論 文 の 内 容 の 要 旨

論文題目　A Categorical and Logical Analysis of the π-calculus

（π計算の圏論的および論理的分析）

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A commonly accepted mathematical formalism for describing and proving properties of concurrent systems is the π-*calculus*. The π-calculus provides various operational techniques for reasoning about the behaviour of systems, and this is one of the reasons why the π-calculus has been widely used.

Although the operational aspects of π-calculus have been well-studied, the connection between the π-calculus and other mathematical objects is not well-understood. This is in contrast to the case of λ-calculus, which is a formal calculus often used as the basis for functional programming. The correspondence between simply typed λ-calculus, cartesian closed categories and intuitionistic logic is well-known. Such correspondence has not been discovered for the π-calculus, except for session-typed variants of the π-calculus in which only a limited form of concurrency is expressible. The lack of such correspondence has been preventing the transfer of established techniques from logic and category theory to the field of concurrency.

This thesis makes an effort to extend the three-way correspondence between computation, categories and logic to the π-calculus (not limited to the session-typed variants).

First, we develop a correspondence between a categorical structure, which we call *compact closed Freyd category*, and a variant of the π-calculus, which we call the π_F-*calculus.* Both the compact closed Freyd category and the π_F-calculus are novel and are carefully defined so that they correspond to each other. Although compact closed Freyd category is introduced for a particular purpose, i.e. establishing a correspondence with the π-calculus, its definition is fairly standard: compact closed Freyd categories combines two well-known structures, namely, closed Freyd category and compact closed category. The former is a model of higher-order effectful language, and the latter describes connections via channels. To demonstrate the relevance of the categorical model, we reconstruct the classical results on the relation between higher-order

languages and the π-calculus by a simple semantic consideration using this model.
The categorical analysis of π_F-calculus reveals a fundamental difficulty in developing a categorical type theory for the π-calculus. We show (modulo some reasonable assumption) that conventional behavioural equivalences for the π-calculus are inherently incompatible with categorical semantics. The root cause of this problem lies in the mismatch between the operational and categorical interpretation of a process called the forwarder. From the operational viewpoint, a forwarder may add an arbitrary delay when forwarding a message, whereas, from the categorical perspective, a forwarder must not add any delay when forwarding a message. As an attempt to overcome this gap, we introduce a novel operational semantics for the π_F-calculus in which forwarders do not cause any delay. We then show that this new operational semantics (i) is compatible with the categorical semantics and (ii) can simulate the standard operational semantics.
As for the relation with logic, we discuss the relation between π_F-calculus and linear logic. The relation between linear logic and the π-calculus has been intensively explored since the early phase of the study of the π-calculus. Among others, Abramsky (1994) and Belling and Scott (1994) showed that linear logic proofs can be interpreted using π-calculus processes. Later Caires and Pfenning (2010) showed that if we limit our focus to session-typed processes, the converse also holds, i.e. session typed processes can be interpreted as linear logic proofs. However, it remained an important open problem whether there exists a proof system that is not only interpretable by π-calculus, but that can also interpret the traditional π-calculus processes. We provide a novel observation to this question by constructing an (inconsistent) proof system that one could reasonably expect to correspond to the π_F-calculus. Our construction makes use of the relation between linear logic and compact closed Freyd category. Since a compact closed Freyd category is a specific instance of a categorical model of linear logic, analysing the difference between general categorical models for linear logic and compact closed Freyd categories leads us to an extension of linear logic that corresponds to π_F-calculus.