

論文の内容の要旨

Exploration of Classical Integrable Systems Assisted by Neural Networks

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Recently, machine learning techniques are significantly developed and applied to various fields. Especially, one of the machine learning techniques called “neural network” achieves prodigious success. In parallel with the success of machine learning, applications to fundamental research are studied energetically. In physics, various applications have also been reported. They include attempts to automatically extract the models and physical concepts from the data by the ability of neural networks. The approaches are expected to be prominently powerful but still developing. In the present thesis, we propose a method to construct the classical integrable systems by neural networks, which has been considered quite challenging so far. In previous studies, integrable systems have been found via ansatz of the Lax pair or assumptions for solutions of the equation of motion. Our method constructs integrable systems neither the Lax pair nor the assumptions of the solutions. Instead, we assume the Hamiltonian described by the action variables and find the natural Hamiltonian, which is consist of a kinetic term and a potential function. In other words, we construct the potential function exhibiting the classical integrability from the reduced Hamiltonian based on neural networks. The action variables are conserved quantities that characterize the integrable systems. We expect that method provides us unrevealed integrable systems that have not been discovered in previous studies. We represent the canonical transformation of the action variables and the potential function by neural networks. The canonical transformation is represented by a particular neural network constructed with invertible neural networks. Our neural network can express the canonical transformation for the Toda lattice, which has not been archived by the previous proposed neural networks. We also invent several novel techniques to achieve our purpose. We apply the adjoint method with the symplectic integrator to calculate the gradient efficiently. Moreover, we introduce some quantities evaluating the validity of the trained neural networks. We introduce various loss functions and structures of the neural networks and discuss appropriate combinations of them for the present purpose. In particular, we adopt two types of the loss function to find integrable systems. The first loss function consisting of just the loss of conservation of the action variables can find some integrable systems. The loss function of conservation of the action variables evaluates the time series generated by the potential function represented by the neural network. The second loss function consists of loss functions about

conservation of the action variables, an equivalence of the energy, and the statistical properties. We demonstrate the loss function with the loss of the energy and the statistical properties find the desired integrable system through an application to the Toda lattice. As a result, we propose an appropriate construction of the loss function and neural networks for the exploration of the integrable systems. We also obtain the potential function which is the integrable system characterized by a reduced Hamiltonian with perturbations. By analyzing the obtained neural networks, we extract insights into candidates of the integrable systems and determine the analytical representation of the suggested integrable system.