論文の内容の要旨

Theoretical Study of Binary Bose-Einstein Condensates under Synthetic Gauge Fields

（人工ゲージ場下の二成分ボース・アインシュタイン凝縮体の理論的研究）

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Bose-Einstein condensates (BECs) in ultracold dilute gases of alkali atoms have been studied intensively. By exploiting lasers and external fields, both the states of atoms and an atom-atom interaction can be controlled. Two-component BECs can be created in a mixture of ^{87}Rb atoms in two different hyperfine spin states and spinor BECs with three or more components have also been realized. When a synthetic magnetic field above a critical value is induced in a BEC by rotating the system, a single vortex appears. As the magnetic field is increased, a vortex lattice where multiple quantized vortices are periodically located appears. Entanglement entropy (EE) and entanglement spectrum (ES) are utilized to explore the properties of the system in condensed-matter systems. For example, the subleading term of the EE logarithmically scales with respect to the boundary size in a three-dimensional system with a continuous symmetry breaking. The ES of the topological system has been found to be proportional to the dispersion relation of the edge state. In a pair of non-chiral coupled Tomonaga-Luttinger liquids (TLLs), the ES has a square-root dispersion relation, which is related to the emergence of a long-range interaction potential in the entanglement Hamiltonian. In this thesis, we study vortex lattices in binary BECs to discuss their ground states and collective modes in a mean-field regime. We also discuss the quantum correction to the ground-state energy. We calculate the intercomponent ES and EE in order to reveal the properties of entanglement of our system.

In Chap. 2, we review binary BECs and synthetic gauge fields. First, we review the ground state of binary BECs where two components are mixed or separated by changing the ratio of the intercomponent interaction to the intracomponent one. We explain a synthetic gauge field induced in a neutral-charged atomic gas by rotating the system. We introduce the lowest-

Landau-level (LLL) approximation in which the ground-state wave function is expanded in terms of the basis states in the LLL manifold. By developing the mean-field theory of binary BECs under the synthetic gauge fields induced by mechanical rotation, we explain that the synthetic magnetic fields are parallel for both BECs and that various configurations of vortex lattices appear, which depend on the ratio of the intercomponent interaction to the intracomponent one. We discuss a synthetic gauge field induced by the spatially dependent interaction of atoms and lasers by using the minimal model. We review an experiment in binary BECs, which indeed realized antiparallel synthetic magnetic fields by using lasers. We explain the equivalence between the ground-state phase diagrams of vortex lattices for parallel and antiparallel fields within the mean-field theory. In the quantum Hall regime where the mean-field theory breaks down due to sufficiently strong magnetic fields, the phase diagrams of binary bosonic systems obtained by exact diagonalization for parallel and antiparallel fields are significantly different. It has been found that for parallel (antiparallel) fields, the product states composed of a pair of various quantum Hall states with the small intercomponent EE appear for an attractive (repulsive) intercomponent interaction while the spin-singlet states with the large intercomponent EE appear for a repulsive (attractive) intercomponent interaction.

In Chap. 3, we present an effective-field theory of vortex lattices for binary BECs. We review the mean-field description of a vortex lattice in a scalar BEC, where the short-distance structure of each vortex is separated from a large-scale structure of vortex lattices, and discuss the renormalization of the coupling constants under the LLL approximation. We derive the effective-field theory by extending the theory for the scalar case to the binary system. We identify the missing term in the elastic energy from a symmetry consideration, which is important in discussing interlaced triangular lattices. We find that the coupling constants in binary BECs under the LLL approximation are renormalized and the renormalization factors depend on the ratio of interactions. By diagonalizing the effective Hamiltonian, we obtain the analytical expressions of dispersion relations, correlation functions and a depleted density. We find the rescaling relations between the dispersion relations for parallel and antiparallel fields. From the rescaling relations, we clarify that the different excited states for parallel and antiparallel fields are indeed related to each other. We demonstrate that the one-particle density matrix (the off-diagonal correlation) shows a quasi-long-range order. From the depleted density, we show a logarithmic increase of the fraction of depletion with respect to the number of fluxes.

