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Appendix for ”Oligopolistic Competition, Price Rigidity, and Monetary Policy”

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Appendix for "Oligopolistic Competition, Price Rigidity, and Monetary Policy"

Kozo Ueda and Kota Watanabe

2023

A Survey of Consumer Goods Manufacturers

A.1 Survey Questions

This appendix provides the English translation of the survey questions.

Introduction

In this questionnaire, we would like to ask you about ________________ of the products you manufacture and sell.

Q1. First, please indicate the brand with the highest sales value for your company in the above category.

Section A: Questions about the outlook for the rate of price increases in the future

According to the Consumer Price Statistics released by the Statistics Bureau of the Ministry of Internal Affairs and Communications, prices in Japan rose by about 0.7% compared with the level of the previous year. Based on this, we would like you to forecast prices in the future.

Q2. **One year from now**, how do you expect prices to change compared with the current level?

Q3. **In five years' time**, how do you expect prices to change compared with the current level?

Q4. The current yen exchange rate is 109 yen to the dollar. How do you expect the yen/dollar exchange rate to change **one year from now**?

Q5. The Bank of Japan believes that deflation (a fall in prices) is undesirable. In April 2013, it began a policy aimed at raising prices by about 2% annually. Do you know about this policy?

Q6. Do you think that it is desirable to have a policy of increasing prices by about 2% every year?

Q7. Do you think that a policy that aims to raise prices by about 2% annually will be successful?

Q8. Are you familiar with the economic policies of the Abe Cabinet (Abenomics)?

Q9. Do you think that Abenomics has been effective in the economic recovery so far?

We would now like to ask you about the prices of your product (the product you provided in Q1).

Q10. **One year from now**, how do you expect the shipping price of this product to change compared with the current level?

Q11. **In five years' time**, how do you expect the shipping price of this product to change compared with the current level?

Q12. If you chose "3", "4", "5", "6," or "7" in Q11, please answer the following questions. Why do you expect shipping prices to increase little or decrease, compared with the current level?

(1) Raw material prices and labor costs are not expected to rise much, so the cost of goods is not expected to rise either. Hence, there is no need to raise shipping prices.

(2) Raw material prices and labor costs are expected to rise, but we will not be able to raise prices because retailers and other distribution firms are opposed to price increases.

(3) Raw material prices and labor costs are expected to rise, but competitors are unlikely to raise their prices, so we will have to match them.

(4) Raw material and labor costs are expected to rise, but consumers are price sensitive, so we will not be able to pass on the price increases.

(5) Raw material prices and labor costs are expected to rise, but there is no need to raise prices because cost-cutting measures can be taken.

(6) Raw material prices and labor costs are expected to rise, but there is no need to raise prices as this can be handled by increasing productivity.

(7) Raw material and labor costs are expected to rise, but there is no need to raise prices because we can respond by downsizing products (reducing capacity or weight).

(8) Other

We would now like to ask you about your company's wage outlook.

Q13. **One year from now**, how do you expect your company's wages to change compared with the current level? Please think about your company's wages as a whole, including part-time workers as well as full-time workers.

Q14. **In five year's time**, how do you expect your company's wages to change compared with the current level? Please think about your company's wages as a whole, including part-time workers as well as full-time workers.

Section B: Questions about the demand for the product you provided in Q1

Please answer the following questions assuming the following situation.

Suppose that for some reason your manufacturing costs have increased by 10%. Your firm is trying to decide whether to raise your shipping price. You know that your competitors will keep their prices the same.

Q15. If only your company raised its shipping price by 10%, how much do you think the sales volume would change?

Now let's say that for some reason your manufacturing costs have dropped by 10%. As before, your firm is trying to decide whether to lower your shipping price. As before, you know that your competitors will keep their prices the same.

Q16. If only your company lowered its shipping price by 10%, how much do you think the sales volume would change?

This question is for those who answered a combination of the following answers in Q15 and Q16.

Combination of choice "2" in Q15 and choice "1" in Q16

Combination of option "3" in Q15 and option "1" or "2" in Q16

Combination of option "4" in Q15 and option "1" or "2" or "3" in Q16

Q17. Please select from the following reasons why you answered that the decrease in the sales volume associated with the increase in shipping prices would be greater than the increase in the sales volume associated with the decrease in shipping prices.

(1) If you raise the price, regular customers of your products will leave. Therefore, raising the price will cause a significant drop in the sales volume. On the contrary, a price cut will not result in a sudden increase in regular customers. Therefore, even if you lower the price, the sales volume will increase little.

(2) A price hike will become a topic of conversation in the mass media and on the Internet, resulting in a large decrease in the sales volume. A price cut, on the other contrary, will not be talked about in the mass media or on the Internet, so a large increase in the sales volume cannot be expected.

(3) A price hike may cause "anger" among some consumers. This explains the large decrease in the sales volume. However, a price reduction does not mean that consumers will praise you. Many consumers take price cuts for granted, so the sales volume will not increase significantly.

(4) If you raise the price, some retailers will stop carrying your products in their stores. This is the cause of the decline in the sales volume. On the contrary, lowering the price will not result in a sudden increase in the number of retailers that carry your products. Therefore, a price reduction will not result in a large increase in the sales volume.

(5) Other

Section C: Questions related to the depreciation of the yen since 2012

With the inauguration of the Abe administration at the end of 2012, the yen began to weaken. In particular, it weakened significantly from 77 yen to the dollar in December 2012 to 125 yen to the dollar in June 2015. This depreciation of the yen is said to have raised the prices of imported products and raw materials. Please answer the following questions about your company's price setting during this period of the depreciation of the yen. Refer to Figure 1 below (not shown here), which shows the trend of the yen/dollar exchange rate during this period.

Q18. The depreciation of the yen between December 2012 and June 2015, expressed as a percentage, is 62%. In a hypothetical situation, do you think that your company would be able to pass on the weaker yen to prices? In that case, to what extent did your company raise prices to reflect the weaker yen?

Q19. In response to the depreciation of the yen between December 2012 and June 2015, did your company actually raise the shipping price of this product?

Q20. Was your company's price increase for this product in response to the depreciation of the yen between December 2012 and June 2015 sufficient?

If you chose "2" or "3" in Q20, please answer the following questions.

Q21. Please explain why you were not able to fully pass on the weaker yen to shipping prices.

(1) Because you expected a significant decrease in the sales volume if you raised prices.

(2) Because distributors (wholesalers and retailers) were strongly opposed to the price increase.

(3) Because other firms in the same industry were expected to keep their prices unchanged.

(4) Because the depreciation of the yen is unlikely to continue forever and we decided to endure it until it ends.

(5) Because you thought that raising the price might cause anger or antipathy among consumers.

(6) Other

If you chose "2" or "3" in Q20, please answer the following questions.

Q22. If it was insufficient to pass on the weaker yen to shipping prices, it would have been necessary to absorb it in some way. Please tell us how you absorbed it.

(1) Thoroughly reduced wasteful expenses.

(2) Reduced labor costs by keeping wages down.

(3) Reduced labor costs by reducing the number of workers and working hours (e.g., overtime hours)

(4) Refrained from making urgent capital investments.

(5) Improved productivity.

(6) Made up for it with profits from strong exports due to the weak yen.

(7) Lowered manufacturing costs by reducing the size (capacity or weight) of products.

(8) We were not able to stop the deterioration of profits.

(9) No special measures were taken, but profits deteriorated little.

(10) Other

If you chose "1" or "2" in Q20, please answer the following questions.

Q23. As a result of passing on the weaker yen to shipping prices, what changes have you seen in the sales volume and end-user prices?

(1) The sales volume decreased. The extent of the decline was in line with our initial forecast.

(2) The sales volume decreased, but not as much as initially expected.

(3) The sales volume decreased more than initially expected.

(4) Other firms in the same industry also raised their shipping prices, so the decrease in your sales volume was mitigated by that.

(5) Since other firms in the same industry kept their shipping prices unchanged, the decrease in your sales volume was larger than expected.

(6) Retailers' in-store prices (end-user prices) increased in line with your price hike.

(7) Although your company raised its shipping prices, the prices in retail stores (end-user prices) increased little.

(8) Other

Section D: Questions about the overseas development of the product you provided in Q1

Q24 to Q30 not shown here (provided upon request)

Section E: Questions about product downsizing

Q31 to Q36 not shown here (provided upon request)

Section F: Questions about the product you provided in Q1

We recognize that this page may contain some questions that are difficult to answer. Please fill out the form to the best of your ability, and if it is difficult, please put a shaded line in parentheses.

Q37. Including your company, how many firms (competitors to your company) are manufacturing and selling the category to which this product belongs?

Q38. What is your company's share of the overall market for the brand to which this product belongs?

Q39. What is the current shipping price of this product (the price at which your company sells it to wholesalers and retailers)?

Q40. Is the current shipping price of this product higher than that of a competitor's product of the same quality? Is it lower? If there is more than one competitor for this product, please compare it with the main competitor's product.

Q41. Please indicate the current margin for this product and average margin over the past 10 years. Margin in this question refers to the company's own margin (shipping price minus manufacturing cost).

Q42. Please indicate the ratio of the cost of raw materials and labor to the manufacturing cost of this product.

We would now like to ask you about the process you use to set the shipping price of this product (the price at which you sell it to wholesalers and retailers).

Q43. How many employees does your company have?

Q44. How many employees of your company are involved in determining the shipping price?

Q45. How often does your company review its current shipping prices to determine if they are appropriate?

Q46. What factors do your company consider when reviewing shipping prices and determining new shipping prices?

(1) Competitors' trends (e.g., whether they have changed their prices)

(2) Changes in the current sales volume (e.g., has the sales volume been decreasing recently)

(3) Consumer trends (e.g., whether consumer confidence is strong)

(4) Information coming from retailers that handle your products.

(5) Exchange rates

(6) Labor costs (e.g., wage increases)

(7) Price of imported raw materials

(8) Price of domestically procured raw materials

(9) World economic situation

(11) Situation of the Japanese economy

(12) Economic policy of the Japanese government

(13) Monetary policy of the Bank of Japan

(14) Your company's stock price

(15) Other

Q47. Who decides if there will be a change in the shipping price of this product?

Last Section

F1. Finally, please fill in the respondent's company name and department name.

F2. The results of this survey will be compiled into a paper and published on the University of Tokyo's website. Do you wish to have the survey results sent to you by mail?

F3. We are planning to conduct post-survey interviews with those who can cooperate in this survey. If you are willing to cooperate, please send us your contact information below.

A.2 Basic Results from the Survey

Table 1 shows the basic statistics of the sample firms. Figure 1 shows the distribution of the expectation of price changes.

