博士論文（要約）

Information-Geometric Construction of Bayesian Methods for Curved Exponential Families

（曲指数型分布族に対するベイズ統計手法の情報幾何を用いた構成）

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Abstract

We investigate Bayesian methods of prediction and estimation for curved exponential families from a perspective of information geometry. The Bayesian predictive density is optimal about the Bayes risk under the Kullback–Leibler divergence, but its representation is complicated and the numerical evaluation is burdensome. We construct a distinct class of predictive densities with simple forms and at the same time with pleasant risk performance. We investigate their information-geometric aspects and also apply them to estimation. Geometric structures and invariance properties of models are utilized for choice of priors and construction of estimators. This thesis consists of three parts.

First, extended plugin densities are proposed as predictive densities for curved exponential families. We embed curved exponential families in larger full exponential families and consider plugin densities in the full exponential families, which we call extended plugin densities. The extended Bayes estimator, which is the Bayes estimator of the full exponential family, is obtained as the posterior mean about the expectation parameter. Several information-geometric results are obtained in parallel with the Bayesian predictive density, and a superharmonic condition is shown for the choice of priors.

Second, Bayesian shrinkage methods with the extended plugin densities are applied to spiked covariance models. Shrinkage estimation of eigenvalues of covariance matrices are known to be effective, and existing research typically employs nonlinear functions of eigenvalues of sample covariance matrices. We propose a shrinkage prior that asymptotically dominates the Jeffreys prior. The extended Bayes estimators are obtained as the posterior mean of the covariance matrices. Numerical studies suggest that our Bayesian method is effective compared to other non-Bayes methods regarding the Kullback-Leibler risk.

Third, we consider estimations for the factor analysis model based on its invariance properties. Factor analysis is a method of multivariate analysis commonly used in psychology and social sciences. We propose a new parametrization and priors respecting the invariance properties about the scaling of observations. The estimator is constructed by projecting the extended Bayes estimator onto the factor analysis model.