

## Characterization of Thermal Stresses in Ceramic/Metal-Joint

by

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Residual thermal stresses in ceramic/metal joint are investigated. As the joint is cooled down from the joining temperature, a thermal stress concentration developed at the edge of the ceramic/metal interface due to mismatch of the thermal expansion coefficients of the constituent materials. The local stress field around the edge of the interface is found to be singular with a singularity of  $r^{1/2}$ , where  $r$  is the distance from the edge and  $\lambda$  depends on Dunder's composite parameters,  $\alpha$  and  $\beta$ . Edge stress intensity factors,  $K_i$  and  $K_b$ , are introduced to characterize the singular behavior of the residual thermal stresses. It is shown by FEM analysis of the residual thermal stresses that there is a certain correlation between the values of the stress intensity factors and the thermoelastic parameters of the joints. A simple method to estimate the magnitude of the edge stress intensity factors of the residual thermal stresses in a given ceramic/metal joint is proposed.

### 1. Introduction

In general ceramics and metals are joined at a high temperature. Residual thermal stresses will develop in ceramic/metal joint during cooling from the bonding temperature due to the difference of the thermal expansion coefficients between ceramics and metals. The thermal stresses will occur also when the joints are used at a high temperature. The residual thermal stresses will lower the strength of the joint and sometimes induce a damage or cracks into the ceramic part.<sup>1)</sup> Therefore characterization and reduction of the residual thermal stresses are one of the most important problems to increase the reliability of ceramic/metal joints.

The purpose of the present paper is to investigate the residual thermal stresses of ceramic/metal joint systematically with taking the stress concentration at the bonded edge into account. To characterize the stress concentration, a kind of stress intensity factor is introduced and its dependence of the thermoelastic parameters and the size of the ceramic/metal joint is examined.

### 2. Analysis of Residual Thermal Stresses

Residual thermal stresses in bonded dissimilar materials can be calculated analytically

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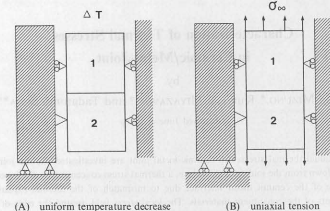


Fig. 1. Boundary condition.

only for limited cases as follows. Consider residual thermal stresses developing in a joint on uniform cooling from the bonding temperature under the boundary condition, that the displacements of the both sides of the joint are restrained, as shown in Fig. 1(a). Let the joint be made up of two planes of materials with thermoelastic constants,  $\alpha_1, E_1, \nu_1$  and  $\alpha_2, E_2, \nu_2$ , where  $\alpha_j$  ( $j$  denotes the material  $j=1, 2$ ) is the thermal expansion coefficient,  $E_j$  the elastic constant,  $\nu_j$  Poisson's ratio and  $\Delta T$  the difference between the bonding temperature and the room temperature. Assuming that there is no dislocation, we have the continuity of strains along the interface:

$$\epsilon_{1x} = \epsilon_{2x} \quad (2.1)$$

Equilibration of the stresses parallel to the interface requires

$$\sigma_{1x} + \sigma_{2x} = 0 \quad (2.2)$$

From the boundary condition, we obtain

$$\sigma_{1y} = \sigma_{2y} = 0 \quad (2.3)$$

The stress component  $\sigma_x$  can be obtained from Hook's law of linear isothermal elasticity:

$$\sigma_{1x} = -\Delta\alpha \cdot E^* \cdot \Delta T / 2, \quad \sigma_{2x} = \Delta\alpha \cdot E^* \cdot \Delta T / 2 \quad (2.4)$$

where

$$\Delta\alpha = \begin{cases} \alpha_1 - \alpha_2 & \text{for plane stress} \\ \frac{\alpha_1 - \alpha_2}{(1 + \nu_1)\alpha_1 - (1 + \nu_2)\alpha_2} & \text{for plane strain} \end{cases}$$

$$E^* = \left[ \frac{(1 + \nu_1)/\mu_1 + (1 + \nu_2)/\mu_2}{16} \right]^{-1}$$

$$\kappa_j = \begin{cases} (3 - \nu_j)/(1 + \nu_j) & \text{for plane stress} \\ 3 - 4\nu_j & \text{for plane strain} \end{cases}$$

and  $\mu_j$  represents the shear modulus of each material.