We examine actual price setting, as shown in Table 2, whereas Table 1 in the main paper concerns the expectation of future price changes. In the survey, we focused on a particular event in which firms were faced with one of the largest cost-push shocks in the last decade. The Abe administration that inaugurated at the end of 2012 conducted so-called Abenomics. One of the three arrows of Abenomics was aggressive monetary policy aiming at achieving the country's 2% inflation target in two years' time. The Japanese yen subsequently weakened from 77 yen to the dollar in December 2012 to 125 yen to the dollar in June 2015. This depreciation of the yen is said to have raised the prices of imported products and raw materials considerably. However, actual prices hardly increased, thereby falling well short of the inflation target. The survey asked firms whether the price increase in response to the depreciation of the yen between December 2012 and June 2015 was sufficient (question 20) and then asked those firms that answered "to some extent, but not sufficiently" or "not at all" the following question: "Why were you not able to fully pass on the weaker yen to shipping prices?" (question 21).

Table 2 shows a similar result to Table 1 in the main paper, illustrating the importance of competitors' prices driving the sluggishness of price increases. Of the 139 firms, 28% and 54% answered that this reason was "highly applicable" and "applicable", respectively, amounting to 82% in total (row (3) in the table). Two other reasons are important as well, namely, opposition from retailers and price-sensitive consumers, for which the answers "highly applicable" and "applicable" amounted to 79% and 83%, respectively. From a survey of Eurozone firms, Fabiani et al. (2006) document that the main source of price rigidity is the customer relationship such that a price increase antagonizes customers.

We explore the survey by investigating whether the competitive environment influences the reasons for price rigidity. We run a regression using the answers on three types of reasons, associated with competitors, consumers, and retailers, in question 12 or 21 as a dependent variable. The variable takes 1, 2, 3 or 4, where 1 represents "highly applicable" and 4 represents "not at all." As explanatory variables, we use the logarithm of the number of competitors (question 37) and market share (question 38), which are self reported by firms.

Table 5 shows the estimation results. In Column (1), the coefficient of the number of competitors is significantly negative at the 1% level. This implies that for their price rigidity, firms tend to blame competitors' sluggishness to adjust their prices more as the number of competitors increases. In Column (2), the coefficient of market share is significantly positive at the 5% level. This implies that for their price rigidity, firms tend to blame competitors' sluggishness to adjust their prices more as market share decreases.[1] Columns (3) to (6) show the estimation results when the reason for price rigidity is attributed to consumers or retailers. Except for Column (5), where the coefficient of the number of competitors is significantly negative at the 5% level, no significant coefficient is obtained. Thus, the competitive environment appears to affect pricing behavior through the strategic consideration of competitors' prices.

Figure 2 illustrates the distribution of actual quantitative price setting in response to the depreciation of the yen between December 2012 and June 2015.

Table 1: Basic Statistics of the Sample Firms

	No. of firms	min	p25	p50	mean	p75	max	SD
Total	176							
No. of competitors	129	1	4	7	81.4	15	4,000	406.2
Market share	123	0	5	15	22.8	37	97	22.0
No. of employees	173	20	150	399	1,056.9	980	30,000	2,718.4

Notes: The data are taken from Q37, Q38, and Q43.

[1] In the regression, we use the number of competitors and market share as the explanatory variables. One may wonder whether these variables are highly correlated, and thus, may cause a multicollinearity problem. Table 3 shows that their correlation is significantly positive, but the R squared value is well below one.

Table 2: Reasons for the Insufficient Price Increases in Response to the Depreciation of the Yen (Q21)

	No of firms	1 Highly applicable	2 Applicable	3 Not very applicable	4 Not at all
(1) Because you expected a significant decrease in the sales volume if you raised prices.	109	34.9	48.6	12.8	3.7
(2) Because distributors (wholesalers and retailers) were strongly opposed to the price increase.	110	28.2	41.8	20.9	9.1
(3) Because other firms in the same industry were expected to keep their prices unchanged.	**110**	**23.6**	**39.1**	**27.3**	**10.0**
(4) Because the depreciation of the yen is unlikely to continue forever and we decided to endure it until it ends.	109	3.7	16.5	45.0	34.9
(5) Because you thought that raising the price might cause anger or antipathy among consumers.	109	11.0	33.0	41.3	14.7
(6) Other	14	85.7	14.3	0.0	0.0

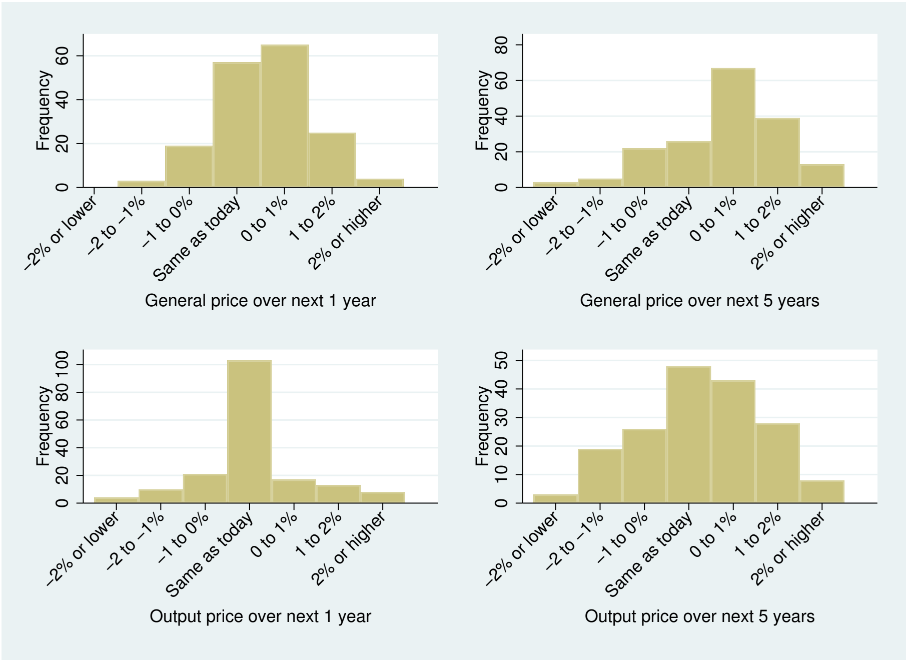
Notes: In the preceding question (Q20), we asked firms, "Was your firm's price increase for this product in response to the depreciation of the yen between December 2012 and June 2015 sufficient?" Then, we asked the firms that chose "to some extent, but not sufficiently" or "not at all" in this question, "Please explain why you were not able to fully pass on the weaker yen to shipping prices." The unit is percent except for the number of firms.

A.3 Relationship between Pricing Behavior and the Competitive Environment

The survey asked the sample firms about their actual price setting practices during rapid yen depreciation between December 2012 and June 2015. The questions included the actual timing (year and month) and size of the shipping price increases (question 19). Using these data, we calculate the number of price changes (frequency) and the total size of price changes (size). We run a regression for these variables using the logarithm of the number of competitors and market share as the explanatory variables.

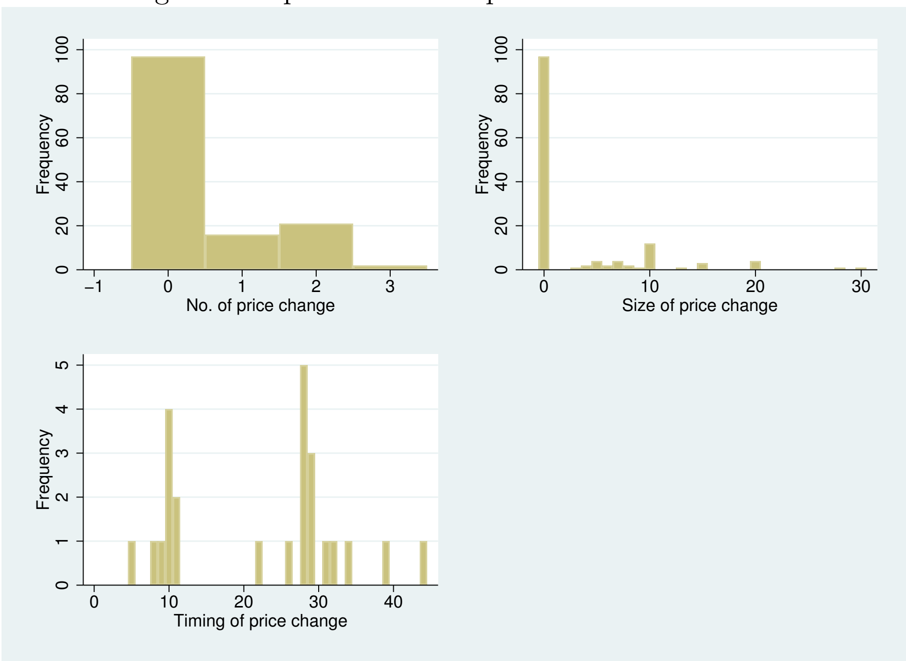
Table 4 shows the estimation results. Column (1) highlights that the coefficient of market share is positive and significant at the 10% level, indicating that the frequency of price changes increases as market share rises. When we incorporate the product category fixed effects in Columns (3) and (4) considering the heterogeneity in price rigidity across product categories, the coefficient of market share is significantly positive at the 5% level, suggesting that both the frequency and the size of price changes increase as market share rises. We also find a similar result when regressing the answers on questions 12 and 21 using the number of competitors and market share (Table 5). Specifically, the coefficients of the number of competitors and market share are negative and positive, respectively, significant at the 5% level, which implies that firms tend to blame competitors' sluggishness to adjust their prices more as the number of competitors increases or as market share decreases. However, the number of observations is small and these estimation results are not robust. In the next subsection, we describe the results of using point-of-sale (POS) scanner data on retailer to

Figure 1: Expectation of Price Changes



Note: The data are taken from Q2, Q3, Q10, and Q11.

Figure 2: Price Changes in Response to the Depreciation of the Yen from December 2012



Note: The data are taken from Q19. The bottom left-hand panel shows when firms raised their prices first in response to the depreciation of the yen that started in December 2012, where the horizontal axis represents months elapsed after December 2012.

Table 3: Relation between the Number of Competitors and Market Share

	(1) Log(no. of competitors)	(2) Log(no. of competitors)
Market share	-0.016***	-0.008
	(0.004)	(0.010)
Constant	2.733***	2.730***
	(0.195)	(0.274)
N	112	42
Category fixed effect	no	yes
No. of categories	–	18
R2	0.071	0.686
Within R2	0.071	0.016

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

increase the number of observations substantially and thus enrich our analyses.

Table 4: Price Changes in Response to the Depreciation of the Yen from December 2012

	(1) Frequency	(2) Size	(3) Frequency	(4) Size
Log(no. of competitors)	0.001	-0.114	-0.207	-1.739
	(0.064)	(0.630)	(0.131)	(1.804)
Market share	0.009*	0.104	0.017**	0.287**
	(0.005)	(0.063)	(0.007)	(0.094)
Constant	0.376	2.835	0.808	4.485
	(0.281)	(2.533)	(0.448)	(5.834)
N	49	49	21	21
Category fixed effect	no	no	yes	yes
No. of categories	–	–	9	9
R2	0.064	0.095	0.627	0.755
Within R2	0.064	0.095	0.322	0.519

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: In Q19, we asked firms, "In response to the depreciation of the yen between December 2012 and June 2015, did your company actually raise the shipping price of this product?" Firms provided the frequency, size, and timing of their price changes.

