# 論文の内容の要旨

Adiabatic Charge Pumping in Quantum Dot Systems (量子ドット系における断熱電荷ポンピングの理論)

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## 1. Background and Motivation

Statistical physics is one of the successful theories, which gives a microscopic understanding and general theoretical framework of thermodynamics in equilibrium. However, construction of a general theory on thermodynamics in nonequilibrium has been a huge challenge in several decades. Nonequilibrium steady state (NESS) is a class of thermodynamic states, which is time-independent but has a constant flow of heat and particles. Since NESS is a simpler state rather than other nonequilibrium states, steady state thermodynamics (SST) has been studied in these decades.

Recently, SST of nanoscale devices, such as quantum dots, has been investigated in both experiments and theories. One of the problems in SST of nanoscale devices is how to treat the hybridization effect due to a strong coupling between the system and the reservoirs. When the system strongly couples to the reservoirs, heat transfer cannot be the described by previous formalism of SST because the coupling terms possess non-negligible energies. A new framework is required to discuss the strong system-reservoir coupling.

Adiabatic pumping is a key physical phenomenon to generalize SST to the strong system-reservoir coupling region. When a quasi-static and time-periodic driving of external fields is applied to the system, extra charges and heat are transferred from one reservoir to the other through the system in addition to a steady charge and heat flow due to the bias between the reservoirs. This transport phenomenon is called adiabatic pumping. From the viewpoint of thermodynamics, adiabatic pumping is equivalent to a quasi-static and cyclic operation, so it is useful to study the thermodynamic property of nanoscale devices with the strong coupling.

In this thesis, we focus on the following two subjects to discuss the hybridization effect on the adiabatic pumping:

#### (i) Relation between the pumped charge and the density of states of the reservoirs

In the strong coupling region, the number of pumped electrons via a quantum dot in one cycle depends on the band structure of the electron reservoirs. In two specific cases, the flat-band and the tight-binding model cases, the number of pumped electrons is reported to be one-half and one, respectively. We clarify the general relation between the density of states of the reservoirs and the

number of pumped electrons.

(ii) Effect of Coulomb interaction on the adiabatic charge pumping

Effect of Coulomb interaction on the adiabatic pumping via a quantum dot becomes significant in particular when the Kondo state appears at low temperature. The Kondo state is a typical quantum many-body state where electron states in the dot and the reservoirs are strongly mixed. Although the adiabatic pumping via the Kondo quantum dot has been discussed in several works, its connection to SST have not been discussed so far. Therefore we discuss a general framework for adiabatic pumping induced by the reservoir parameter driving, such as temperature and chemical potential driving.

### Almost topological pumping in non-interacting quantum dot system

Pumped charge in one cycle is calculated by the surface integral of the Berry curvature on a two dimensional area surrounded by a closed driving contour. When the Berry curvature has a sharp peak, the numbers of pumped electrons by the different driving contours become almost the same as long as the contours do not impinge on the peak. This pumping is referred to as "almost" topological pumping.

Almost topological pumping can be observed in a single-level quantum dot without Coulomb interaction. When the tunnel couplings between the dot and the reservoirs are driven, the number of



Fig. 1 A single-level quantum dot with the timedependent tunnel coupling,  $K_L(t)$  and  $K_R(t)$ . The time-dependence of the tunnel couplings is introduced by the time-dependent gate voltage  $V_L(t)$  and  $V_R(t)$ .

pumped electrons is almost quantized to fractional values depending on the density of states of the reservoirs.

In the quantum dot system, the energy level of the quantum dot is shifted and broadened by the hybridization effect due to the tunnel coupling between the dot and the reservoirs. The density of states of the reservoirs affects this energy shift and broadening, called Lamb shift and level broadening, respectively. Therefore, it is expected that the number of pumped electrons can be characterized by the energy shift and broadening.

We consider a single-level and non-interacting quantum dot system with time-dependent tunnel couplings (Fig. 1) and discuss the relation between the number of pumped electrons and the density of states of the reservoirs. As a result, we found that the number of pumped electrons is almost quantized to fractional values between zero and one and its quantized value is decided by one parameter,  $\lambda$ , the ratio of the Lamb shift and the level broadening at the Fermi level. When  $\lambda$  is positive, the height of the Berry curvature is quite low. In this case, the number of pumped electron approaches to zero. On the other hand, when  $\lambda$  is negative, the height of the Berry curvature increases and the number of pumped electrons becomes one (Fig. 2).