It is shown by eq. (2.4) that the stress  $\sigma_x$  is tensile in the material of higher thermal expansion and compressive in the material of lower thermal expansion on cooling from the bonding temperature. Eq. (2.4) gives an approximation to  $\sigma_x$  in the middle of the bonded interface.

Examine a joint subjected to a uniform tension  $\sigma_x$  and to the same boundary condition for constraining of the displacements as the above case, as shown in Fig. 1(b). The stresses parallel to the interface are obtained as,

$$\sigma_{1x} = -E^* \sigma_x / (2\Delta E), \quad \sigma_{2x} = -\sigma_{1x} \quad (2.5)$$

where

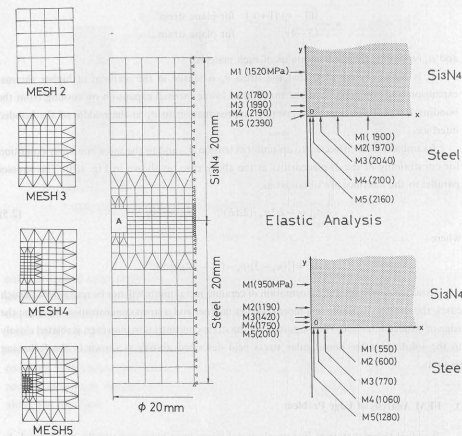
$$\Delta E = \left[ \frac{(\kappa_1 - 3)/\mu_1 - (\kappa_2 - 3)/\mu_2}{8} \right]^{-1}$$

In actual condition, the deformation of ceramic/metal joints will not be restrained. In such cases, the stresses distribute in a more complex manner and a stress concentration occurs at the edge of bonded interface. The magnitude of the stress concentration, however, is related closely to the solution of the nonsingular stress field described above, as shown in the following sections.

### 3. FEM Analysis of Edge Problem

Residual thermal stresses in ceramic/metal joints have been numerically investigated. In most studies,<sup>2,3)</sup> it is assumed that the stresses have a finite value in the bonded edge, whether they are elastic or elasto-plastic analyses. The maximum principal stress obtained numerically has been used to characterize the magnitude of residual thermal stresses. It has been also said that the maximum principal stress of the joint is located not at the bonded edge but in the ceramic part.

However, it is known that when the external loads are applied on the joint, the stress field has a singularity at the bonded edge.<sup>4)</sup> Therefore the thermal stresses are expected also to be infinite at the bonded edge. In order to make sure whether the maximum stress occurs at the bonded edge or not, and whether the value of the maximum stress at the bonded edge is finite or infinite, residual thermal stresses of the joint were calculated by means of FEM analysis. It was assumed in the calculations that 1) two isotropic materials are bonded perfectly to each other along their straight bonded interface, 2) the temperature of the joint is uniformly decreased by  $\Delta T = -1000$  K from the bonding temperature, and 3) the thermoelastic parameters of the constituent materials are independent of the temperature. The residual thermal stresses were calculated for a cylindrical joint of  $\text{Si}_3\text{N}_4$ /steel for both elastic and elasto-plastic



Element patterns around A

Fig. 2. Finite element divisions (MESH5 elements = 394, nodes = 1149).

Table 1. Material Properties of the Two Components

Si <sub>3</sub> N <sub>4</sub>	Steel
$E = 314 \text{ GPa}$	$E = 210 \text{ GPa}$
$\nu = 0.28$	$\nu = 0.3$
$\alpha = 2.7 \times 10^{-6} \text{ K}^{-1}$	$\alpha = 12 \times 10^{-6} \text{ K}^{-1}$
	$\sigma_s = 406 \text{ MPa}$
	$n = 0.25$
	$C = 1937 \text{ MPa}$

cases. The size of the joint is  $\phi 20 \times 40 \text{ mm}$ . Several finite element divisions with different mesh size shown in Fig. 2 are used to examine the effects of the mesh size. The values of the parameters used in the calculations are compiled in Table 1.