Table 5: Reasons for Price Rigidity in Relation to Competition

	(1) Q12 (competitors)	**(2) Q21 (competitors)**	(3) Q12 (consumers)	(4) Q21 (consumers)	(5) Q12 (retailers)	(6) Q21 (retailers)
Log(no. of competitors)	**-0.119*****	**-0.058**	-0.104	-0.108	-0.140**	-0.084
	(0.043)	**(0.081)**	(0.074)	(0.067)	(0.062)	(0.075)
Market share	**0.000**	**0.011****	-0.000	-0.002	-0.003	0.002
	(0.004)	**(0.005)**	(0.004)	(0.004)	(0.004)	(0.006)
Constant	**2.116*****	**2.222*****	2.202***	2.199***	2.284***	2.214***
	(0.165)	**(0.265)**	(0.228)	(0.257)	(0.219)	(0.275)
N	**87**	**76**	87	75	86	76
R2	**0.051**	**0.094**	0.033	0.038	0.047	0.023

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: In Q12, we asked firms, "Why do you expect that shipping prices will increase little compared with the current level or will decrease?" In Q21, we asked firms, "Please explain why you were not able to fully pass on the weaker yen to shipping prices." For each question, we asked whether the reason is that competitors are unlikely to raise their prices (competitors), consumers are price sensitive (consumers), or retailers are opposed to price increases (retailers). Firms choose an answer from 1 to 4 for each reason, where 1 is "highly applicable," 2 is "applicable", 3 is "not very applicable," and 4 is "not at all."

B Scanner Data from Retailers

B.1 Identification of Products and Manufacturers

In our dataset, each product is uniquely identified by the Japanese Article Number (JAN) and Nikkei Inc.'s codes. The JAN code obeys the Global Trade Item Number (GTIN) standard and it is composed of manufacturer and product codes. While a manufacturer code, the so-called GS1 code, is specified by GS1 Japan, a product code is specified by the product manufacturer. Since a product code is sometimes reused for different products (e.g., for old and new products with the same product name), Nikkei Inc. provides a code to distinguish products that have the same JAN code.

The manufacturer of each product is identified by using the GS1 code and JAN Item Code File Service (JICFS) data. The GS1 code is a 6- to 10-digit number in the 8- or 13-digit JAN code. Since a firm may have more than one GS1 code, we additionally use the JICFS data provided by GS1 Japan. These data provide information on the main GS1 code for each firm and its branch codes.

In this study, we do not use records in the POS data if a category code cannot be identified by the item master or a GS1 code cannot be identified from the JICFS data.

B.2 Definition of the Variables

We denote the sales amount, quantity purchased, and price purchased for product i at retailer s on day t by s_{ist}, q_{ist}, and $p_{ist} = s_{ist}/q_{ist}$, respectively. The unit price in month m, p_{ism}, is calculated as $p_{ism} = \sum_{t \in M} p_{ist} q_{ist} / \sum_{t \in M} q_{ist}$, where M denotes the set of days in month m.

Regular Price We calculate the regular price for product i at retailer s in month m as

$$\overline{p}_{ism} = mode(p_{ist} | t \in M \text{and} s_{ist} > 0) \tag{1}$$

for the price observed (i.e., $s_{ist} > 0$) for 14 days or more. We select the highest price for $\overline{p}_{ism}$ if two or more modes share the same frequency.

Sales Share and Herfindahl–Hirschman Index (HHI) We calculate the sales share r_{kcy} for each firm k and the firm-level HHI HHI_{cy} for each product category c in year y as

$$r_{kcy} = \frac{\sum_{i \in K \cap C} \sum_s \sum_{t \in Y} p_{ist} q_{ist}}{\sum_{i \in C} \sum_s \sum_{t \in Y} p_{ist} q_{ist}}, \tag{2}$$

$$HHI_{cy} = \sum_k r_{kcy}^2, \tag{3}$$

where K, C, and Y are the set of products produced by firm k, the set of products in category c, and the set of days in year y, respectively. We also calculate the product-level, rather than firm-level, sales share and HHI. In this case, firm k is replaced by product i.

Frequency of Regular Price Changes A regular price change is recorded when such a change is more than two yen. We calculate the frequency of regular price changes for each product i in category c in year y as

$$\begin{aligned} fr_{icy} &= \frac{\sum_s \sum_{m\in Y} \sum_{i\in C\cap\Theta^s_{m-1,m}} \omega_{ism} 1\{\, \overline{p}_{ism} - \overline{p}_{ism-1} > 2\}}{\sum_s \sum_{m\in Y} \sum_{i\in C\cap\Theta^s_{m-1,m}} \omega_{ism}} \\ &\quad + \frac{\sum_s \sum_{m\in Y} \sum_{i\in C\cap\Theta^s_{m-1,m}} \omega_{ism} 1\{\, \overline{p}_{ism} - \overline{p}_{ism-1} < -2\}}{\sum_s \sum_{m\in Y} \sum_{i\in C\cap\Theta^s_{m-1,m}} \omega_{ism}} \\ &= fr^+_{icy} + fr^-_{icy}, \end{aligned} \tag{4}$$

where ω_{ism} is the weight function given by $(s_{ism-1} + s_{ism})/2$ and $\Theta^s_{m-1,m}$ denotes the product set sold at retailer s in both months $m-1$ and m.

At the firm level, the frequency of regular price changes for each firm k in category j in year y is defined as

$$\begin{aligned} fr_{kcy} &= \frac{\sum_s \sum_{m\in Y} \sum_{i\in K\cap C\cap\Theta^s_{m-1,m}} \omega_{ism} 1\{\, \overline{p}_{ism} - \overline{p}_{ism-1} > 2\}}{\sum_s \sum_{t\in Y} \sum_{i\in K\cap C\cap\Theta^s_{m-1,m}} \omega_{ism}} \\ &\quad + \frac{\sum_s \sum_{m\in Y} \sum_{i\in K\cap C\cap\Theta^s_{m-1,m}} \omega_{ism} 1\{\, \overline{p}_{ism} - \overline{p}_{ism-1} < -2\}}{\sum_s \sum_{t\in Y} \sum_{i\in K\cap C\cap\Theta^s_{m-1,m}} \omega_{ism}} \\ &= fr^+_{kcy} + fr^-_{kcy}. \end{aligned} \tag{5}$$

Price Changes for Each Firm We calculate the month-to-month Tornqvist price change for firm k in category c in month m as

$$\pi_{kcm} = \frac{1}{2} \sum_s \sum_{i\in K\cap C\cap\Theta^s_{m-1,m}} \left(\frac{s_{ism-1}}{\sum_s \sum_{i\in K\cap C\cap\Theta^s_{m-1,m}} s_{ism-1}} + \frac{s_{ism}}{\sum_s \sum_{i\in K\cap C\cap\Theta^s_{m-1,m}} s_{ism}} \right) \log\left(\frac{p_{ism}}{p_{ism-1}}\right), \tag{6}$$

where $s_{ism} = \sum_{t\in M} p_{ist} q_{ist}$.

B.3 Further Estimation Results

B.3.1 Relationship between the Frequency of Regular-Price Changes and Competitive Environment

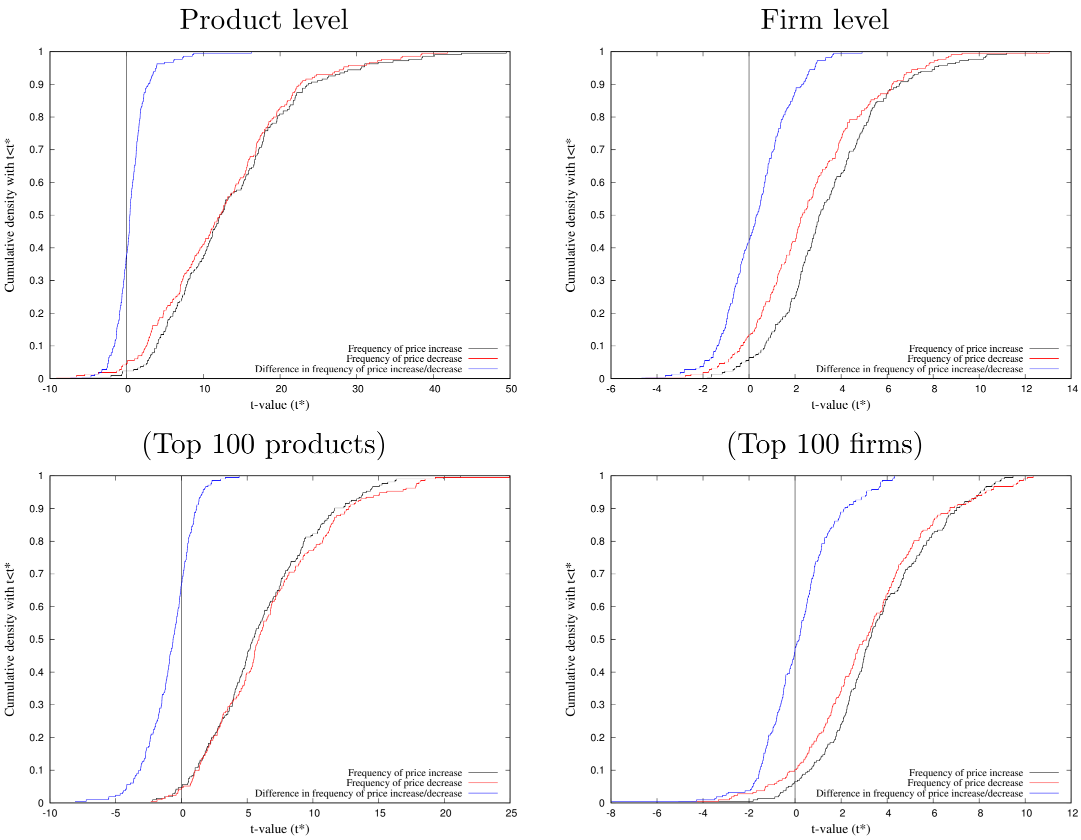
We investigate the relationship between market competitiveness and price setting behaviors, particularly the frequency of regular-price changes. For each product category c, we regress the frequency of regular price increases/decreases for each product/firm i on its market share as

$$fr^X_{icy} = \alpha \log s_{icy} + \sum_{y=1}^{31} \beta_y d^{year}_y + \sum_{k=1}^{K-1} m_k d^{firm}_k + \varepsilon_{icy}, \tag{7}$$

where fr^X_{icy} and s_{icy} represent the frequency of regular-price changes (X represents the direction of the change $\{+,-\}$) and the sales share of product/firm i in product category c in year y, respectively. We include the period and firm fixed effects by adding the dummy variables d_y^{year} and d_k^{firm}, where the time subscript y takes an integer from 1 to 31 (each representing the year $1988 + y$) and K denotes the number of firms. In the firm-level regression, i equals k.

