論文の内容の要旨

論文題目

Quantum metrology with superpositions of macroscopically distinct states

（マクロに異なる量子状態の重ね合わせを用いた量子計測）

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In this thesis, we show that superpositions of macroscopically distinct states are useful in quantum metrology. Quantum metrology is a field in which high-precision measurement of a target parameter is pursued using quantum systems. Theoretical background has roots in classical estimation theory, which was generalized to quantum mechanical formulation by Holevo and Helstrom from late 1960's. Practically, improvement of the performance of various sensors such as electric sensors and thermometer is the goal of quantum metrology. Magnetic field sensing is particularly well-studied for its wide application in, for example, biology, geology, and physics. In this thesis, we focus on magnetometry using N spins with spin-1/2 as a probe system.

The uncertainty, which is the inverse of the sensitivity, of a quantum sensor has two bounds, the standard quantum limit (SQL) and the Heisenberg scaling. The former is set by the central limit theorem, and it provides the limit for quantum sensors using separable states and classical sensors. The SQL is proportional to $N^{-1/2}$. The latter is set by quantum mechanics, which is proportional to N^{-1}. Hence the Heisenberg scaling offers the potential improvement from the SQL by $\Theta(N^{1/2})$ times. It is known that the Greenberger-Horne-Zeilinger (GHZ) state can achieve the Heisenberg scaling. It is a superposition of all up state and all down state, regarded a Schrödinger's cat state.

Schrödinger's cat state, "cat state" in short, is a superposition of macroscopically distinct states. Possibility of such a peculiar state was first questioned by Schrödinger in

1930's, and since then the state has attracted much attention. Since the notion "superposition of macroscopically distinct states" is ambiguous, several criteria for characterizing cat state were proposed. Among them, we focus on an index named q. Unlike other criteria that are, for example, defined for only superposition of two states (proposed by Dür, Simon, and Cirac in 2002), pure states (proposed by Shimizu and Miyadera in 2002), or defined from applicational perspective (proposed by Fröwis and Dür in 2012), index q can be used for mixed states and extracts the quantum coherence between macroscopically distinct states, genuinely detecting superposition. It is defined as

$$\max\left\{\max_{\hat{A},\hat{\eta}} \mathrm{Tr}\left[\hat{\rho}[\hat{A},[\hat{A},\hat{\eta}]]\right], N\right\} = \Theta(N^q),$$

where $\hat{A}$ is an additive observable, i.e., a sum of local observables, and $\hat{\eta}$ is a projection operator. By definition, $1 \le q \le 2$. According to Shimizu and Morimae, states with $q = 2$ are regarded as superposition of macroscopically distinct states. We call a state with $q = 2$ a *generalized cat state*.

In this thesis, we first prove that all the states that can be regarded as a generalized cat state achieves the Heisenberg scaling sensitivity in the ideal situation where there is no noise. We consider Ramsey-type sensing, which is done as follows. Step 1, prepare the probe system in a state that is suitable for sensing. In our case, prepare a generalized cat state that satisfies $\mathrm{Tr}\left[\hat{\rho}[\hat{A},[\hat{A},\hat{\eta}] = \Theta(N^2)$ for some $\hat{A}$ and $\hat{\eta}$. Step 2, let the system evolve in the presence of the target magnetic field for time t. We assume that the Hamiltonian can be expressed as $\omega\hat{A}$, where ω is the parameter to be estimated. Step 3, readout by projection. We assume the readout is done by $\hat{\eta}$. Step 4, repeat Step 1 to Step3 for T/t times. Here T is the total measurement time, and we assumed that the time to perform both Step 1 and Step 3 is much shorter than t. With such a sensing protocol, we prove that the uncertainty $\delta\omega$ can achieve the Heisenberg scaling. It is worthwhile to compare the index q with the quantum Fisher information (QFI). The QFI is known to give the lower bound of the uncertainty as $\delta\omega \ge (\mathrm{QFI})^{-1/2}$. For the reason that the improvement of the sensitivity beyond the SQL should be caused by macroscopic quantum effect instead of an accumulation of microscopic quantum effects, the QFI is considered to be able to judge the quantum macroscopicity. Although the use of the QFI was further extended as a criterion to characterize cat states, the relation with the index q remained unknown. In this thesis, we proved all the states with $q = 2$ guarantee the Heisenberg scaling, hence we were successful in relating these two quantities. There is an advantage in using q because we adopted the Ramsey-type protocol and have information on what kind of projection should be done in the readout process, while the QFI does not provide what kind of measurement should be done in order to obtain the Heisenberg scaling.