The maximum principal stresses  $\sigma_1$  in the ceramic and metal part are shown in Fig. 3 for the different finite element meshes. Fig. 3 indicates that the finer mesh is used, the higher value

maximum principal stresses takes and the closer to the bonded edge it is located. These results confirm the existence of a stress singularity at the bonded edge. Therefore the value of the maximum principal stress is not enough to characterize the residual thermal stresses of the joint. Fig. 3 shows also that the maximum principal stress takes no limited value at the bonded edge even in the case of the elasto-plastic analysis. This is because a power law is assumed for the strain hardening characteristic of the metal and therefore the value of the equivalent stresses of the metal increases to an unlimited extent after yielding. Thus, the edge effects must be considered even in the case of the elasto-plastic analysis.

#### 4. Eigenfunction Approach to Singular Thermal Stress Field

We consider two dimensional bonded quarter planes of two dissimilar isotropic elastic materials, as shown in Fig. 4. Let  $(r, \theta)$  be polar coordinates of a point with rectangular Cartesian coordinates  $(x, y)$ . Suppose each material is subjected to an initial strain  $\epsilon_0$ . On introducing the complex valuable  $z = x + iy = re^{i\theta}$ , the basic equations of elasticity are given by

$$\begin{aligned} \sigma_{\theta\theta} - i\tau_{\theta z} &= \phi_j'(z) + \bar{\phi}_j'(\bar{z}) + z\bar{\phi}_j''(\bar{z}) + \bar{\psi}_j'(\bar{z}), \\ \sigma_{zz} + i\tau_{z\theta} &= \phi_j'(z) + \bar{\phi}_j'(\bar{z}) - z\bar{\phi}_j''(\bar{z}) - \bar{\psi}_j'(\bar{z}), \end{aligned} \quad (4.1)$$

where  $j$  denotes the material  $j$  ( $j=1, 2$ ), and  $\phi_j$  and  $\psi_j$  are the Muskhelishvili complex stress functions.<sup>5)</sup> Taking into account the initial strain  $\epsilon_0$ , the relationship between stresses and strains can be given by

$$\begin{aligned} 2\mu_j \left( \frac{\partial u_j}{\partial x} - \epsilon_0 \right) &= \frac{\kappa_j + 1}{4} (\sigma_{xx} + \sigma_{yy}) - \sigma_{xx}, \\ 2\mu_j \left( \frac{\partial v_j}{\partial y} - \epsilon_0 \right) &= \frac{\kappa_j + 1}{4} (\sigma_{xx} + \sigma_{yy}) - \sigma_{yy}, \end{aligned} \quad (4.2)$$

where  $\epsilon_0 = \alpha_j \Delta T$  for plane stress and  $\epsilon_0 = (1 + \nu_j) \alpha_j \Delta T$  for plane strain. By integrating eq. (4.2)

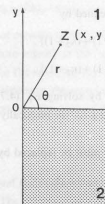


Fig. 4. Bonded quarter planes.

with respect to  $z$ , it is found that the displacements  $u_j$  and  $v_j$  are written by  $\phi f(z)$  and  $\psi f(z)$  as,

$$2\mu_j(u_j + iv_j - v_{j0}z) = \kappa_j \phi f(z) - z \overline{\phi'(z)} - \overline{\psi f(z)}. \quad (4.3)$$

Considering the continuity of displacements and the equilibrium of forces across the interface, we meet the boundary conditions:

$$\begin{aligned} (u_1 + iv_1)_{z=0} &= (u_2 + iv_2)_{z=0}, \\ (\sigma_{1y} - i\tau_{1x})_{z=0} &= (\sigma_{2y} - i\tau_{2x})_{z=0}, \\ (\sigma_{1x} + i\tau_{1y})_{z=0} &= (\sigma_{2x} + i\tau_{2y})_{z=0} - \kappa_2 = 0. \end{aligned} \quad (4.4)$$

According to the eigenfunction expansion method by Williams,<sup>6)</sup> the stress functions are given by the eigenfunction series of  $z$  as

$$\phi f(z) = \sum_n a_n z^{2n}, \quad \psi f(z) = \sum_n b_n z^{2n} \quad j = 1, 2. \quad (4.5)$$