Figure B.3.1 shows the results of the above regression at the product/firm level. The curves represent the cumulative distribution, where the vertical axis is the proportion of the product categories in which the t-value of the coefficient of market share, α, takes a value less than or equal to t^* (horizontal axis). The intercepts of the curves at $t^* = 0$ are less than 0.1, which suggests that α is positive for more than 90% of the product categories for both regular price increases and decreases. Furthermore, the curves in the left-hand panels take lower values than 0.2 at $t^* = 2$, which suggests that α is significantly positive at the 5% level for more than 80% of the product categories when we investigate the relationship between market competitiveness and the frequency of regular price changes at the product level. When we investigate this relationship at the firm level (see the right-hand panels), the results are weaker; however, α is significantly positive for the majority of the product categories. Comparing regular price increases and decreases, we find that the frequency of price increases is more strongly associated with a firm's market share than that of price decreases. Additionally, we run the regression by using the frequency of price increases minus that of price decreases for the dependent variable. The figure shows no significant relationship between market competitiveness and the difference in the frequency of regular price increases and decreases. In sum, the estimation results suggest that the frequency of regular price changes tends to increase as the market share of the product/firm is large. The market leader, rather than market followers, changes its prices frequently.

Figure 3: Cumulative Distribution of the t-value of the Coefficient of Market Share



Notes: We run the regression for the frequency of regular price increases/decreases on market share for each product category. The vertical axis represents the proportion of the product categories in which the t-value of the coefficient of market share takes a value less than or equal to t^* (horizontal axis). The left- and right-hand panels show the results for the product- and firm-level regressions, respectively. The upper and lower panels show the results when using all the products/firms and top 100 products/firms in terms of market share $\times$ observation periods, respectively.

C Input Prices

C.1 Construction of the Category-Level Input Prices

We use the input/output (IO) table for the basic sector (narrowest category) as of 2011. Let us denote the IO matrix as A_{ij}, where i and c represent a good and service category for inputs and outputs, respectively ($i = 1, 2, \cdots, 518$ and $c = 1, 2, \cdots, 397$). While it would be desirable to use an IO table valued at purchase prices for the analysis of input prices, we use an IO table valued at producer prices because a table for the narrowest category is valued only at producer prices. The reason we use the IO table as of 2011 is that the Corporate Goods Price Index (CGPI), more specifically, Input-Output Price Index (IOPI), compiled by the Bank of Japan uses 2011 as a base year. The IOPI provides us with an output price index p_{it} for a good and service category c in month t, where category c is virtually the same as that for the inputs in the IO table.

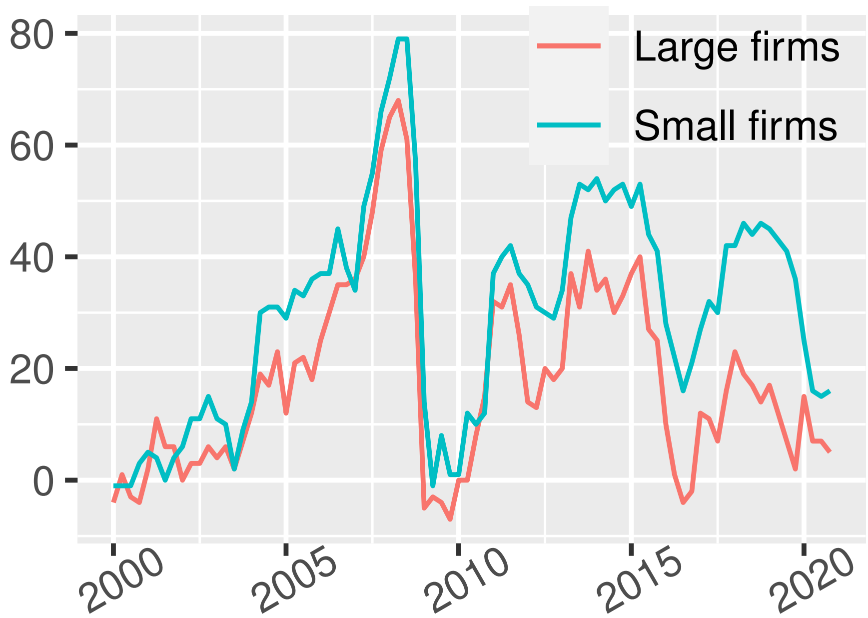
The input price for category c, p_{cm}^{input}, is defined as

$$p_{cm}^{input} = \sum_i A_{ic} p_{im} / \sum_i A_{ic}. \tag{8}$$

C.2 Changes in Input Prices for Large and Small Firms

The Bank of Japan conducts a quarterly survey, Tankan, in which it asks firms about their judgment on the changes in the yen-based purchase prices of the main raw materials (including processing fees for subcontractors) and/or the main merchandise paid by responding firms during the three months before the survey. Firms choose an answer from rise, unchanged, and fall, and their answers are aggregated into the diffusion index (DI). Figure 4 shows the changes in input prices based on DI for large and small firms.

Figure 4: Input Prices for Large and Small Firms



Note: In the Tankan, firms with capital of 1 billion yen or more are large, whereas those with capital from 20 million yen to less than 100 million yen are small. Source: the Bank of Japan "Tankan"

D Model

The model is extended from that in Ueda (2023) to incorporate firm asymmetry.

D.1 Demand System

We consider an arbitrary invertible demand system $x_t^i = x^i(p_t^i, p_t^{-i}; M_t) = x^i(p_t^i/M_t, p_t^{-i}/M_t)$ for firm i. Firm profit is given by $\Pi_t^i = (p_t^i - W_t/\phi^i)x^i(p_t^i/M_t, p_t^{-i}/M_t)$, where $\phi^i > 0$ represents firm-specific time-invariant productivity. We assume that nominal spending must be equal to the money supply, which yields $P_tC_t = M_t = W_t$. If firms are symmetric, demand is shared equally between them, which equals $x^i \equiv M_t/(nP_t) = 1/(np)$ in the steady state. We define the demand elasticities as

$$\Psi^i \equiv \frac{\partial \log x^i(p^i/M, p^{-i}/M)}{\partial \log(p^i/M)} \tag{9}$$

$$\Psi^{-i} \equiv \frac{\partial \log x^i(p^i/M, p^{-i}/M)}{\partial \log(p^{-i}/M)} \tag{10}$$

$$\Psi^{i,i} \equiv \frac{\partial \Psi^i}{\partial \log(p^i/M)} \tag{11}$$

$$\Psi^{-i,-i} \equiv \frac{\partial \Psi^{-i}}{\partial \log(p^{-i}/M)} \tag{12}$$

$$\Psi^{i,-i} \equiv \frac{\partial \Psi^{-i}}{\partial \log(p^i/M)} = \frac{\partial \Psi^i}{\partial \log(p^{-i}/M)}. \tag{13}$$

Similarly, the demand elasticities for firm $-i$, which is the competitor of firm i, are defined by using an asterisk such as

$$\Psi^{-i*} \equiv \frac{\partial \log x^{-i*}(p^{-i}/M, p^i/M)}{\partial \log(p^{-i}/M)}. \tag{14}$$

CES and Oligopolistic Competition We assume that the number of firms is finite given by n ($n = 2, 3, \cdots$). In the case of CES preferences, for each product line $j \in [0, 1]$, consumption is aggregated following the CES form of aggregation: $c_t^j = \left\{\sum_{i=1}^n \left(b^i/n\right)^{1/\sigma} \left(x_t^i\right)^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}$, where $\sum_{i=1}^n \frac{b^i}{n} = 1$. Parameter b^i captures consumers' taste for the good produced by firm i. A high b^i implies the high competitiveness (profitability) of firm i.

This formulation yields demand $x_t^i = \frac{b^i}{n}\left(\frac{p_t^i}{P_t}\right)^{-\sigma}\frac{M_t}{P_t}$ and $P_t = \left\{\sum_{i=1}^n \frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$. Thus, we obtain $\log x_t^i = \log(b^i/n) - \sigma(\log(p_t^i/M_t) - \log(P_t/M_t)) - \log(P_t/M_t) = \log(b^i/n) - \sigma\log(p_t^i/M_t) - \log\left\{\sum_{i=1}^n \frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}$ and, in turn,

$$\begin{aligned}
\Psi^i &= -\sigma - \left\{\sum_{i=1}^n \frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}^{-1}\frac{b^i}{n}(1-\sigma)\left(p_t^i/M_t\right)^{-\sigma}\left(p_t^i/M_t\right) \\
&= -\sigma - \left\{\sum_{i=1}^n \frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}^{-1}\frac{b^i}{n}(1-\sigma)\left(p_t^i/M_t\right)^{1-\sigma} \\
&= -\sigma - \frac{b^i}{n}(1-\sigma)\left\{P^{1-\sigma}\right\}^{-1}\left(p^i\right)^{1-\sigma} \\
&= -\sigma - \frac{b^i}{n}(1-\sigma)(p^i/P)^{1-\sigma},
\end{aligned}$$

$$
\begin{aligned}
\Psi^{-i} &= -\left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}^{-1}\frac{b^{-i}}{n}(1-\sigma)\left(p_t^{-i}/M_t\right)^{1-\sigma}\\
&= \frac{b^{-i}}{n}(\sigma-1)(p^{-i}/P)^{1-\sigma},
\end{aligned}
$$

$$
\begin{aligned}
\Psi^{i,i} &= \left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}^{-2}\left\{\frac{b^i}{n}(1-\sigma)\left(p_t^i/M_t\right)^{1-\sigma}\right\}^2\\
&- \left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}^{-1}\frac{b^i}{n}(1-\sigma)(1-\sigma)\left(p_t^i/M_t\right)^{1-\sigma}\\
&= \left\{P^{1-\sigma}\right\}^{-2}\left\{\frac{b^i}{n}(1-\sigma)\left(p^i\right)^{1-\sigma}\right\}^2 - \left\{P^{1-\sigma}\right\}^{-1}\frac{b^i}{n}(1-\sigma)^2\left(p^i\right)^{1-\sigma}\\
&= \left\{\frac{b^i}{n}(1-\sigma)(p^i/P)^{1-\sigma}\right\}^2 - \frac{b^i}{n}(1-\sigma)^2(p^i/P)^{1-\sigma}\\
&= -\left\{1-\frac{b^i}{n}(p^i/P)^{1-\sigma}\right\}\left\{\frac{b^i}{n}(1-\sigma)^2(p^i/P)^{1-\sigma}\right\},
\end{aligned}
$$

$$
\begin{aligned}
\Psi^{-i,-i} &= \left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}^{-2}\left\{\frac{b^{-i}}{n}(1-\sigma)\left(p_t^{-i}/M_t\right)^{1-\sigma}\right\}^2\\
&- \left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}^{-1}\frac{b^{-i}}{n}(1-\sigma)(1-\sigma)\left(p_t^{-i}/M_t\right)^{-\sigma}\\
&= \left\{P^{1-\sigma}\right\}^{-2}\left\{\frac{b^{-i}}{n}(1-\sigma)\left(p^{-i}\right)^{1-\sigma}\right\}^2 - \left\{P^{1-\sigma}\right\}^{-1}\frac{b^{-i}}{n}(1-\sigma)^2\left(p^{-i}\right)^{1-\sigma}\\
&= \frac{b^{-i}}{n}(1-\sigma)^2(p^{-i}/P)^{1-\sigma}\left\{\frac{b^{-i}}{n}(p^{-i}/P)^{1-\sigma}-1\right\},
\end{aligned}
$$

$$
\begin{aligned}
\Psi^{i,-i} &= \left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i/M_t\right)^{1-\sigma}\right\}^{-2}\frac{b^i}{n}(1-\sigma)\left(p_t^i/M_t\right)^{1-\sigma}\frac{b^{-i}}{n}(1-\sigma)\left(p_t^{-i}/M_t\right)^{1-\sigma}\\
&= \left\{P^{1-\sigma}\right\}^{-2}\left\{\frac{b^i}{n}(1-\sigma)\right\}^2 b^i b^{-i}\left(p^i\right)^{1-\sigma}\left(p^{-i}\right)^{1-\sigma}\\
&= \left(\frac{1-\sigma}{n}\right)^2 b^i b^{-i}(p^i/P)^{1-\sigma}(p^{-i}/P)^{1-\sigma},
\end{aligned}
$$

where we use $d(x)/dlogx = d(x)/(dx/x) = x$.

