12* (4), 447–494.

Pal, Debashis and Mark D White, "Mixed Oligopoly, Privatization, and Strategic Trade Policy," *Southern Economic Journal*, 1998, pp. 264–281.

Rochet, Jean-Charles and Jean Tirole, "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 06 2003, *1* (4), 990–1029.

_ **and** _ , "Two-Sided Markets: a Progress Report," *The RAND Journal of Economics*, 2006, *37* (3), 645–667.

Rysman, Marc, "Competition Between Networks: A Study of the Market for Yellow Pages," *The Review of Economic Studies*, 04 2004, *71* (2), 483–512.

_ , "An Empirical Analysis of Payment Card Usage," *The Journal of Industrial Economics*, 2007, *55* (1), 1–36.

Sato, Susumu, "Monopoly Regulation in the Presence of Consumer Demand-Reduction," *Economics Letters*, 2018, *173*, 61–64.

_ , "Horizontal Mergers in the Presence of Network Externalities," Working Paper 2019.

_ **and Toshihiro Matsumura**, "Dynamic Privatization Policy," *The Manchester School*, 2019, *87* (1), 37–59.

Singh, Nirvikar and Xavier Vives, "Price and Quantity Competition in a Differentiated Duopoly," *RAND Journal of Economics*, 1984, *15* (4), 546–554.

Stiglitz, Joseph E, *Economics of the Public Sector*, New York, Norton, 1988.

Tan, Guofu and Junjie Zhou, "Price Competition in Multi-Sided Markets," Working Paper 2017.

Vickers, John, "Delegation and the Theory of the Firm," *The Economic Journal*, 1985, *95* (Supplement), 138–147.

Vives, Xavier, *Oligopoly Pricing: Old Ideas and New Tools*, MIT press, 2001.

Weyl, E. Glen, "A Price Theory of Multi-Sided Platforms," *American Economic Review*, 2010, *100* (4), 1642–72.

Whinston, Michael D., "Chapter 36 Antitrust Policy toward Horizontal Mergers," in M. Armstrong and R. Porter, eds., *M. Armstrong and R. Porter, eds.*, Vol. 3 of *Handbook of Industrial Organization*, Elsevier, 2007, pp. 2369–2440.

Williamson, Oliver E, "Economies as an Antitrust Defense: The Welfare Tradeoffs," *American Economic Review*, 1968, *58* (1), 18–36.

Xu, Lili, Sang-Ho Lee, and Leonard FS Wang, "Free Trade Agreements and Privatization Policy with an Excess Burden of Taxation," *Japan and the World Economy*, 2016, *37*, 55–64.

_ , _ , and Toshihiro Matsumura, "Ex-ante versus Ex-Post Privatization Policies with Foreign Penetration in Free-Entry Mixed Markets," *International Review of Economics & Finance*, 2017, *50*, 1–7.