Substituting eq. (4.5) into eq. (4.1), the boundary conditions eq. (4.4) are represented by  $a_n, b_n$  and  $z^{2n}$ . From comparison of the coefficients of the terms  $z^{2n}$ , we obtain in the case of  $\lambda_n \neq 1$

$$\begin{aligned} k(\kappa_1 a_{1n} - \lambda_n \overline{a_{1n}} - \overline{b_{1n}}) &= \kappa_2 a_{2n} - \lambda_n \overline{a_{2n}} - \overline{b_{2n}}, \\ \lambda_n \overline{a_{1n}} + a_{1n} + \overline{b_{1n}} &= \lambda_n \overline{a_{2n}} + a_{2n} + \overline{b_{2n}}, \\ \lambda_n \overline{a_{1n}} - \overline{b_{1n}} - a_{1n} e^{i\lambda_n} &= 0, \\ \lambda_n \overline{a_{2n}} - \overline{b_{2n}} - a_{2n} e^{-i\lambda_n} &= 0, \end{aligned} \quad (4.6)$$

where  $k = \mu_2/\mu_1$ . For non zero solution of the coefficients  $a_n$  and  $b_n$ , the determinant of the coefficient matrix of eq. (4.6),  $\Delta(\lambda_n, \alpha, \beta)$ , must be zero:

$$\begin{aligned} \Delta(\lambda_n, \alpha, \beta) &= \lambda_n^2 (\lambda_n^2 - 1)^2 x^2 + 2\lambda_n^2 (\sin^2(\pi\lambda_n/2) - \lambda_n^2) \beta^2 \\ &\quad + (\sin^2(\pi\lambda_n/2) - \lambda_n^2)^2 \beta^2 + \sin^2(\pi\lambda_n/2) \cos^2(\pi\lambda_n/2) = 0, \end{aligned} \quad (4.7)$$

where  $\alpha$  and  $\beta$  are Dundur's composite parameters<sup>7)</sup> defined by

$$\begin{aligned} \alpha &= \{k(\kappa_1 + 1) - (\kappa_2 + 1)\} / \{k(\kappa_1 + 1) + (\kappa_2 + 1)\}, \\ \beta &= \{k(\kappa_1 - 1) - (\kappa_2 - 1)\} / \{k(\kappa_1 + 1) + (\kappa_2 + 1)\}. \end{aligned} \quad (4.8)$$

The eigenvalues of  $\lambda_n$  can be obtained numerically by solving eq. (4.7). The minimum eigenvalue of  $\lambda_1$  is shown in Fig. 5 as a function of  $\alpha$  and  $\beta$  for the physically relevant range of the Poisson's ratio, i.e.  $0 < \nu_j \leq 1/2$ .

In the case of  $\lambda_n = 1$ , the first equation of eq. (4.6) must be replaced by,

$$k(\kappa_1 a_{1n} - \overline{a_{1n}} - \overline{b_{1n}}) = (\kappa_2 a_{2n} - a_{2n} - \overline{b_{2n}}) = 2\mu_2(\epsilon_{20} - \epsilon_{10}), \quad (4.9)$$

and the coefficients  $a_n$  and  $b_n$  can be easily obtained as

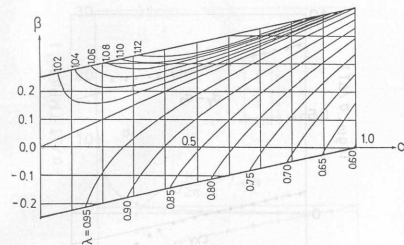


Fig. 5. Dependence of  $\lambda_1$  on composite parameters  $\alpha$  and  $\beta$ .

$$\begin{aligned} a_{1n} &= a_{2n} = b_{1n}/2 = b_{2n}/2 \\ &= 2(\epsilon_{20} - \epsilon_{10}) / \{(\kappa_1 - 3)/\mu_1 - (\kappa_2 - 3)/\mu_2\}. \end{aligned} \quad (4.10)$$

The local stress fields in the bonded edge are derived from eq. (4.1), eq. (4.5) and eq. (4.10) as follows:

$$\begin{aligned} \sigma_{yy} - i\tau_{xy} &= \sum_{n=1}^{\infty} f(\lambda_n, \theta) r^{\lambda_n-1} + \sigma_0, \\ \sigma_{xx} + i\tau_{yx} &= \sum_{n=1}^{\infty} g(\lambda_n, \theta) r^{\lambda_n-1}, \end{aligned} \quad (4.11)$$

where

$$\begin{aligned} f(\lambda_n, \theta) &= \lambda_n \{[\overline{a_{1n}} + \overline{b_{1n}} + (\lambda_n - 1)\overline{a_{1n}} e^{2i\theta}] e^{-i\lambda_n - 1} + a_{1n} e^{i\lambda_n - 1}\}, \\ g(\lambda_n, \theta) &= \lambda_n \{[\overline{a_{1n}} - \overline{b_{1n}} + (\lambda_n - 1)\overline{a_{1n}} e^{2i\theta}] e^{-i\lambda_n - 1} + a_{1n} e^{i\lambda_n - 1}\}, \\ \sigma_0 &= (2a_{1n} + b_{1n})_{\lambda_n=1} = -\Delta\alpha\Delta T. \end{aligned}$$

