論 文 の 内 容 の 要 旨

論文題目　Learning Algebraic Varieties under Noise
（ノイズ下での代数多様体学習）

氏　　名　　計良　宥志

How can we retrieve the relations between variables from noisy observation? This has been a fundamental problem in various fields. It has been shown that if data lie on am algebraic variety, which is described by a set of polynomials, we can enjoy several strong theoretical statements in supervised classification, semi-supervised learning, and clustering.

In the present thesis, we address the problem of computing a polynomial system from data. In particular, we would like to obtain a *nice* polynomial system from *noisy* data. In the last decade, this problem has been addressed in the context of the basis construction of the approximate vanishing. However, most existing algorithms are heavily dependent on the use of the monomial order, which is problematic prior information in various applications. Without using the monomial order, existing algorithms encounter various theoretical issues, resulting in a low quality of the output polynomial system. The main contribution of the present thesis is to realize a monomial-order-free basis construction algorithm that overcomes the following issues which the existing algorithms suffer from.

The spurious vanishing problem---A polynomial can approximately vanish for a point not because the point is close to the roots of the polynomial but merely because the polynomial is close to the zero polynomial.

Redundancy in the basis set---The output basis set can contain polynomials that are redundant because they can be generated by other lower-degree polynomials. For example, a basis polynomial is redundant if its polynomial factor is included

in the basis set. Determining the redundancy usually needs exponentially costly symbolic procedures and is also unreliable in our approximate setting.

Inconsistency of the basis set with respect to input transformation---Given translated or scaled data points, the output of the basis construction can drastically change in terms of the number of polynomials and their nonlinearity.

For the first issue, we design a monomial-order-free algorithm with normalization, which is proven valid, stable, and optimal. We consider coefficient normalization and gradient normalization. In particular, the gradient normalization is the first to realize the polynomial-time normalization in monomial-order-free basis construction. For the second issue, we reveal that the gradient of basis polynomials reflects the symbolic relation between polynomials. With this result, a basis reduction method is proposed to remove redundant basis polynomials. The last issue is resolved by the gradient normalization. We prove that with the gradient normalization, one can realize a sort of invariance of the basis set with respect to input transformation.

With these results, we establish an efficient basis construction framework that is accompanied by a rich theoretical foundation even without using the monomial order.