After proving the utility of generalized cat states in the absence of noise, we consider a more realistic case where noise is present. In general, noise degrades the sensitivity, and it is known that even the GHZ state cannot achieve the Heisenberg scaling with the Ramsey-type protocol. Instead, the ultimate scaling is known to be $\Theta(N^{-3/4})$. This scaling is obtained when non-Markovian independent dephasing is assumed. Such an assumption is reasonable because non-Markovian independent dephasing is known to be the major cause of the degradation of the sensitivity of sensors in solid-state systems. By non-Markovian, we mean that the focus is on the regime where the phase accumulation time t is smaller than the correlation time of the environment. Within such a regime, the decay of the coherence is slower than the phase accumulation, hence resulting in beating the SQL. Using this idea, we analyzed the sensitivity of a generalized cat state sensor in the presence of non-Markovian independent dephasing, and obtained the ultimate scaling $\Theta(N^{-3/4})$. Since generalized cat states include mixed states with low purity, the proof of all the generalized cat states achieving the ultimate scaling in the presence of a realistic dephasing may broaden the experimental possibility.

As an example of a generalized cat state with low purity, we consider a generalized cat state that is a mixture of an exponentially large number of states. We call it Mamineko, which stands for <u>ma</u>ximally <u>mi</u>xed <u>neko</u> (cat). We show that such a state can be obtained by performing a single global projection. In the recipe of Mamineko requires just two steps: (1) prepare spins that are macroscopically polarized along z axis, i.e., $\langle \hat{M}_z \rangle = \Theta(N)$. (2) measure the magnetization of spins along x axis and post-select the case $\hat{M}_x = M \pm Theta(1)$, where $M \neq \pm N + o(N)$. We denote the projection that projects the spins onto the $\hat{M}_x = M \pm \Theta(1)$ subspace as $\hat{\eta}_x$. Then Mamineko $\hat{\rho}$ satisfying $\mathrm{Tr}(\hat{\rho}[\hat{M}_z, [\hat{M}_z, \hat{\eta}_x]])$ is obtained (Fig. **??**). If the initial state is a thermal equilibrium state at finite temperature, then this post-measurement state is a mixture of exponentially large number of states for the following reasons. The initial state is a Gibbs state with purity as small as $\exp(-\Theta(N))$. Also, $\hat{\eta}_x$ is a projection onto $\exp(-\Theta(N))$ dimensional space, keeping the purity exponentially low. Since we have shown any generalized cat state can achieve the ultimate scaling sensitivity, Mamineko should be useful as well. To check if there is indeed advantage in using such a mixed state, we numerically estimated the sensitivity under the assumption that Mamineko is created in a silicon substrate. Using realistic parameters, we obtained a sensitivity that is about 20 times better than a separable state sensor in the same setup. Hence generalized cat state sensor seems promising in application.

Provided with the recipe and the numerical evidence that the generalized cat state with low purity has advantage over separable state sensors, it seems worthwhile to discuss the implementation of the recipe more specifically. We consider a hybrid system of nitrogen-vacancy (NV) centers in diamond and a superconducting flux qubit. An NV center is a defect in diamond, having effectively spin-1/2. It has a long coherence time, thus is a suitable system for creation of Mamineko. We consider reading out the NV ensemble's magnetization, i.e., to implement the projection $\hat{\eta}_x$, by a superconducting flux qubit. It is a superconducting loop with Josephson junctions and can be regarded as a spin-1/2 system. It can be used as a magnetic field sensor, and the coupling between NV ensemble is already experimentally realized. However, one readout only gives a binary information +1 or −1, which is far from the projection $\hat{\eta}_x$. Hence we consider repetitive measurements. The probability of obtaining +1 or −1 at each readout is encoded with the information of the magnetization of NV ensemble, and with the number of measurements m, the uncertainty decreases as $1/\sqrt{m}$. Using the feature that the repetitive measurements asymptotically approaches to the projection needed for the creation of Mamineko, we can follow how the state of the NV ensemble changes after each readout. We numerically observed the drastic change of q depending on m using realistic parameters, and obtained up to $q = 1.86$ state. We discuss how useful the states with $1 \leq q < 2$ are, and roughly show that while the states with the states with $1 \leq q \leq 1.5$ does not guarantee an improvement over the SQL, the states with $1.5 < q < 2$ can beat the SQL, though not achieving the ultimate scaling sensitivity. Obtaining a metrologically useful state, the implementation with the flux qubit and NV centers is promising.

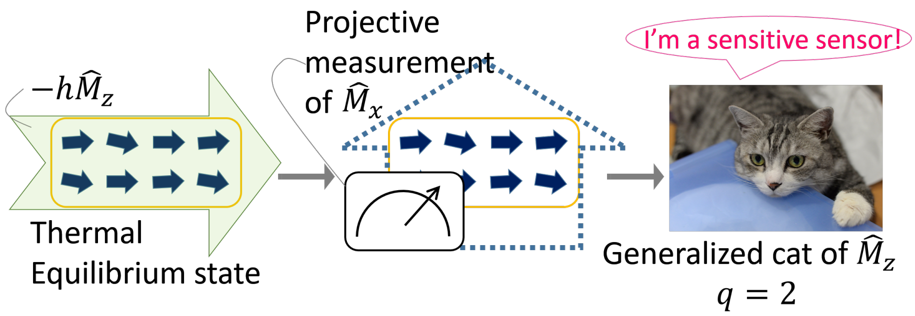

Figure 1: Mamineko can be generated by performing a projection measurement on a thermal equilibrium state, and it is a sensitive magnetic field sensor.