In many cases of ceramic/metal joints, the value of  $\lambda_1$  is smaller than unit. Therefore the stress components go to infinity at the bonded edge as  $r$  approaches zero.

Similarly to the crack tip stress field, a kind of stress intensity factor,  $K_I$  and  $K_{II}$ , can be introduced to represent the singular stress field in the edge region. Using the stress intensity factor, the stresses in the edge of the bonded interface are given by

$$(\sigma_y - i\tau_{xy})_{r=0} = \frac{K_I - iK_{II}}{\sqrt{2\pi}} r^{\lambda_1-1} + \sigma_0. \quad (4.12)$$

The residual thermal stresses were calculated by FEM to verify their singularity at the bonded edge. The calculations were carried out on a joint consist of  $\text{Al}_2\text{O}_3$  and Cu under the

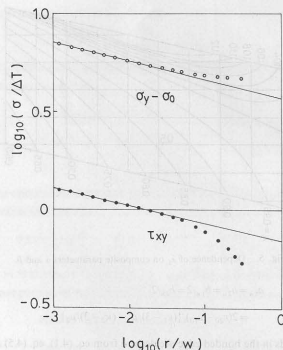


Fig. 6. Relationship between  $\log_{10}[-(\sigma_y - \sigma_0)/\Delta T]$ ,  $\log_{10}(\tau_{xy}/\Delta T)$  and  $\log_{10}(r/w)$ .

assumption of the plane strain condition. Fig. 6 shows  $\log_{10}(\sigma_y - \sigma_0)$  and  $\log_{10} \tau_{xy}$  as a function of  $\log r$ . As  $r$  approaches zero, the curves become linear with a slope of  $1/4$ . These results indicate that the stress field in the edge region has the singularity of  $r^{1/4}$ . The stress intensity factors of the edge stress field defined by eq. (4.12) can be calculated by the equations:

$$K_I = \sqrt{2\pi} \lim_{r \rightarrow 0} r^{1/4} (\sigma_y - \sigma_0), \quad (4.13)$$

$$K_{II} = \sqrt{2\pi} \lim_{r \rightarrow 0} r^{1/4} \tau_{xy},$$

Fig. 7 shows the asymptotic behavior of the interface stresses which are calculated by the edge stress intensity factors according to eq. (4.12). Equation (4.12) is valid for the near region within a range of several percent of the width of the joint, the strength of which determines the ultimate strength of the joint.

### 5. Effects of Size and Thermoelastic Constants of Joint

Consider two rectangular joints, 1 and 2, of similar forms and different sizes. Since the deformation of the similar joints will also be similar, the residual thermal stress  $\sigma_1$  at a point

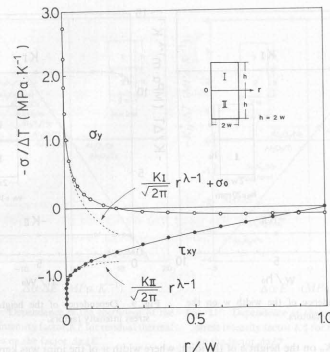


Fig. 7. Asymptotic behavior of the interface stresses calculated by the stress intensity factors.

$(x, y)$  in the joint 1 takes the same value as the residual thermal stress  $\sigma_2$  at the similar point  $(ax, ay)$  in the joint 2<sup>h</sup>:

$$\sigma_1(x, y) = \sigma_2(ax, ay), \quad (5.1)$$

where  $a$  is the ratio of the width  $w_1$  and  $w_2$ , and the heights  $h_1$  and  $h_2$  of the joints:

$$a = w_2/w_1 = h_2/h_1. \quad (5.2)$$

Therefore, the stress intensity factor of the joint 2,  $K_2$ , is related to that of the joint 1,  $K_1$ , by the equation:

$$K_2 = a^{1/4} K_1. \quad (5.3)$$

The value of the stress intensity factor of a larger joint is higher than that of a smaller one. It means that a joint of a larger size is more difficult to be produced than that of a smaller size, as can be observed often in practice.