An increase in b^i decreases the absolute value of own elasticity $|\Psi^i|$, increases the cross elasticity Ψ^{-i}, and increases the cross superelasticity $\Psi^{i,-i}$ for firm i.

A decrease in p^i relative to P (due to an increase in firm i's relative productivity) decreases the absolute value of own elasticity $|\Psi^i|$, decreases the cross elasticity Ψ^{-i}, and increases the cross superelasticity $\Psi^{i,-i}$ for firm i when $\sigma > 1$.

When the number of firms is infinite ($n \to \infty$), the demand elasticities equal

$$\Psi^i = -\sigma,$$

$$\Psi^{-i} = \Psi^{i,i} = \Psi^{-i,-i} = \Psi^{i,-i} = 0.$$

No strategic consideration is taken into account. The elasticities are independent of b^i.

Specifically, we consider two firms A and B ($n = 2$). From $\sum_{i=1}^{2} \frac{b^i}{n} = 1$, we assume $b^B = 2 - b^A \equiv 2 - b$. Then, we have

$$\Psi^A = -\sigma - \frac{b}{2}(1-\sigma)(p^A/P)^{1-\sigma},$$

$$\Psi^B = \frac{2-b}{2}(\sigma-1)(p^B/P)^{1-\sigma},$$

$$\Psi^{AA} = -\left\{1 - \frac{b}{2}(p^A/P)^{1-\sigma}\right\}\left\{\frac{b}{2}(1-\sigma)^2(p^A/P)^{1-\sigma}\right\},$$

$$\Psi^{BB} = -\frac{2-b}{2}(1-\sigma)^2(p^B/P)^{1-\sigma}\left\{1 - \frac{2-b}{2}(p^B/P)^{1-\sigma}\right\},$$

$$\Psi^{AB} = \Psi^{BA} = (1-\sigma)^2\frac{b(2-b)}{4}(p^A/P)^{1-\sigma}(p^B/P)^{1-\sigma},$$

$$\Psi^{B*} = -\sigma - \frac{2-b}{2}(1-\sigma)(p^B/P)^{1-\sigma},$$

$$\Psi^{A*} = \frac{b}{2}(\sigma-1)(p^A/P)^{1-\sigma},$$

$$\Psi^{BB*} = -\left\{1 - \frac{2-b}{2}(p^B/P)^{1-\sigma}\right\}\left\{\frac{2-b}{2}(1-\sigma)^2(p^B/P)^{1-\sigma}\right\},$$

$$\Psi^{AA*} = -\frac{b}{2}(1-\sigma)^2(p^A/P)^{1-\sigma}\left\{1 - \frac{b}{2}(p^A/P)^{1-\sigma}\right\},$$

$$\Psi^{BA*} = \Psi^{AB*} = (1-\sigma)^2\frac{b(2-b)}{4}(p^A/P)^{1-\sigma}(p^B/P)^{1-\sigma},$$

where $P = \left\{\frac{b}{2}\left(p^A\right)^{1-\sigma} + \frac{2-b}{2}\left(p^B\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$. Note that these elasticities satisfy $\Psi^A + \Psi^B = \Psi^{A*} + \Psi^{B*} = -1$,

$$\begin{aligned}
\Psi^A(\Psi^A+1) - \Psi^{AA} &= \left\{-\sigma - \frac{b}{2}(1-\sigma)(p^A/P)^{1-\sigma}\right\}\left\{-\sigma - \frac{b}{2}(1-\sigma)(p^A/P)^{1-\sigma} + 1\right\} \\
&\quad + \left\{1 - \frac{b}{2}(p^A/P)^{1-\sigma}\right\}\left\{\frac{b}{2}(1-\sigma)^2(p^A/P)^{1-\sigma}\right\} \\
&= \sigma(\sigma-1)\left\{1 - \frac{b}{2}(p^A/P)^{1-\sigma}\right\},
\end{aligned}$$

$$\begin{aligned}
\Psi^{AB} - \Psi^B &= \left(\frac{1-\sigma}{2}\right)^2 b(2-b)(p^A/P)^{1-\sigma}(p^B/P)^{1-\sigma} - \frac{2-b}{2}(\sigma-1)(p^B/P)^{1-\sigma} \\
&= \frac{(2-b)(\sigma-1)}{2}(p^B/P)^{1-\sigma}\left\{\frac{\sigma-1}{2}b(p^A/P)^{1-\sigma} - 1\right\}.
\end{aligned}$$

Nested CES and Oligopolistic Competition Motivated by Atkeson and Burstein (2008), we consider the following demand function: $x_t^i = \frac{b^i}{n}\left(\frac{p_t^i}{P_t}\right)^{-\sigma}\left(\frac{P_t}{P_t^Y}\right)^{-\eta} Y_t$, where $1 < \eta < \sigma$ and $P_t = \left\{\sum_{i=1}^{n} \frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$. Final consumption Y_t and price P_t^Y are taken as given. We obtain

$\log x_t^i = constant - \sigma \log(p_t^i) + (\sigma - \eta)\log(P_t) = constant - \sigma \log(p_t^i) - \frac{\sigma-\eta}{\sigma-1}\log\left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}$ and, in turn,

$$\begin{aligned}
\Psi^i &= -\sigma - \frac{\sigma-\eta}{\sigma-1}\left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}^{-1}\frac{b^i}{n}(1-\sigma)\left(p_t^i\right)^{-\sigma}\left(p_t^i\right)\\
&= -\sigma - \frac{\sigma-\eta}{\sigma-1}\left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}^{-1}\frac{b^i}{n}(1-\sigma)\left(p_t^i\right)^{1-\sigma}\\
&= -\sigma - \frac{\sigma-\eta}{\sigma-1}\frac{b^i}{n}(1-\sigma)\left\{P^{1-\sigma}\right\}^{-1}(p^i)^{1-\sigma}\\
&= -\sigma + (\sigma-\eta)\frac{b^i}{n}(p^i/P)^{1-\sigma},
\end{aligned}$$

$$\begin{aligned}
\Psi^{-i} &= -\frac{\sigma-\eta}{\sigma-1}\left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}^{-1}\frac{b^{-i}}{n}(1-\sigma)\left(p_t^{-i}\right)^{1-\sigma}\\
&= \frac{b^{-i}}{n}(\sigma-\eta)(p^{-i}/P)^{1-\sigma},
\end{aligned}$$

$$\begin{aligned}
\Psi^{i,i} &= \frac{\sigma-\eta}{\sigma-1}\left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}^{-2}\left\{\frac{b^i}{n}(1-\sigma)\left(p_t^i\right)^{1-\sigma}\right\}^2\\
&- \frac{\sigma-\eta}{\sigma-1}\left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}^{-1}\frac{b^i}{n}(1-\sigma)(1-\sigma)\left(p_t^i\right)^{1-\sigma}\\
&= \frac{\sigma-\eta}{\sigma-1}\left\{P^{1-\sigma}\right\}^{-2}\left\{\frac{b^i}{n}(1-\sigma)\left(p^i\right)^{1-\sigma}\right\}^2 - \frac{\sigma-\eta}{\sigma-1}\left\{P^{1-\sigma}\right\}^{-1}\frac{b^i}{n}(1-\sigma)^2\left(p^i\right)^{1-\sigma}\\
&= \frac{\sigma-\eta}{\sigma-1}\left\{\frac{b^i}{n}(1-\sigma)(p^i/P)^{1-\sigma}\right\}^2 - \frac{\sigma-\eta}{\sigma-1}\frac{b^i}{n}(1-\sigma)^2(p^i/P)^{1-\sigma}\\
&= -\frac{\sigma-\eta}{\sigma-1}\left\{1-\frac{b^i}{n}(p^i/P)^{1-\sigma}\right\}\left\{\frac{b^i}{n}(1-\sigma)^2(p^i/P)^{1-\sigma}\right\},
\end{aligned}$$

$$\begin{aligned}
\Psi^{i,-i} &= \frac{\sigma-\eta}{\sigma-1}\left\{\sum_{i=1}^{n}\frac{b^i}{n}\left(p_t^i\right)^{1-\sigma}\right\}^{-2}\frac{b^i}{n}(1-\sigma)\left(p_t^i\right)^{1-\sigma}\frac{b^{-i}}{n}(1-\sigma)\left(p_t^{-i}\right)^{1-\sigma}\\
&= \frac{\sigma-\eta}{\sigma-1}\left\{P^{1-\sigma}\right\}^{-2}\left\{\frac{b^i}{n}(1-\sigma)\right\}^2 b^i b^{-i}\left(p^i\right)^{1-\sigma}\left(p^{-i}\right)^{1-\sigma}\\
&= \left(\frac{\sigma-\eta}{n}\right)^2 b^i b^{-i}(p^i/P)^{1-\sigma}(p^{-i}/P)^{1-\sigma}.
\end{aligned}$$

An increase in b^i decreases the absolute value of own elasticity $|\Psi^i|$, increases the cross elasticity Ψ^{-i}, and increases the cross superelasticity $\Psi^{i,-i}$ for firm i.

