論 文 の 内 容 の 要 旨

論文題目　Computing Valuations of Determinants via Combinatorial Optimization: Applications to Differential Equations
（組合せ最適化による行列式の付値計算：微分方程式への応用）

氏　　名　　大城　泰平

Degrees of determinants of polynomial matrices often appear as an algebraic formulation of weighted combinatorial optimization problems. For example, weighted Edmonds' problem (WEP), which is to compute the degree of the determinant of a polynomial matrix having symbols, reduces to the weighted bipartite matching problem and the weighted linear matroid intersection and parity problems depending on symbols' pattern. Conversely, the degree of the determinant of an arbitrary polynomial matrix serves as a lower bound on the maximum weight of a perfect matching in the associated edge-weighted bipartite graph. Based on this relation, the combinatorial relaxation algorithm of Murota (1995) computes the degree of the determinant of a polynomial matrix by iteratively solving the weighted bipartite matching problem.

The above property on degrees of determinants extends to valuations of determinants of matrices over valuation fields, or more generally, to valuations of the Dieudonné determinants of matrices over valuation skew fields. In combinatorial optimization, valuations of the Dieudonné determinants arise from a noncommutative version of WEP (nc-WEP). An algebraic abstraction of linear differential and difference equations gives rise to skew polynomials, which are a noncommutative generalization of polynomials. Valuations of Dieudonné determinants of skew polynomial matrices provide information on dimensions of solution spaces of linear differential and difference equations.

The combinatorial relaxation is of importance to preprocessing of differential-algebraic equations (DAEs). In numerical analysis of DAEs, consistent initialization and index reduction are necessary preprocessing prior to the numerical integration. Popular preprocessing methods of Pantelides (1988), Mattsson-Söderlind (1993), and Pryce (2001) are based on the assignment problem on a bipartite graph that represents variable occurrences in equations. The structural methods, however, fail for some DAEs due to inherent numerical or symbolic cancellations. The combinatorial relaxation provides a framework of modifying a DAE into another DAE to which the structural methods are applicable, whereas modification method used in the framework should be appropriately chosen according to the target DAEs.

In the first half of this thesis, we propose two algorithms for computing valuations of the Dieudonné determinants of matrices over valuation skew fields. The algorithms are extensions of the combinatorial relaxation of Murota and the matrix expansion by Moriyama–Murota (2013), both of which are based on combinatorial optimization. We show that the skew polynomials arise as the most general algebraic structure to which these algorithms admit natural extensions. Applications are presented for the nc-WEP and analysis of linear differential and difference equations.

The last half of this thesis is devoted to DAEs' modification methods based on the combinatorial relaxation. This thesis presents three methods for modifying DAEs into other DAEs to which the preprocessing methods can be applied. One method is for linear DAEs whose coefficient matrices are mixed matrices, which are matrices having symbols representing physical quantities. We develop an efficient algorithm that relies on graph and matroid algorithms but not on symbolic computation. Other two deal with general nonlinear DAEs with the aid of symbolic computation engines to manipulate nonlinear formulas. In addition to theoretical guarantees, we conduct numerical experiments on real instances to present practical efficiency.