# 4. Charge pumping in interacting quantum dot system

To discuss the adiabatic pumping induced by the driving of reservoir parameters, a temperature and an electrochemical potential, we consider a single-level quantum dot with Coulomb interaction (Fig. 3). To describe time-dependent temperature of the reservoirs, we introduce thermomechanical field, an artificial field which mimics time-dependent temperature by driving the energy



(b) No Lamb shift:  $\lambda = 0$  (uniform reservoir density of states) Volume under 0.06 surface=1/2  $\Pi(V_{\rm L},V_{
m R})$ 10  $e \alpha_{\rm L} \alpha_{\rm R}$  $\alpha_{
m R} V_{
m R}$ 0.00 -10  $\alpha_{\rm L} V_{\rm L}$ -10 10 (c) Negative Lamb shift:  $\lambda =$ -3/olume under surface=0.993 0.5  $\Pi(V_{\rm L}, V_{\rm R})$  $e \alpha_{\rm L} \alpha_{\rm R}$ 10  $\alpha_{\rm R} V_{\rm R}$ 0.0 -10 n

Fig. 2 The Berry curvature,  $\Pi(V_L, V_R)$ , shown in the  $V_L - V_R$  space for (a)  $\lambda = 3$ , (b)  $\lambda = 0$ , (c)  $\lambda = -3$ .  $\alpha_L$  and  $\alpha_R$  are normalization constants of gate voltages.

 $\alpha_{\rm L} V_{\rm L}$ 

10 -10

scale of the reservoirs. Using the thermomechanical field, we formulate a theoretical framework of the adiabatic charge pumping induced by reservoir parameter driving.

Based on our framework, we clarify the mechanism of the charge pumping in the present model. When one drives the reservoir parameters, the Fermi distribution of electrons in the reservoirs are also driven. Then, quantum states of electrons in the dot, such as an occupation number, responses to the change of the Fermi distribution through the tunnel coupling with a delay time. This delayed response can be described by an effective RC circuit and rectifies the charge current, which produced an extra transfer of electrons in a cycle.

We also estimate Coulomb interaction effect on the adiabatic charge pumping in the present model. We employ the renormalized perturbation theory and discuss U-dependence of the amount of the pumped charge, where U is a strength of Coulomb interaction. Figure 4 shows the pumped charge as a function of the energy level of the dot for different values of U. As Coulomb interaction becomes stronger, the peak grows and becomes broad. When Coulomb interaction exists, the energy level of the quantum dot is renormalized to the effective energy level due to the Coulomb repulsion. The effective energy level is insensitive to a shift of the original energy level of the dot; when the original energy level is raised, it decreases the occupation number of the quantum dot and counters the energy shift. This stable feature of the effective energy level can be regarded as pinning of the occupation number. For strong Coulomb interaction, the effective energy level is almost fixed, so the adiabatic pumping becomes less dependent on the energy level of the quantum dot, which is observed as the broadening of the peak in Fig. 4.

#### 5. Summary and Perspectives

We mainly study the adiabatic charge pumping via a single-level quantum dot and obtained two results : (i) The number of pumped electrons via a non-interacting quantum dot strongly depends on the density of states of the reservoirs and can be characterized by the ratio between the Lamb shift and the level broadening. (ii) The adiabatic charge pumping induced by reservoir parameter driving via an interacting quantum dot is caused by rectification due to the delayed response of the dot and strongly reflects the Coulomb interaction effect, such as the pinning effect.

Our results provide a clear understanding of adiabatic charge pumping in the strong system-reservoir coupling region. Now, we are ready to study adiabatic heat pumping. An important future problem of this study is to define a heat transfer properly and construct SST in the strong system-reservoir coupling regime.



Fig. 3 A schematic of our model. Temperature  $T_L(t)$ ,  $T_R(t)$  and electrochemical potential  $\mu_L(t)$ ,  $\mu_R(t)$  are time-dependent.



Fig. 4 The pumped charge induced by temperature driving as a function of the energy level of the dot. The horizontal axis denotes the number of pumped charge in one cycle.