論文の内容の要旨

On the Possibility of Application of Weak Value Amplification to Precision Measurement

(精密測定への弱値増幅の応用可能性について)

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1 General introduction

Quantum mechanics has been applied to microscopic physical systems such as atoms and nuclei from an early stage and contributed to the understanding of these physical systems. On the other hand, quantum mechanics has changed our understanding of physics including measurement itself.

In fact, in quantum mechanics, a change of the state is inevitable not only by time evolution but also by the process of measurement. Taking account of this, a novel expectation value of an observable $\hat{A}$ has been proposed in 1988 called *weak value*,

$$A_{\mathrm{w}} := \frac{\langle\phi|\hat{A}|\psi\rangle}{\langle\phi|\psi\rangle}, \tag{1}$$

under the state $|\psi\rangle$ before the measurement together with the state $|\phi\rangle$ after the measurement. These states, $|\psi\rangle$ and $|\phi\rangle$, are called the *preselected state* and the *postselected state*, respectively. Since the weak value is an expectation value of an observable $\hat{A}$, one would expect that it must take a real value and stay between the maximum and the minimum eigenvalues of the observable $\hat{A}$. However, it is obvious from its form (1) that the weak value can take an arbitrary complex number.

This property has attracted a lot of researchers for its applicability in precision measurement. In fact, the weak value may take a value much larger than the typical value of $\hat{A}$. This is called *weak value amplification* (WVA) and we expect to improve the sensitivity by the amplification in some cases.

Issues to be resolved for the application of the WVA

Indeed, the success of a series of experiments for precision measurement using the WVA method, beginning with the detection of the spin Hall effect of light by Hosten and Kwiat in 2008, has proven that the WVA is practically useful. However, there are two issues that need to be resolved before extending the applicability of the WVA beyond optical systems for which it has been primarily applied. One is that we still do not know under what conditions or situations the WVA can be effective, and the other is that we also do not know for which systems the process of the WVA can be implemented.

As to the first issue, it is known that the WVA may pose problems with measurement accuracy, statistically or otherwise. For example, the statistical error may become large due to the decrease in the number of signals caused by the postselection, which is required to realize the postselected state $|\phi\rangle$. Due to this postselection, a substantial decrease in the number of signals is unavoidable in using the WVA. It has been pointed out that the effectiveness of the WVA cannot be evaluated unless the problems and other problems arising in the actual experiments are comprehensively addressed.

As to the second issue, it is necessary to consider the conditions for implementing the process of the WVA. For example, in order for the weak value to be made large, we need to ensure that a proper postselection can be implementable in which one realizes a desired postselected state $|\phi\rangle$ by measurement. In other words, it is required that the basis states in the Hilbert space corresponding to the measurement for the postselection can be freely prepared. Quantum theory can be applied virtually to any conceivable physical systems, but there may be physical systems in which the postselection cannot be freely implemented.

2 Strategies of our analysis

In this thesis, we have addressed the above two issues separately. Our strategy for each of these issues is as follows.

On the effectiveness of the WVA

To clarify when the WVA is effective, we have adopted an earlier proposal by Lee and Tsutsui which presented a scheme to discuss the uncertainty involved in the WVA systematically. We have improved their scheme and also confirmed that the condition given in the scheme is indeed responsible for the improvement of the accuracy of the experiments using the WVA method. To demonstrate the usefulness of our improved scheme, we have analyzed two celebrated experiments of the WVA including Hosten's mentioned above.

On the implementability of the WVA

As a first step to studying the implementability of the WVA, we have considered the system of B mesons in which the postselection is normally dictated by the process of particle interaction, not by our free choice. We argue that, despite this obstacle, it is possible to implement the postselection by finding alternative final states which correspond to the postselected states one wishes to realize. Another obstacle related to coherence length of the system can also be resolved by adopting a different scheme in which the weak value appears directly in the system under consideration.

3 Main Results

On the effectiveness of the WVA

As a result of our analysis of the two experiments, we confirmed that the WVA is effective in reducing the level of uncertainty. Specifically, we found that the actual errors in the experiments are much larger than the expected elements of the uncertainty caused by the WVA.

We also found that the experimental parameters employed in these experiments were actually close to the cases of the minimum relative uncertainty, even though the parameters were chosen prior to our analysis.

On the implementability of the WVA

First, when considering the decay process of B mesons, we found that the alternative selection of states for decayed particles can in principle be used for the desired postselection in the B meson systems.

Second, we also found that, if we choose the Hamiltonian operator for the observable $\hat{A}$, the imaginary part of the weak value is related to the lifetime and that this lifetime can be extended, in effect, by 2.6 times compared to the original lifetime of the B meson.

Third, our Monte-Carlo simulation shows that the accuracy of the estimation of the CP violating parameter may be improved by 20%.

4 Conclusion of this thesis

Combining all of these, we conclude that the WVA can indeed be applied for various physical systems including those which do not admit the postselection directly to improve the measurement accuracy.