In Chap. 4, we formulate the Bogoliubov theory of vortex lattices in binary BECs under the LLL approximation. We introduce the LLL magnetic Bloch states which constitute a complete orthonormal basis of the LLL manifold and have a periodic pattern of zeros, to expand the bosonic field operator. We employ the Bogoilubov approximation to derive the Bogoliubov Hamiltonian under the LLL approximation. By performing the Bogoliubov transformation to diagonalize the Hamiltonian, we obtain the energy spectra, the quantum depletion and the correction to the ground-state energy due to zero-point fluctuations for parallel and antiparallel fields. Through a numerical calculation, we obtain the energy spectra with quadratic and linear dispersion relations for parallel and antiparallel fields. We find that the rescaling relations hold for every phase and reveal that the problem in the previous research, where the rescaling relations only hold for overlapping triangular lattices, can be resolved by the renormalization of the coupling constants. By calculating the energy spectra at low energies along the circular path around the Gamma point, we confirm that an anisotropy of the energy spectra is consistent with the effective-field theory. By fitting the analytical expressions of the energy spectra to the numerical results, we obtain the elastic constants as a function of the ratio of the interactions, so that all the parameters introduced in the effective-field theory are determined. We find that the quantum depletion for parallel (antiparallel) fields is larger than that for antiparallel (parallel) fields for a repulsive (attractive) intercomponent interaction. We also confirm the logarithmic divergence of the quantum depletion with respect to the number of fluxes, which indicates the enhancement of the quantum correction in a large system. By calculating the quantum correction to the ground-state energy by the Bogoliubov theory, we find that the transition points shift from those obtained by the mean-field theory, so that the distinction between the phase diagrams for parallel and antiparallel fields appears. We note the result is unique to the binary system since parallel and antiparallel fields only appear in our case. We show that both an inner angle for rhombic lattices and an aspect ratio for rectangular lattices as a function of the ratio of the coupling constants also change from the mean-field results due to the quantum correction. Since shifts of the transition points are larger in parallel fields, we find that the increase of the quantum depletion enhances the shifts.

In Chap. 5, we study the intercomponent ES and EE for vortex lattices in binary BECs. We review the EE and ES in condensed-matter systems. The leading term of the EE is proportional to L^{d-1} where L is the length scale of the subsystem in a variety of quantum many-body systems. This is known as the "area law". In a d-dimensional system with a continuous symmetry breaking, we find that the EE has a subleading term which logarithmically scales with respect to L^{d-1} due to the Nambu-Goldstone modes and the "tower of states". In two non-chiral TLLs coupled by the interaction, the ES has been found to show a square-root or gapped dispersion relation when the energy spectra of the bulk are partially gapless (i.e. one of the symmetric and antisymmetric channels is gapless) or fully gapless. This interesting feature of the ES is related to the emergence of a long-range interaction in the entanglement Hamiltonian. We derive analytical expressions and numerical results of the intercomponent ES and EE of vortex lattices from the effective-field theory and the Bogoliubov theory under the LLL approximation, respectively, on the basis of the phase diagram in the mean-field theory. We obtain the ES with a square-root dispersion relation for every phase and the rescaling relation between the ES for parallel and antiparallel fields in a manner similar to the energy spectra. We discuss which excitation modes make large contributions to the EE from the ES. We also obtain the analytical expression of the long-range interaction in the entanglement Hamiltonian. We find that the intercomponent EE has the leading term which is proportional to the area of the system and the subleading term which scales logarithmically with respect to the number of fluxes. By calculating the EE as a function of the ratio of the interactions, we show that it tends to be larger for a repulsive (attractive) intercomponent interaction in the case of parallel (antiparallel) fields. Since both the EE and the ground states with the quantum correction are obtained by the excitation spectra, the EE is expected to reflect the corrected ground states. Therefore, we can restrict the ground states which appear although the rigorous ground states are hard to obtain. This interesting behavior is qualitatively consistent with the ground-state phase diagram in the quantum Hall regime discussed in Chap. 2, so that the feature found here is supposed to appear in a wide regime. By plotting the EE as a function of the number of fluxes, we find its leading term proportional to the number of fluxes and the subleading term which logarithmically scales with respect to the number of fluxes, which is similar to the EE for systems with a continuous symmetry breaking. We show that the coefficient of the logarithmic term is close to 1/4 for both parallel and antiparallel fields. We find the subleading term is determined by the Nambu-Goldstone modes, which suggests an interesting relation between the symmetries spontaneously broken and the EE.

In Chap. 6, we conclude the present thesis and discuss the outlook.