論文の内容の要旨

Combinatorial Realization of Minimal Models for Gaiotto-Rapcak's W-algebras (Gaiotto-RapcakによるW代数のミニマル模型の組み合わせ論的実現)

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In this thesis, we study the minimal models for the W-algebras proposed by Gaiotto and Rapcak.

W-algebras originally appeared in the study of two-dimensional conformal field theories (CFTs). They are the generalization of Virasoro algebra and describe a larger symmetry than Virasoro symmetry. That allows us to extract further information on physical quantities and we can sometimes solve the model exactly. The typical examples are minimal models. They possess a salient feature that we can completely determine all of the correlation function. For the construction of these theories, it is essential to understand the representation theory of the W-algebras. However, that is, in general, a difficult problem because the algebraic structure of W-algebras is very complicated.

W-algebras become increasingly important because they play a crucial role not only in 2d CFTs but also in higher dimensional field theories. In particular, significant progress has been made in AdS_3/CFT_2 and 4d supersymmetric gauge theories. It was proposed that higher spin theories on AdS_3 are dual to minimal models living at the 2d boundary. W-algebras also appear in the context of 4d supersymmetric gauge theories. It was discovered that the partition function of 4d $\mathcal{N} = 2$ supersymmetric gauge theories is equal to the correlation function of W-algebras. This duality is now called Alday-Gaiotto-Tachikawa correspondence and has been intensively studied so far.

These studies also developed the technique to deal with W-algebras. It has been gradually recognized that W_N algebras, the most fundamental W-algebras, can be uniformly dealt with as the truncation of much larger W-algebra. Such algebras are described well

by quantum groups referred to as affine Yangians rather than W-algebras themselves. The affine Yangian realizes W-algebras in a totally different way from conventional approaches, which shed new light on the study of the representation theory of W-algebras.

Recently, Gaiotto and Rapcak constructed a new class of W-algebras from supersymmetric gauge theories associated with type IIB string theory. Interestingly, the vacuum character of these W-algebras can be interpreted in terms of a plane partition, which suggests that they are truncations of the affine Yangian. One of the desirable features in this construction is that one can construct infinitely many W-algebras. In this construction, the affine Yangian serves as a building block. Therefore, it is natural to expect that we can study the representation theory of various W-algebras by using plane partitions.

In this thesis, we study the representation theory for the minimal models of these W-algebras by using plane partitions. We first study the W_N minimal model by using a single plane partition. We find that a certain condition imposed on plane partition correctly provides the Hilbert space for the W_N minimal model. As the next step, we consider the generalization to a wider class of W-algebras. We mainly focus on $\mathcal{N} = 2$ unitary minimal models because they play an important role in string theory and we can check whether our construction is correct through the comparison with the known results. We find that we can reproduce the correct conformal dimension, U(1) charge and character from a combination of plane partitions. Finally, we study Bershadsky-Polyakov algebra, which is the simplest example of a non-principal W-algebra. We find that our result reproduces the correct modules.