# 論文の内容の要旨

# Study of the sign problem based on the generalized Lefschetz thimble method using gradient flows without blow-up
# (ブローアップのないグラディエントフローを用いた一般化レフシェッツシンブル法による符号問題の研究)

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In quantum physics, physical quantities are calculated by the path integral. The path integral can be regarded as the multi-dimensional integral, and the stochastic methods such as the Monte Carlo are used to compute it. The point of this method is that the integrand of the path integral is considered as the Boltzmann weight, and the configurations are generated according to the probability distribution proportional to the weight. Then we obtain the expectation value of an observable effectively by the average over the samples. In quantum field theories, especially, this method is a powerful tool to obtain non-perturbative results, and it has been succeeded to study hadron physics using lattice quantum chromodynamics (QCD). However, when the action is a complex number, we no longer can interpret the integrand as the probability, and the direct Monte Carlo method cannot be applied. This problem is called the sign problem. One may consider the imaginary part of the action as a phase factor and include it in the observable. However, as the volume or the inverse temperature increase, the phase fluctuates rapidly, and the path integral gives an exponentially small phase average. Thus in the numerical simulation, an exponentially huge number of configurations are required, and the sign problem is not resolved. Some of the physically interesting systems, for example finite-

density QCD and real-time dynamics, have the sign problem. Therefore the development of the approaches solving the sign problem is important.

In this dissertation, we study the generalized Lefschetz thimble method using gradient flows without the blow-up, in order to solve the sign problem. To solve the sign problem, the generalized Lefschetz thimble method is proposed. The idea of the generalized Lefschetz thimble method is complexifying the integration variables and deforming the integration contour owing to Cauchy's integral theorem. In the generalized Lefschetz thimble method, the gradient flow is used to deform integration contour and suppress the fluctuation of the imaginary part of the action. However, generally the action goes to infinity in a finite time by the gradient flow. The divergence of the action is called the blow-up. The blow-up causes a separation of the integration contour and the ergodicity is broken. To circumvent the blow-up the modification of the gradient flow is proposed. We construct the hybrid Monte Carlo algorithm on the flowed integration contour using the gradient flow without the blow-up. In order to test the validity of the gradient flow without the blow-up, We apply the algorithm to the $(0+1)$-dimensional massive Thirring model at finite density, which is a fermionic model with auxiliary fields. Then we show that the sign problem is mild in the algorithm and the result we obtain agree with the analytic result.