論文の内容の要旨

Quantum estimation theory for continuous data
(連続的データの量子推定理論)

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Measurement is a crucial step for the understanding of the world we live in. We struggle to acquire accurate pieces of information from the observation under inevitable noises. Probability theory and statistics have developed various techniques to distinguish signals from noises, while they have revealed fundamental limits on the accuracy which cannot be surpassed due to the inherent stochastic noises.

In quantum mechanics, measurement is a more intricate process than just a probabilistic process. It often does not obey the rules of classical statistics as typified by the uncertainty relation and the violation of Bell's inequality. It can be a bliss, however, since quantum measurement does not necessarily respect the limits set by the classical statistics. This is the case when both the target system and the measurement probes involve quantumness in certain senses. Quantum metrology is a field of study to exploit the benefit of such quantum measurements: With a classical resource, the measurement error is subject to the *standard quantum limit* (SQL) of $O(N^{-1/2})$ where N generically represents the amount of resource. With a quantum resource, it can be lower than the SQL, where the new limit of $O(N^{-1})$ called the *Heisenberg limit* can be reached.

Although the distinction between the SQL and the Heisenberg limit represents a huge impact of quantum measurements, existing theories of quantum metrology does not cover all realistic problems. Continuous data is an example — an important example seen in many situations. To be more concrete, continuously monitored signals vary over time, and images from telescopes and microscopes spread in two-dimensional areas. Interesting phenomena emerge as functional structures in such continuous data as characteristic peaks in the signal or unusual holes in the image.

Such continuous data can be regarded as functions, and can be handled by advanced mathematics. In particular, due to its infinite degrees of freedom, estimation of an unknown function cannot be done without assuming certain regularity. This regularity can be defined in function analysis as a bound on the vector norm of the function, which must appropriately be chosen to perform analytic calculations on the error bound. Theory of signal processing offers important techniques as well, including the wavelet transform that is useful for the edge detection.

In this thesis, we explore the theory of quantum estimation on continuous data. We especially study when and how quantum metrology still benefits for the estimation on systems with unknown functions. Our studies consist of three parts: a preliminary study on multiparameter estimation, a fundamental study on function estimation, and its application to edge detection.

In the preliminary study, we compare the sequential and parallel schemes of multiparameter metrology, which is based on the quantum limits upon general Hamiltonian models derived in the author's master thesis. After a series of operator-algebraic arithmetics, we show the error bounds on these estimation schemes are both $O(md/T)$, where m is the number of parameters, d is the dimension of the base Hilbert space, and T is the total time cost. We note that the total time counts the multiplicity of the parallel scheme, and therefore quantum metrology can be parallelized without any cost except for an overhead factor independent of the size of the Hamiltonian model.

In the fundamental study, we derive the theoretical limits on function estimation with quantum metrology incorporated. We consider an unknown function $\varphi(x)$ in a coherent quantum system as a unitary gate with phase factor $e^{i\varphi(x)}$ varying continuously in space, and the accuracy of the function estimation is measured as the root-mean-square error. We define the regularity of the function by a modified q-Hölder norm, where q is the number that determines the degree of the smoothness. The theoretical limits are derived by reducing the problem to the multiparameter estimation, which we can treat with the results of the preliminary study. The error bound is of $O(N^{-q/(2q+1)})$ with classical scheme of $O(N^{-q/(q+1)})$ with quantum scheme, which can be regarded as the SQL and the Heisenberg limit on function estimation. While they scale slower than the corresponding limits on parameter estimation, the latter can be reproduced from the former by formally taking the smooth limit $q \to \infty$. We have also investigated the methodology to achieve the quantum limits. Notably, one may choose either position- or momentum-localized states for the probes: The local linear smoothing can be used for the localized states; the postselected quantum tomography can be used for the momentum-localized states. Both strategies saturate the SQL or the Heisenberg limit, except that function estimation by momentum-localized states is ensured only for $0 < q \leq 1$.

In the last part of our study, we apply the function estimation to the realistic problem of edge detection. Continuing with the position-dependent phase-shift gate, we consider a modified version of ghost imaging as a tool for edge detection. In this strategy, one lets wave packets undergo the phase-shift gate, when the wavelet transform can be measured by detecting the momentum of the output state. The edge of a fixed lengthscale can be detected as the extrema of the momentum. We have shown that the probability distribution of the momentum is subject to a quasi-Markovian process. By this, the estimation error can be decomposed into two parts, each contributed by the momentum and the position. As a result, the error bounds can be determined from the position-momentum uncertainty relation. Imaging strategy with multi-mode wave packets can be analyzed in the same way, and the Heisenberg-limited edge detection is found to be performed by Gaussian beams with negative quantum correlations in momentum. Notably, although the quantum limits for a single wavelet scale are the same as those of the multiparameter estimation, it is consistent with the function estimation when all wavelet scales are taken into consideration.

Through the studies above, we connect multiparameter quantum metrology to the new theory of the function estimation, and then to its application for the edge detection. These constitute a series of theories on how far quantum mechanics can be exploited for the better estimation of continuous data in the real world.