Fig. 8 shows the dependence of the edge stress intensity factors  $K_I$  and  $K_{II}$  on the width of  $\text{Al}_2\text{O}_3$ /steel joints, where the height of the joint was kept constant as  $h_0 = 20$  mm. The stress intensity factors  $K_I$  and  $K_{II}$  are independent of the width  $w$  for  $w \geq 2h_0$ , while they decrease as the value of the width becomes small. Fig. 9 shows the dependence of the stress intensity

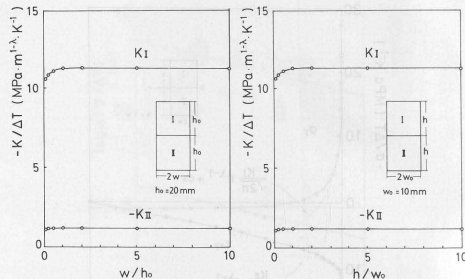


Fig. 8. Dependence of the width  $w$  on the stress intensity factors.

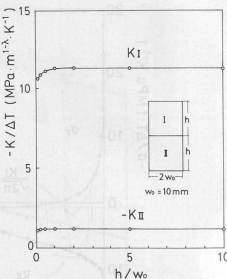


Fig. 9. Dependence of the height  $h$  on the stress intensity factors.

factors  $K_I$  and  $K_{II}$  on the height  $h$  of the joint, where width  $w$  of the joint was kept constant as  $w_0 = 10$  mm. Similarly to the case of the width, the stress intensity factors are independent of the height  $h$  for  $h \geq 2w_0$ , while they decrease as the value of the height  $h$  becomes small.

Effects of the thermoelastic constants of the constituent materials on the edge stress intensity factors were examined for square joints ( $w = 2h$ ). The stress intensity factors can be expressed by the following equations.

$$K_I = K_I - iK_{II} = (K_I^* - iK_{II}^*)F(w), \quad (5.4)$$

$$F(w) = (w/10)^{-\lambda_1}, \quad (5.5)$$

where  $K_I^*$  and  $K_{II}^*$  are coefficients which depend on the thermoelastic parameters and  $F(w)$  is a size function.

The coefficients of the stress intensity factors,  $K_I^*$  and  $K_{II}^*$ , for various ceramic/metal joints are plotted in Figs. 10 and 11 against  $\Delta\alpha\Delta EAT$  and  $\Delta\alpha E^*\Delta T$ , respectively. As can be seen,  $K_I^*$  is proportional to  $\Delta\alpha\Delta EAT$  and  $K_{II}^*$  is proportional to  $\Delta\alpha E^*\Delta T$ . The sign of  $\sigma_x$  at the bonded edge is determined by  $K_I$ . For example of  $E_1 > E_2$ ,  $\alpha_1 < \alpha_2$  and  $\Delta T < 0$ ,  $\sigma_x$  takes a positive value and diverges at the edge of the joint.

The edge stress intensity factors of square joints subjected to a uniform tension  $\sigma_\infty$  are shown in Figs. 12 and 13. Fig. 12 shows that  $K_I^*$  is independent of  $\lambda_1$ , and Fig. 13 shows that  $K_{II}^*$  is proportional to  $\sigma_\infty E^*\Delta E$ .

The results are summarized as follows. The coefficient of the stress intensity factor  $K_I^*$  depends on  $\Delta\alpha\Delta EAT$  for residual thermal stress and on  $\sigma_\infty$  for the uniform tension. Those

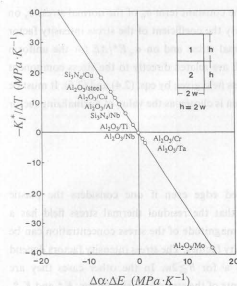


Fig. 10. Dependence of the coefficient of the stress intensity factor  $K_I^*$  for residual thermal stresses on the factor  $\Delta\alpha\Delta E$ .

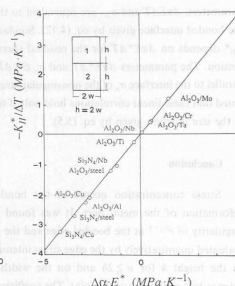


Fig. 11. Dependence of the coefficient of the stress intensity factor  $K_{II}^*$  for residual stresses on the factor  $\Delta\alpha E^*$ .