Hotelling's Address Model and Duopolistic Competition In Hotelling's model, demand is given by $x_t^A = \left(\frac{\delta}{2} - \frac{\log(p_t^A/M_t)-\log(p_t^B/M_t)}{2\tau}\right)\frac{M_t}{p_t^A}$ and $x_t^B = \left(\frac{2-\delta}{2} - \frac{\log(p_t^B/M_t)-\log(p_t^A/M_t)}{2\tau}\right)\frac{M_t}{p_t^B}$. Thus, $\log x_t^A = \log\left(\frac{\delta}{2} - \frac{\log(p_t^A/M_t)-\log(p_t^B/M_t)}{2\tau}\right) - \log(p_t^A/M_t)$,

$$\begin{aligned}
\Psi^A &= \left(\frac{\delta}{2} - \frac{\log(p_t^A/M_t)-\log(p_t^B/M_t)}{2\tau}\right)^{-1}\left(-\frac{1}{2\tau}\right) - 1\\
&= -\frac{1}{\tau}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-1} - 1,
\end{aligned}$$

$$\Psi^{B} = \left(\frac{\delta}{2} - \frac{\log(p_t^A/M_t) - \log(p_t^B/M_t)}{2\tau}\right)^{-1}\left(\frac{1}{2\tau}\right)$$
$$= \frac{1}{\tau}\left(\delta - \frac{\log(p^A) - \log(p^B)}{\tau}\right)^{-1},$$

$$\Psi^{AA} = -\left(\frac{\delta}{2} - \frac{\log(p_t^A/M_t) - \log(p_t^B/M_t)}{2\tau}\right)^{-2}\left(-\frac{1}{2\tau}\right)\left(-\frac{1}{2\tau}\right)$$
$$= -\frac{1}{\tau^2}\left(\delta - \frac{\log(p^A) - \log(p^B)}{\tau}\right)^{-2},$$

$$\Psi^{BB} = -\left(\frac{\delta}{2} - \frac{\log(p_t^A/M_t) - \log(p_t^B/M_t)}{2\tau}\right)^{-2}\left(\frac{1}{2\tau}\right)\left(\frac{1}{2\tau}\right)$$
$$= -\frac{1}{\tau^2}\left(\delta - \frac{\log(p^A) - \log(p^B)}{\tau}\right)^{-2},$$

$$\Psi^{AB} = \Psi^{BA} = -\left(\frac{\delta}{2} - \frac{\log(p_t^A/M_t) - \log(p_t^B/M_t)}{2\tau}\right)^{-2}\left(-\frac{1}{2\tau}\right)\left(\frac{1}{2\tau}\right)$$
$$= \frac{1}{\tau^2}\left(\delta - \frac{\log(p^A) - \log(p^B)}{\tau}\right)^{-2}.$$

Similarly, the demand elasticity for firm B, denoted with an asterisk, is as follows:

$$\Psi^{B*} = \left(\frac{2-\delta}{2} - \frac{\log(p_t^B/M_t) - \log(p_t^A/M_t)}{2\tau}\right)^{-1}\left(-\frac{1}{2\tau}\right) - 1$$
$$= -\frac{1}{\tau}\left(2 - \delta - \frac{\log(p^B) - \log(p^A)}{\tau}\right)^{-1} - 1,$$

$$\Psi^{A*} = \left(\frac{2-\delta}{2} - \frac{\log(p_t^B/M_t) - \log(p_t^A/M_t)}{2\tau}\right)^{-1}\left(\frac{1}{2\tau}\right)$$
$$= \frac{1}{\tau}\left(2 - \delta - \frac{\log(p^B) - \log(p^A)}{\tau}\right)^{-1},$$

$$\Psi^{BB*} = -\left(\frac{2-\delta}{2} - \frac{\log(p_t^B/M_t) - \log(p_t^A/M_t)}{2\tau}\right)^{-2}\left(-\frac{1}{2\tau}\right)\left(-\frac{1}{2\tau}\right)$$
$$= -\frac{1}{\tau^2}\left(2 - \delta - \frac{\log(p^B) - \log(p^A)}{\tau}\right)^{-2},$$

$$\Psi^{AA*} = -\left(\frac{2-\delta}{2} - \frac{\log(p_t^B/M_t) - \log(p_t^A/M_t)}{2\tau}\right)^{-2}\left(\frac{1}{2\tau}\right)\left(\frac{1}{2\tau}\right)$$
$$= -\frac{1}{\tau^2}\left(2 - \delta - \frac{\log(p^B) - \log(p^A)}{\tau}\right)^{-2},$$

$$\Psi^{BA*} = \Psi^{AB*} = -\left(\frac{2-\delta}{2} - \frac{\log(p_t^B/M_t) - \log(p_t^A/M_t)}{2\tau}\right)^{-2}\left(-\frac{1}{2\tau}\right)\left(\frac{1}{2\tau}\right)$$
$$= \frac{1}{\tau^2}\left(2 - \delta - \frac{\log(p^B) - \log(p^A)}{\tau}\right)^{-2}.$$

An increase in δ decreases the absolute value of own elasticity $|\Psi^A|$, the cross elasticity Ψ^B, and the cross superelasticity Ψ^{AB} for firm A. This response is different from that in the CES preferences model.

A decrease in p^A relative to p^B (due to an increase in firm A's relative productivity) decreases the absolute value of own elasticity $|\Psi^A|$, the cross elasticity Ψ^B, and the cross superelasticity Ψ^{AB} for firm A.

These elasticities satisfy $\Psi^A + \Psi^B = \Psi^{A*} + \Psi^{B*} = -1$,

$$\begin{aligned}\Psi^A(\Psi^A+1) - \Psi^{AA} &= \left\{-\frac{1}{\tau}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-1} - 1\right\}\left\{-\frac{1}{\tau}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-1}\right\} \\ &+ \frac{1}{\tau^2}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-2} \\ &= \frac{1}{\tau}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-1}\left\{1 + \frac{2}{\tau}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-1}\right\},\end{aligned}$$

$$\begin{aligned}\Psi^{AB} - \Psi^B &= \frac{1}{\tau^2}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-2} - \frac{1}{\tau}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-1} \\ &= \frac{1}{\tau}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-1}\left\{\frac{1}{\tau}\left(\delta - \frac{\log(p^A)-\log(p^B)}{\tau}\right)^{-1} - 1\right\}.\end{aligned}$$

D.2 Steady State without Price Stickiness

Before we introduce price stickiness, we consider the steady-state equilibrium. The first-order condition with respect to p_t^i yields

$$\begin{aligned}\frac{\partial \Pi_t^i}{\partial p_t^i} &= \frac{\partial}{\partial p_t^i}\Big((p_t^i - W_t/\phi^i)x^i(p_t^i/M_t, p_t^{-i}/M_t)\Big) \\ &= x^i(p_t^i/M_t, p_t^{-i}/M_t) + \frac{p_t^i - W_t/\phi^i}{M_t}\frac{\partial x^i}{\partial p_t^i} = 0.\end{aligned}$$

In the steady state with $W = M = 1$, it becomes

$$\begin{aligned}0 &= x^i + (p^i - 1/\phi^i)\frac{x^i \partial \log x^i(p^i, p^{-i})}{p^i \partial \log p^i} \\ 0 &= 1 + (p^i - 1/\phi^i)\frac{\partial \log x^i}{p^i \partial \log p^i} \\ &= 1 + (p^i - 1/\phi^i)\frac{\Psi^i}{p^i}.\end{aligned}$$

This leads to

$$p^i = \frac{\Psi^i}{\Psi^i + 1}\frac{1}{\phi^i}. \tag{15}$$

By differentiating with respect to $\log(p^{-i})$, we obtain the best response of $\log(p^i)$ to $\log(p^{-i})$ as

$$0 = \frac{\partial p^i}{\partial \log(p^{-i})} + \frac{\partial p^i}{\partial \log(p^{-i})}\Psi^i + \Big(p^i - 1/\phi^i\Big)\frac{\partial \Psi^i}{\partial \log p^{-i}}.$$

$$\frac{\partial \log p^i}{\partial \log p^{-i}} = \frac{\Psi^{i,-i}}{\Psi^i(1+\Psi^i)}. \tag{16}$$

D.3 Pricing under Calvo-type Price Stickiness

When firm i has a chance to set its price at t, it sets $\bar{p}_t^i$ to maximize

$$\begin{aligned}&\max \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t\left[\Big(\bar{p}_t^i - W_{t+k}/\phi^i\Big)\theta_{-i}^{k+1} x^i(\bar{p}_t^i/M_{t+k}, p_{t-1}^{-i}/M_{t+k})\right]\cdot \frac{\Lambda_{t+k}}{\Lambda_t}\frac{P_t}{P_{t+k}} \\ &+ \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t\left[\Big(\bar{p}_t^i - W_{t+k}/\phi^i\Big)\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'} x^i(\bar{p}_t^i/M_{t+k}, p_{t+k'}^{-i}/M_{t+k})\right]\cdot \frac{\Lambda_{t+k}}{\Lambda_t}\frac{P_t}{P_{t+k}}.\end{aligned} \tag{17}$$

The first-order condition for the optimal $\bar{p}^i_t$ is given by

$$\begin{aligned}
0 =& \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left[\theta_{-i}^{k+1} x^i(\bar{p}^i_t/M_{t+k}, p^{-i}_{t-1}/M_{t+k})\right] \cdot \frac{\Lambda_{t+k}}{\Lambda_t}\frac{P_t}{P_{t+k}} \\
&+ \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left[\sum_{k'=0}^{k} (1-\theta)\theta_{-i}^{k-k'} x^i(\bar{p}^i_t/M_{t+k}, p^{-i}_{t+k'}/M_{t+k})\right] \cdot \frac{\Lambda_{t+k}}{\Lambda_t}\frac{P_t}{P_{t+k}} \\
&+ \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left(\bar{p}^i_t - M_{t+k}/\phi^i\right) \left[\theta_{-i}^{k+1} \frac{\partial x^i(\bar{p}^i_t/M_{t+k}, p^{-i}_{t-1}/M_{t+k})}{\partial \bar{p}^i_t}\right] \cdot \frac{\Lambda_{t+k}}{\Lambda_t}\frac{P_t}{P_{t+k}} \\
&+ \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left(\bar{p}^i_t - M_{t+k}/\phi^i\right) \left[\sum_{k'=0}^{k} (1-\theta_{-i})\theta_{-i}^{k-k'} \frac{\partial x^i(\bar{p}^i_t/M_{t+k}, p^{-i}_{t+k'}/M_{t+k})}{\partial \bar{p}^i_t}\right] \cdot \frac{\Lambda_{t+k}}{\Lambda_t}\frac{P_t}{P_{t+k}} \\
&+ \sum_{k=1}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left(\bar{p}^i_t - M_{t+k}/\phi^i\right) \left[\sum_{k'=1}^{k} (1-\theta_{-i})\theta_{-i}^{k-k'} \frac{\partial x^i(\bar{p}^i_t/M_{t+k}, p^{-i}_{t+k'}/M_{t+k})}{\partial p^{-i}_{t+k'}} \frac{\partial p^{-i}_{t+k'}}{\partial \bar{p}^i_t}\right] \cdot \frac{\Lambda_{t+k}}{\Lambda_t}\frac{P_t}{P_{t+k}}.
\end{aligned}$$

