論 文 の 内 容 の 要 旨

論文題目　Quantum information-theoretic study of complexity in chaotic many-body dynamics

（多体カオスのダイナミクスにおける複雑性の量子情報理論的研究）

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Bridging the gap between macroscopic thermodynamics and microscopic quantum dynamics is one of the most fundamental problems in statistical physics, dating back to von Neumann. Recently, non-equilibrium dynamics of isolated quantum many-body systems have been observed in highly controllable quantum systems, such as ultracold atoms, superconducting qubits, and trapped ions. Both theoretical and experimental studies have revealed that isolated many-body systems thermalize under the unitary time evolution, which triggered progress in understanding mechanisms of thermalization. A widely accepted mechanism of thermalization is based on a conjecture called the eigenstate thermalization hypothesis (ETH). The ETH states that all the energy eigenstates of a chaotic Hamiltonian are indistinguishable from the thermal states and is regarded as a counterpart of ergodicity in quantum systems. The ETH has been numerically confirmed for many nonintegrable systems, while the ETH is violated in integrable systems and localized systems.

Another essential concept of thermodynamics is the second law, which is formulated in several ways, such as the entropy increase and the impossibility of the perpetual motion of the second kind. The second law in quantum systems has been studied based on quantum information theory, while it has not yet been much studied in light of chaos of many-body dynamics. Correspondingly, our understanding of the second law in quantum systems is not as established as that of thermalization for isolated many-body quantum systems.

More recently, high-energy physics provided new insights into quantum chaotic dynamics. In the context of the black hole information problem, a notion of information scrambling is proposed as a new characterization of quantum many-body chaos. Information scrambling represents the spreading of localized quantum information over the entire system. Moreover, information scrambling is closely related to a notion of quantum pseudorandomness, which is an important resource of quantum information tasks.

The present thesis is devoted to two issues regarding chaotic many-body dynamics. First, we consider the second law at the level of individual energy eigenstates to establish thermodynamics of isolated quantum many-body systems. Especially, we investigate both numerically and analytically the possibility of work extraction by cyclic Hamiltonian dynamics. We propose a second-law analog of the ETH from numerical results (i.e., the eigenstate second law), and rigorously prove a weaker version of the eigenstate second law. Second, we focus on yet another aspect of chaotic quantum systems beyond conventional thermodynamics, especially the universal structure of entanglement and information scrambling. We propose a generalization of the ETH to characterize higher-order complexity of quantum chaos at the level of a single energy eigenstate. In addition, we attempt to bridge the gap between chaotic many-body dynamics and intrinsically random unitary dynamics, by introducing a new quantum information-theoretic concept regarding partial pseudorandomness.

The first issue is the possibility of work extraction from quantum many-body systems. The concept of work is essentially important in classical thermodynamics. In fact, one cannot extract positive work from a single heat bath, which is manifestation of the second law in the version of the Kelvin–Planck statement. Establishing the second law from microscopic dynamics is one of the central issues in statistical physics. Motivated by researches on designing quantum heat engines and nanomachines, this problem is also actively investigated for isolated quantum systems, and there have been several important developments these days. For a given Hamiltonian, a quantum state is said to be passive, if its energy cannot be decreased by any cyclic unitary operations. We note that the complete characterization of passive states has already obtained without any assumption on the Hamiltonian. For example, statistical mechanical ensembles such as the Gibbs states are passive, while pure states are not passive in general. However, this result is not enough to conclude that the second law is already established from quantum mechanics, especially given that a pure state also describes thermal equilibrium.

Since one has only limited controllability of large-scale systems, a natural question is whether one can extract positive work by a limited class of unitary operations. In this thesis, we numerically investigate this question and show that integrability plays an important role. By using numerical exact diagonalization, we calculate the extracted work from individual energy eigenstates by quench protocols of the mixed-field Ising model with changing its integrability. We show that one cannot extract positive work from any energy eigenstate, if the initial Hamiltonian or the protocol has nonintegrability. This result leads us to an analogy between thermalization and the second law. We thus conjecture the eigenstate second law, which states that all the energy eigenstates satisfy the second law under experimentally realistic quench protocols. In fact, we show that the eigenstate second law follows from the ETH for a non-local operator corresponding to the energy of the final state. On the other hand, we numerically show the violation of the eigenstate second law in an integrable system, which is the counterpart of the violation of the ETH in integrable systems. In addition, we prove that for arbitrary fixed unitary operation, one cannot extract positive work from almost all energy eigenstates. This claim is regarded as a weaker version of the eigenstate second law and holds even in integrable systems as long as the Boltzmann's formula is true. This weaker version of the second law is analogous to the weaker version of the ETH.

The second issue is how to characterize complexity and randomness of quantum chaotic dynamics. The complexity of a quantum state increases in time until it saturates at the maximum value. Thermalization and information scrambling are related to the growth of complexity under Hamiltonian time evolutions. Characterization of the complexity of chaotic many-body dynamics would be expected to provide a unified description of thermalization and information scrambling. Moreover, quantum chaos has attracted attention in high-energy physics and quantum information theory. In the context of the blackhole information paradox, the out-of-time-ordered correlator (OTOC) has been proposed as an indicator of information scrambling, and the decay of the OTOC characterizes quantum chaos. Furthermore, it has been argued that blackholes are quantum chaotic and the most powerful information scrambler. These researches suggest that quantum chaotic dynamics including blackholes are closely related to random unitary dynamics, which is well known as a good scrambler. Random unitaries are also important in quantum information theory, but we need a large number of quantum gates to realize exact random unitaries. This fact has brought up the research of quantum pseudo-randomness that approximates random unitaries. Quantum pseudo-randomness is formulated as a unitary k-design, which simulates the kth moment of random unitaries. However, the relationship between chaotic Hamiltonian dynamics and unitary k-designs has not been fully addressed in previous studies.

Combining the notions of the ETH and unitary k-designs, we introduce a higher-order generalization of the ETH, named the k-ETH (k=1, 2,⋯). The lowest order ETH (i.e., the 1-ETH), reproduces the conventional ETH. The higher-order ETH is a higher-order counterpart and characterizes higher-order randomness. The k-ETH is formulated by considering the k-replicated system in the same way as the definition of a unitary k-design. The k-ETH (k≥2) also implies that the kth Renyi entanglement entropy of an energy eigenstate follows a universal system-size dependence, called the Page curve. The volume law of the eigenstate entanglement can be explained by the conventional ETH, while the Page correction intrinsically originates from the higher-order ETH. In addition, we numerically verified that the 2-ETH approximately holds for a nonintegrable spin model, but does not hold for an integrable model.

In order to clarify the connection between the k-ETH and unitary k-design, we introduce a new information-theoretic concept named a partial unitary k-design (PU k-design). A PU k-design is an approximation of random unitaries in which one focuses only on a limited number of observables. A conventional unitary k-design is a special case of a PU k-design where all the observables are accessible. The k-ETH is also a special case of a PU k-design where the ensemble is given by the random-time sampling of Hamiltonian time evolution operators. On the other hand, we found that its relationship to information scrambling is not straightforward as follows. The exact scrambling, i.e., the exact agreement of the late-time OTOC with its random-unitary average, requires the exact 2-ETH, but the approximate decay of the OTOC in the thermodynamic limit follows only from the approximate 1-ETH.

Our results shed new light on the foundation of statistical mechanics and characterization of chaos in quantum many-body systems. Moreover, we expect that our approach would contribute not only to statistical mechanics but also to various fields related to quantum many-body dynamics, including high-energy physics and quantum information theory. Investigating the possibility of experimental verification of our results with quantum simulators is an important future issue.