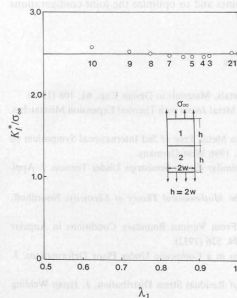


Fig. 12. Dependence of the coefficient of the stress intensity factor  $K_I^*$  for edge stress on the factor  $\lambda_1$  in the case of the uniaxial tension.

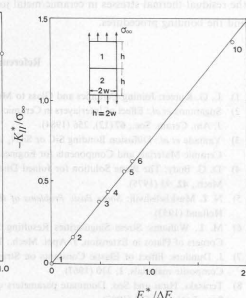


Fig. 13. Dependence of the coefficient of the stress intensity factor  $K_{II}^*$  for edge stress on the factor  $E^*\Delta E$  in the case of the uniaxial tension.



parameters,  $\Delta\alpha E\Delta T$  and  $\sigma_{\alpha\alpha}$ , are equivalent to the constant term  $\sigma_0$  of the normal stress  $\sigma_x$  on the bonded interface given by eq. (4.12). Similarly the coefficient of the stress intensity factor  $K_{II}^*$  depends on  $\Delta\alpha E^* \Delta T$  for the residual thermal stress and on  $\sigma_{\alpha\alpha} E^* / \Delta E$  for the uniform tension. The parameters  $\Delta\alpha E^* \Delta T$  and  $\sigma_{\alpha\alpha} E^* / \Delta E$  are related directly to the stress component parallel to the interface  $\sigma_x$  of the nonsingular stress field given by eqs. (2.4) and (2.5). It must be noted that these linear correlations hold only if  $\alpha$  is chosen as the value of normalizing factor in the size function given by eq. (5.5).

## 6. Conclusion

Stress concentration occurs at the bonded edge even if one considers the plastic deformation of the metal part. It was found that the residual thermal stress field has a singularity of  $r^{1/2}$  at the bonded edge and the magnitude of the stress concentration can be evaluated quantitatively by the edge stress intensity factors. The stress intensity factors depend on the height  $h$  for  $w \geq 2h$  and on the width  $w$  for  $h \geq 2w$ . In the other cases they are independent of the size of the joint. The coefficients of the stress intensity factor,  $K_I^*$  and  $K_{II}^*$ , are proportional to the thermoelastic parameters,  $\Delta\alpha E\Delta T$  and  $\Delta\alpha E^* \Delta T$  respectively. Thus, if once the thermoelastic constants and the size of the joint are given, the local stress field of the edge region can be determined by eq. (4.12). These results will help to predict and characterize the residual thermal stresses in ceramic/metal joints and to optimize the joint configurations and the bonding procedures.

## References

- 1) L. G. Kutzer: Joining Ceramics and Glass to Metals, *Materials in Design Eng.*, **61**, 106 (1965).
- 2) Suganuma *et al.*: Effect of Interlayers in Ceramic-Metal Joints with Thermal Expansion Mismatches, *J. Am. Ceram. Soc.*, **67**(12), 256 (1984).
- 3) Yamada *et al.*: Diffusion Bonding SiC or Si<sub>3</sub>N<sub>4</sub> to Metal, *Proc of 2nd International Symposium on Ceramic Materials and Components for Engines*, 1986 West Germany.
- 4) D. G. Boley: The Plane Solution for Joined Dissimilar Elastic Semistrips Under Tension, *J. Appl. Mech.*, **42**, 93 (1975).
- 5) N. I. Muskhelishvili: *Some Basic Problems of the Mathematical Theory of Elasticity*, Noordhoff, Holland (1953).
- 6) M. L. Williams: Stress Singularities Resulting From Various Boundary Conditions in Angular Corners of Plates in Extension, *J. Appl. Mech.*, **74**, 526 (1952).
- 7) J. Dundurs: Effect of Elastic Constants on Stress in a Composite Under Plane Deformations, *J. Composite materials*, **1**, 310 (1967).
- 8) Terasaki, Hirai and Seo: Dominate parameters of Residual Stress Distribution, *J. Japan Welding Soc.*, **5** (No. 4), 533 (1987).