論文の内容の要旨

Quantum algorithms for higher-order quantum transformations of universal unitary operations

（普遍的なユニタリ操作に対する高階量子変換の量子アルゴリズム）

董 青秀雄

Quantum mechanics provides a new understanding of the fundamental physical principles of our world. At the same time, new phenomena discovered in quantum mechanics have been used for enhancing technologies. Quantum information processing is one of such emerging technologies aiming to achieve more efficient information processing than conventional ones. Developing technologies for utilizing quantum systems is essential for quantum information processing, and so are discovering and understanding quantum algorithms. Quantum information processing is usually described as transformations of quantum states, which are quantum operations, but not limited to them. In this thesis, we study *higher-order quantum transformations*, namely, transformations of quantum operations. Higher-order quantum transformations describe relationships between quantum operations or quantum dynamics, and are expected to bring further insights to quantum mechanics. Moreover, they are expected to provide an alternative way of quantum programming and a comprehensive implementation of quantum algorithms. In Chapter 2, we provide an introduction to the basics of quantum information theory and higher-order quantum transformations, and a review of higher-order quantum transformations related to our main results.

Higher-order quantum transformations yield various applications if they are implemented, but it is not obvious how higher-order quantum transformations can be implemented in quantum mechanics and provide advantages. In this thesis, we focus on higher-order quantum transformations on unitary operations instead of general quantum operations, as unitary operations are an essential component in quantum algorithms, and general quantum operations can be represented by unitary operations acting on an extended quantum system.

The first higher-order quantum transformation we focus on is *quantum switch*. Quantum switch is a well-studied higher-order quantum transformation for understanding the causal structure in quantum mechanics. It transforms a pair of input unitary operations into a superposition of two differently ordered concatenations of the pair of input unitary operations, and introduces a coherent superposition of different causal orders which is not exhibited in classical mechanics. The definition of quantum switch extends naturally to general quantum operations beyond unitary operations. Based on this definition, quantum switch is shown to provide several advantages in quantum communication and quantum computation, especially when it acts on general quantum operations. On the other hand, while the definition of quantum switch on unitary operations has a clear operational characterization, it is not clear when it is defined on general quantum operations. In Chapter 3, we investigate the definition of quantum switch on general quantum operations, and show that the action of quantum switch on general quantum operations can be uniquely determined from its action on unitary operations by adding two natural assumptions. This result strengthens the theoretical background for the studies on quantum switch, and provides an operational interpretation of the experimental realization of quantum switch.

The second higher-order quantum transformation we focus on is controllization, transforming a unitary operation into the corresponding controlled unitary operation. Controlled unitary operations are the quantum counterpart of "if-operation" in classical computing, and controllization can be considered to be a quantum counterpart of the "if-clause". Controllization has been studied in various contexts, and there are a few quantum algorithms for controllization proposed under certain assumptions. In Chapter 4, we seek a general framework for understanding and analyzing controllization, and propose a new quantum algorithm for approximate controllization without using an auxiliary system based on the framework. We first extend the definition of a controlled version of general quantum operations beyond unitary operations by extending the definition of controlled unitary operations. We consider three definitions based on two possible physical implementations and one axiomatic approach, and we show that all the three converge to the same definition. We further extend this definition to a controlled version of higher-order quantum transformations, and controllization can be regarded as a controlled version of a certain class of higher-order quantum transformations called *neutralization*. Moreover, we provide a method to reproduce and understand previously known quantum algorithms for controllization, and propose a new quantum algorithm for approximate controllization without using an auxiliary system.

In the third topic, we propose a new structure for higher-order quantum transformations named *success-or-draw* for probabilistic higher-order quantum transformations, which allows a repeat-until-success strategy for implementing them. When a probabilistic algorithm is available, a straightforward method for enhancing the success probability is to perform the same algorithm multiple times until success, i.e., a repeat-until-success strategy. Such a strategy requires the input state to be suitably prepared every time when the algorithm is repeated. In order to perform a higher-order quantum transformation, the input state on which the output operation is applied is necessary in addition to the input operations. However, the input state cannot be universally cloned in quantum mechanics, and moreover, the input state is usually transformed to another state regardless of the success or failure of the probabilistic higher-order quantum transformation. Thus, the applicability of a repeat-until-success strategy is not straightforward for probabilistic higher-order quantum transformations. In Chapter 5, we mathematically identify the structure that guarantees the preservation of the input state on failure as the success-or-draw structure. With the definition of the success-or-draw structure, we prove that this structure is compatible with a large class of higher-order quantum transformations. Especially, any probabilistic higher-order quantum transformation that transforming unitary operations into deterministic quantum operations can obtain the success-or-draw structure by adding the number of uses of the input unitary operation. We also analyze the problem of unitary inversion in terms of success-or-draw, and propose a protocol with a higher success probability than previously known ones.

In Chapter 6, we summarize our results, and propose the possible future scope for further study of higher-order quantum transformations and their applications for quantum algorithms.