論文の内容の要旨

Algebraic Proof of S-Duality Formula in Refined Topological Vertex
(位相的頂点におけるS双対性の代数的証明)

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We deal with the five dimensional $\mathcal{N}=1$ super Yang-Mills theories. Let $\mathcal{Z}_{\text{inst.}}^{(A_N,A_M)}$ be the instanton partition function of the A_M quiver gauge theory with A_N gauge group with $N_F = 2(N+1)$ matters. We set all the Chern-Simons levels to be zero. Then, the S-duality claims the invariance of $\mathcal{Z}_{\text{inst.}}^{(A_N,A_M)}$ under the exchange between N and M. The main object of the present thesis is to prove this claim. By rewriting the equality in terms of the topological vertex, we obtain the duality formula under changing the preferred directions.

The key ingredient of the proof is the operator realization of the topological vertex. This is achieved by the intertwiners of the Ding-Iohara-Miki algebra. By gluing the intertwiners, we can realize what we call the Mukadé operator $\mathcal{V}(x)$. Let us define the Mukadé operator. Let $\mathcal{F}_{\boldsymbol{u}} = \widetilde{\bigotimes}_{j=1}^{N} \mathcal{F}_{u_j}$ be the N-fold tensor space of the Fock spaces.

Definition. *Let $\mathcal{V}(x) : \mathcal{F}_{\boldsymbol{u}} \to \mathcal{F}_{\boldsymbol{v}}$ be a linear map satisfying the commutation relations*

$$\left(1 - \frac{x}{z}\right) X^{(i)}(z)\mathcal{V}(x) = \left(1 - (t/q)^i \frac{x}{z}\right) \mathcal{V}(x) X^{(i)}(z) \qquad (i = 1, \ldots, N)\,, \tag{0.0.1}$$

and the normalization condition $\langle \mathbf{0} | \mathcal{V}(x) | \mathbf{0} \rangle = 1$. We refer to this operator as the Mukadé operator. Here $|\mathbf{0}\rangle$ (resp. $\langle \mathbf{0}|$) is the vacuum (resp. dual vacuum) state.

Roughly speaking, $X^{(i)}(z)$'s are the generating currents of q-deformed $\mathcal{W}$-algebra for $\mathfrak{g} = A_{N-1}$. Then, we compute the matrix elements of the Mukadé operator with respect to the generalized Macdonald functions. For N-tuples of partitions $\boldsymbol{\lambda}$, we denote by $|K_{\boldsymbol{\lambda}}\rangle = |K_{\boldsymbol{\lambda}}(\boldsymbol{u})\rangle$, the integral forms of the generalized Macdonald functions. Then, the following theorem is our final result.

Theorem. *We have*

$$\langle K_{\boldsymbol{\lambda}}(\boldsymbol{v})|\, \mathcal{V}(x)\, |K_{\boldsymbol{\mu}}(\boldsymbol{u})\rangle = \frac{\left((-\gamma^2)^N e_N(\boldsymbol{u}) x\right)^{|\boldsymbol{\lambda}|}}{(\gamma^2 x)^{|\boldsymbol{\mu}|}} \prod_{i=1}^{N} \frac{u_i^{|\mu^{(i)}|} g_{\mu^{(i)}}}{\left(v_i^{|\lambda^{(i)}|} g_{\lambda^{(i)}}\right)^{N-1}} \cdot \prod_{i,j=1}^{N} N_{\lambda^{(i)},\mu^{(j)}}(qv_i/tu_j)\,.$$

Here, $\gamma = (t/q)^{1/2}$, $e_N(\boldsymbol{u}) = u_1 \cdots u_N$ *and* $N_{\lambda,\mu}$ *is the Nekrasov factor.* $g_\lambda = q^{n(\lambda')} t^{-n(\lambda)}$ *and* $n(\lambda) = \sum_{i\geq 1}(i-1)\lambda_i$.

Our main claim follows easily from this theorem.

Moreover, the Mukadé operator reduces to the primary fields of the Virasoro algebra, under the $q, t \to 1$ limit. In the gauge theory terminology, this limit corresponds to the reduction to the four dimension. Then, the matrix elements formula of the Mukadé operator can be interpreted as the proof of the five dimensional analogue of the Alday-Gaiotto-Tachikawa correspondence.