Given the Markov perfect equilibrium, the log-linearized optimal reset price is expressed in the following form:

$$\hat{p}^i_t = \Gamma^{ii}\hat{p}^i_{t-1} + \Gamma^{i-i}\hat{p}^{-i}_{t-1} + \Gamma^{\varepsilon}\varepsilon_t, \tag{18}$$

$$\hat{p}^{i*}_t = \Gamma^{ii*}\hat{p}^{i*}_{t-1} + \Gamma^{i-i*}\hat{p}^{-i*}_{t-1} + \Gamma^{\varepsilon*}\varepsilon_t, \tag{19}$$

$$\partial \log \bar{p}^{-i}_{t+k}/\partial \log \bar{p}^i_t = \Gamma^{*i-i} \text{ for } k \geq 1, \tag{20}$$

where $\bar{p}^i_t \equiv pM_t e^{\hat{p}^i_t}$. Equation (19) in the second line indicates the log-linearized optimal reset price set by the competitor, which we denote using an asterisk. The third line shows that from the standpoint of firm i, a marginal change in its reset price ($\partial \log \bar{p}^i_t$) induces the competitor $-i$ to change its price by Γ^{*i-i} from equation (19). When firms A and B exist ($n = 2$) in each product line, we express the above equations as

$$\hat{p}^A_t = \Gamma^{AA}\hat{p}^A_{t-1} + \Gamma^{AB}\hat{p}^B_{t-1} + \Gamma^{A\varepsilon}\varepsilon_t, \tag{21}$$

$$\hat{p}^B_t = \Gamma^{BB}\hat{p}^B_{t-1} + \Gamma^{BA}\hat{p}^A_{t-1} + \Gamma^{B\varepsilon}\varepsilon_t, \tag{22}$$

$$\partial \log \bar{p}^B_{t+k}/\partial \log \bar{p}^A_t = \Gamma^{BA} \text{ for } k \geq 1. \tag{23}$$

Proposition 1 *The method of undetermined coefficients enables us to solve* p^A, Γ^{AA}, Γ^{AB}, $\Gamma^{A\varepsilon}$, p^B, Γ^{BB}, Γ^{BA}, *and* $\Gamma^{B\varepsilon}$ *from the coefficients of* 1, $\hat{p}^A_{t-1}$, $\hat{p}^B_{t-1}$, *and* ε_t *in the following two equations:*

$$
\begin{aligned}
0 =& \frac{1}{1-\theta_A\beta} \\
&- \Psi^A \frac{\rho}{1-\rho}\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\beta\rho}{1-\theta_A\beta\rho}\right)\varepsilon_t + \Psi^A \frac{1}{1-\theta_A\beta}\left(\Gamma^{AA}\hat{p}^A_{t-1} + \Gamma^{AB}\hat{p}^B_{t-1} + \Gamma^{A\varepsilon}\varepsilon_t\right) \\
&- \Psi^B \frac{1}{1-\rho}\theta_B\left(\frac{1}{1-\theta_A\theta_B\beta} - \frac{\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t + \Psi^B \frac{\theta_B}{1-\theta_A\theta_B\beta}\hat{p}^B_{t-1} \\
&+ \Psi^B \frac{1}{(1-\rho)(1-\rho/\theta_B)}\left[\frac{1-\rho}{1-\theta_A\beta\rho} - \frac{\theta_B-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_B}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
&+ \Psi^B\left(\mathbb{\Gamma}^{BA}_{k0}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{BB}_{k0}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{B\varepsilon}_{k0}\varepsilon_t\right) \\
&+ (1-\frac{1}{p^A\phi^A})\Psi^A\frac{1}{1-\theta_A\beta} \\
&- \left(\frac{\Psi^A}{p^A\phi^A} + (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^A + \Psi^{AA})\right)\frac{\rho}{1-\rho}\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\beta\rho}{1-\theta_A\beta\rho}\right)\varepsilon_t \\
&+ \left(\frac{\Psi^A}{p^A\phi^A} + (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^A + \Psi^{AA})\right)\frac{1}{1-\theta_A\beta}\left(\Gamma^{AA}\hat{p}^A_{t-1} + \Gamma^{AB}\hat{p}^B_{t-1} + \Gamma^{A\varepsilon}\varepsilon_t\right) \\
&- (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\frac{1}{1-\rho}\theta_B\left(\frac{1}{1-\theta_A\theta_B\beta} - \frac{\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t \\
&+ (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\frac{\theta_B}{1-\theta_A\theta_B\beta}\hat{p}^B_{t-1} \\
&+ (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\frac{1}{(1-\rho)(1-\rho/\theta_B)}\left[\frac{1-\rho}{1-\theta_A\beta\rho} - \frac{\theta_B-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_B}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
&+ (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})(\mathbb{\Gamma}^{BA}_{k0}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{BB}_{k0}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{B\varepsilon}_{k0}\varepsilon_t) \\
&+ (1-\frac{1}{p^A\phi^A})\left(\frac{-1}{1-\theta_A\theta_B\beta} + \frac{1}{1-\theta_A\beta}\right)\Psi^B\Gamma^{BA} \\
&- \left(\frac{1}{p^A\phi^A}\Psi^B + (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\right)\Gamma^{BA}\left\{\frac{\rho}{1-\rho}\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\beta\rho}{1-\theta_A\beta\rho}\right)\varepsilon_t - \frac{\rho}{1-\rho}\left(\frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta} - \frac{\theta_A\theta_B\beta\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t\right. \\
&+ \left(\frac{1}{p^A\phi^A}\Psi^B + (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\right)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\left(\Gamma^{AA}\hat{p}^A_{t-1} + \Gamma^{AB}\hat{p}^B_{t-1} + \Gamma^{A\varepsilon}\varepsilon_t\right) \\
&+ (1-\frac{1}{p^A\phi^A})\Gamma^{BA}(\Psi^B\Psi^B + \Psi^{BB})\frac{\theta_A\beta\rho}{(1-\rho)(1-\rho/\theta_B)}\left[\frac{1-\rho}{1-\theta_A\beta\rho} - \frac{\theta_B-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_B}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
&+ (1-\frac{1}{p^A\phi^A})\Gamma^{BA}(\Psi^B\Psi^B + \Psi^{BB})(\mathbb{\Gamma}^{BA}_{k1}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{BB}_{k1}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{B\varepsilon}_{k1}\varepsilon_t),
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
0 =& \frac{1}{1-\theta_B\beta} \\
& - \Psi^{B*}\frac{\rho}{1-\rho}\left(\frac{\theta_B\beta}{1-\theta_B\beta} - \frac{\theta_B\beta\rho}{1-\theta_B\beta\rho}\right)\varepsilon_t + \Psi^{B*}\frac{1}{1-\theta_B\beta}\left(\Gamma^{BB}\hat{p}^B_{t-1} + \Gamma^{BA}\hat{p}^A_{t-1} + \Gamma^{B\varepsilon}\varepsilon_t\right) \\
& - \Psi^{A*}\frac{1}{1-\rho}\theta_A\left(\frac{1}{1-\theta_A\theta_B\beta} - \frac{\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t + \Psi^{A*}\frac{\theta_A}{1-\theta_A\theta_B\beta}\hat{p}^A_{t-1} \\
& + \Psi^{A*}\frac{1}{(1-\rho)(1-\rho/\theta_A)}\left[\frac{1-\rho}{1-\theta_B\beta\rho} - \frac{\theta_A-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_A}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
& + \Psi^{A*}\left(\mathbb{\Gamma}^{AB}_{k0}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{AA}_{k0}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{A\varepsilon}_{k0}\varepsilon_t\right) \\
& + (1-\frac{1}{p^B\phi^B})\Psi^{B*}\frac{1}{1-\theta_B\beta} \\
& - \left(\frac{\Psi^{B*}}{p^B\phi^B} + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{B*} + \Psi^{BB*})\right)\frac{\rho}{1-\rho}\left(\frac{\theta_B\beta}{1-\theta_B\beta} - \frac{\theta_B\beta\rho}{1-\theta_B\beta\rho}\right)\varepsilon_t \\
& + \left(\frac{\Psi^{B*}}{p^B\phi^B} + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{B*} + \Psi^{BB*})\right)\frac{1}{1-\theta_B\beta}\left(\Gamma^{BB}\hat{p}^B_{t-1} + \Gamma^{BA}\hat{p}^A_{t-1} + \Gamma^{B\varepsilon}\varepsilon_t\right) \\
& - (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*} + \Psi^{BA*})\frac{1}{1-\rho}\theta_A\left(\frac{1}{1-\theta_A\theta_B\beta} - \frac{\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t \\
& + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*} + \Psi^{BA*})\frac{\theta_A}{1-\theta_A\theta_B\beta}\hat{p}^A_{t-1} \\
& + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*} + \Psi^{BA*})\frac{1}{(1-\rho)(1-\rho/\theta_A)}\left[\frac{1-\rho}{1-\theta_B\beta\rho} - \frac{\theta_A-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_A}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
& + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*} + \Psi^{BA*})(\mathbb{\Gamma}^{AB}_{k0}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{AA}_{k0}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{A\varepsilon}_{k0}\varepsilon_t) \\
& + (1-\frac{1}{p^B\phi^B})\left(\frac{-1}{1-\theta_A\theta_B\beta} + \frac{1}{1-\theta_B\beta}\right)\Psi^{A*}\Gamma^{AB} \\
& - \left(\frac{1}{p^B\phi^B}\Psi^{A*} + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*} + \Psi^{BA*})\right)\Gamma^{AB}\left\{\frac{\rho}{1-\rho}\left(\frac{\theta_B\beta}{1-\theta_B\beta} - \frac{\theta_B\beta\rho}{1-\theta_B\beta\rho}\right)\varepsilon_t - \frac{\rho}{1-\rho}\left(\frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta} - \frac{\theta_A\theta_B\beta\rho}{1-\theta_A\theta_B\beta\rho}\right.\right. \\
& + \left(\frac{1}{p^B\phi^B}\Psi^{A*} + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*} + \Psi^{BA*})\right)\left(\frac{\theta_B\beta}{1-\theta_B\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{AB}\left(\Gamma^{BB}\hat{p}^B_{t-1} + \Gamma^{BA}\hat{p}^A_{t-1} + \Gamma^{B\varepsilon}\varepsilon_t\right) \\
& + (1-\frac{1}{p^B\phi^B})\Gamma^{AB}(\Psi^{A*}\Psi^{A*} + \Psi^{AA*})\frac{\theta_B\beta\rho}{(1-\rho)(1-\rho/\theta_A)}\left[\frac{1-\rho}{1-\theta_B\beta\rho} - \frac{\theta_A-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_A}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
& + (1-\frac{1}{p^B\phi^B})\Gamma^{AB}(\Psi^{A*}\Psi^{A*} + \Psi^{AA*})(\mathbb{\Gamma}^{AB}_{k1}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{AA}_{k1}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{A\varepsilon}_{k1}\varepsilon_t), \qquad (25)
\end{aligned}
$$

where $\mathbb{\Gamma}_{k0}$ *and* $\mathbb{\Gamma}_{k1}$ *are defined as*

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \mathbb{\Gamma}_{k0}^{BA} & \mathbb{\Gamma}_{k0}^{BB} & \mathbb{\Gamma}_{k0}^{B\varepsilon} \\ \cdot & \cdot & \cdot \end{pmatrix} \equiv (1-\theta_B)\Gamma\left[I-(\mathbb{\Gamma}/\theta_B)\right]^{-1}\left[\frac{1}{1-\theta_A\theta_B\beta}I-\mathbb{\Gamma}/\theta_B\left[I-(\theta_A\beta\mathbb{\Gamma})\right]^{-1}\right],$$

$$\begin{pmatrix} \mathbb{\Gamma}_{k0}^{AA} & \mathbb{\Gamma}_{k0}^{AB} & \mathbb{\Gamma}_{k0}^{A\varepsilon} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \equiv (1-\theta_A)\Gamma\left[I-(\mathbb{\Gamma}/\theta_A)\right]^{-1}\left[\frac{1}{1-\theta_A\theta_B\beta}I-\mathbb{\Gamma}/\theta_A\left[I-(\theta_B\beta\mathbb{\Gamma})\right]^{-1}\right],$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \mathbb{\Gamma}_{k1}^{BA} & \mathbb{\Gamma}_{k1}^{BB} & \mathbb{\Gamma}_{k1}^{B\varepsilon} \\ \cdot & \cdot & \cdot \end{pmatrix} \equiv (1-\theta_B)\Gamma\left[I-(\mathbb{\Gamma}/\theta_B)\right]^{-1}\mathbb{\Gamma}/\theta_B\left[\frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}I-\theta_A\beta\mathbb{\Gamma}\left[I-(\theta_A\beta\mathbb{\Gamma})\right]^{-1}\right],$$

$$\begin{pmatrix} \mathbb{\Gamma}_{k1}^{AA} & \mathbb{\Gamma}_{k1}^{AB} & \mathbb{\Gamma}_{k1}^{A\varepsilon} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \equiv (1-\theta_A)\Gamma\left[I-(\mathbb{\Gamma}/\theta_A)\right]^{-1}\mathbb{\Gamma}/\theta_A\left[\frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}I-\theta_B\beta\mathbb{\Gamma}\left[I-(\theta_B\beta\mathbb{\Gamma})\right]^{-1}\right],$$

$$\text{where } \mathbb{\Gamma} = \begin{pmatrix} \Gamma^{AA} & \Gamma^{AB} & \Gamma^{A\varepsilon} \\ \Gamma^{BA} & \Gamma^{BB} & \Gamma^{B\varepsilon} \\ 0 & 0 & \rho \end{pmatrix}.$$

(Proof) The term $\frac{\Lambda_{t+k}}{\Lambda_t}\frac{P_t}{P_{t+k}}\frac{M_{t+k}}{M_t}$ equals one because $P_tC_t = M_t$. Thus,

$$\begin{aligned}
0 =& \sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left(\frac{M_t}{M_{t+k}}\right)\left[\theta_{-i}^{k+1}x^i(\bar{p}_t^i/M_{t+k}, p_{t-1}^{-i}/M_{t+k})\right] \\
&+\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left(\frac{M_t}{M_{t+k}}\right)\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}x^i(\bar{p}_t^i/M_{t+k}, p_{t+k'}^{-i}/M_{t+k})\right] \\
&+\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left(\frac{\bar{p}_t^i}{M_{t+k}}-1/\phi^i\right)\frac{M_t}{M_{t+k}}\left[\theta_{-i}^{k+1}\frac{\partial x^i(\bar{p}_t^i/M_{t+k}, p_{t-1}^{-i}/M_{t+k})}{\partial(\bar{p}_t^i/M_{t+k})}\right] \\
&+\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left(\frac{\bar{p}_t^i}{M_{t+k}}-1/\phi^i\right)\frac{M_t}{M_{t+k}}\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\frac{\partial x^i(\bar{p}_t^i/M_{t+k}, p_{t+k'}^{-i}/M_{t+k})}{\partial(\bar{p}_t^i/M_{t+k})}\right] \\
&+\sum_{k=1}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left(\frac{\bar{p}_t^i}{M_{t+k}}-1/\phi^i\right)\frac{M_t}{M_{t+k}}\left[\sum_{k'=1}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\frac{\partial x^i(\bar{p}_t^i/M_{t+k}, p_{t+k'}^{-i}/M_{t+k})}{\partial(p_{t+k'}^{-i}/M_{t+k})}\frac{\partial p_{t+k'}^{-i}}{\partial\bar{p}_t^i}\right].
\end{aligned}$$

In log-linearization, each term in the above equation is given by

$$\begin{aligned}
x^i(\bar{p}_t^i/M_{t+k}, p_t^{-i}/M_{t+k}) =& x^i \\
&+\frac{\partial\log x^i(\bar{p}^i/M, p^{-i}/M)}{\partial\log(\bar{p}^i/M)}x^i d\log(\bar{p}_t^i/M_{t+k}) \\
&+\frac{\partial\log x^i(\bar{p}^i/M, p^{-i}/M)}{\partial\log(p^{-i}/M)}x^i d\log(p_t^{-i}/M_{t+k}) \\
=& x^i\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_t/M_{t+k})+\hat{p}_t^{-i})\right\},
\end{aligned}$$

$$\begin{aligned}
\frac{\partial x^i(\bar{p}_t^i/M_{t+k}, p_{t-1}^{-i}/M_{t+k})}{\partial(\bar{p}_t^i/M_{t+k})} =& \frac{x_{t+k}^i}{\bar{p}_t^i/M_{t+k}}\frac{\partial\log x^i(\bar{p}_t^i/M_{t+k}, p_{t-1}^{-i}/M_{t+k})}{\partial\log(\bar{p}_t^i/M_{t+k})} \\
=& x^i\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_{t-1}/M_{t+k})+\hat{p}_{t-1}^{-i})\right\} \\
&\cdot\frac{\partial\log x^i(\bar{p}_t^i/M_{t+k}, p_{t-1}^{-i}/M_{t+k})}{(\bar{p}_t^i/M_{t+k})\partial\log(\bar{p}_t^i/M_{t+k})} \\
=& x^i\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_{t-1}/M_{t+k})+\hat{p}_{t-1}^{-i})\right\} \\
&\cdot\frac{M_{t+k}}{p^iM_te^{p_t^{i*}}}\cdot\left\{\Psi^i+\Psi^{i,i}(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{i,-i}(\log(M_{t-1}/M_{t+k})+\hat{p}_{t-1}^{-i})\right\},
\end{aligned}$$

$$\frac{\partial x^i(\bar{p}_t^i/M_{t+k}, p_{t+k'}^{-i}/M_{t+k})}{\partial(p_{t+k'}^{-i}/M_{t+k})}\frac{\partial p_{t+k'}^{-i}}{\partial \bar{p}_t^i} = \frac{x_{t+k}^i}{\bar{p}_{t+k'}^{-i}/M_{t+k}}\frac{\partial \log x^i(\bar{p}_t^i/M_{t+k}, \bar{p}_{t+k'}^{-i}/M_{t+k})}{\partial \log(\bar{p}_{t+k'}^{-i}/M_{t+k})}\frac{p_{t+k'}^{-i}}{\bar{p}_t^i}\frac{\partial \log p_{t+k'}^{-i}}{\partial \log \bar{p}_t^i}$$

$$= x^i\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_{t+k'}/M_{t+k})+p_{t+k'}^{*-i})\right\}$$

$$\cdot\left\{\Psi^{-i}+\Psi^{i,-i}(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i,-i}(\log(M_{t+k'}/M_{t+k})+p_{t+k'}^{*-i})\right\}$$

$$\cdot\frac{M_{t+k}}{p^iM_te^{p_t^{i*}}}\cdot\Gamma^{i-i*}.$$

Thus, the first-order condition becomes

$$0 = \sum_{k=0}^{\infty}\theta_i^k\theta_{-i}^{k+1}\beta^k\mathbb{E}_t\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_{t-1}/M_{t+k})+\hat{p}_{t-1}^{-i})\right\}$$

$$+\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_{t+k'}/M_{t+k})+p_{t+k'}^{*-i})\right\}\right]$$

$$+\sum_{k=0}^{\infty}\theta_i^k\theta_{-i}^{k+1}\beta^k\mathbb{E}_t\left(\frac{p^iM_te^{p_t^{i*}}}{M_{t+k}}-1/\phi^i\right)$$

$$\cdot\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_{t-1}/M_{t+k})+\hat{p}_{t-1}^{-i})\right\}$$

$$\cdot\frac{M_{t+k}}{p^iM_te^{p_t^{i*}}}\cdot\left\{\Psi^i+\Psi^{i,i}(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{i,-i}(\log(M_{t-1}/M_{t+k})+\hat{p}_{t-1}^{-i})\right\}$$

$$+\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left(\frac{p^iM_te^{p_t^{i*}}}{M_{t+k}}-1/\phi^i\right)$$

$$\cdot\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_{t+k'}/M_{t+k})+p_{t+k'}^{*-i})\right\}\right.$$

$$\left.\cdot\frac{M_{t+k}}{p^iM_te^{p_t^{i*}}}\cdot\left\{\Psi^i+\Psi^{i,i}(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{i,-i}(\log(M_{t+k'}/M_{t+k})+\hat{p}_{t+k'}^{-i})\right\}\right]$$

$$+\sum_{k=1}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left(\frac{p^iM_te^{p_t^{i*}}}{M_{t+k}}-1/\phi^i\right)$$

$$\cdot\left[\sum_{k'=1}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\left\{1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i}(\log(M_{t+k'}/M_{t+k})+p_{t+k'}^{*-i})\right\}\right.$$

$$\left.\cdot\left\{\Psi^{-i}+\Psi^{i,-i}(\log(M_t/M_{t+k})+p_t^{i*})+\Psi^{-i,-i}(\log(M_{t+k'}/M_{t+k})+p_{t+k'}^{*-i})\right\}\right]$$

$$\cdot\frac{M_{t+k}}{p^iM_te^{p_t^{i*}}}\cdot\Gamma^{i-i*}.$$

$$
\begin{aligned}
0 =& \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left[1 + \Psi^i (\log(M_t/M_{t+k}) + p_t^{i*}) \right] \\
&+ \sum_{k=0}^{\infty} \theta_i^k \theta_{-i}^{k+1} \beta^k \mathbb{E}_t \left[\Psi^{-i} (\log(M_{t-1}/M_{t+k}) + \hat{p}_{t-1}^{-i}) \right] \\
&+ \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left[\sum_{k'=0}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} \left\{ \Psi^{-i} (\log(M_{t+k'}/M_{t+k}) + p_{t+k'}^{*-i}) \right\} \right] \\
&+ \sum_{k=0}^{\infty} \theta_i^k \theta_{-i}^{k+1} \beta^k \mathbb{E}_t \left\{ 1 - \frac{1}{p^i \phi^i} + \frac{1}{p^i \phi^i} (\log(M_t/M_{t+k}) + p_t^{i*}) \right\} \\
&\cdot \left\{ \Psi^i + (\Psi^i \Psi^i + \Psi^{i,i})(\log(M_t/M_{t+k}) + p_t^{i*}) + (\Psi^i \Psi^{-i} + \Psi^{i,-i})(\log(M_{t-1}/M_{t+k}) + \hat{p}_{t-1}^{-i}) \right\} \\
&+ \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left\{ 1 - \frac{1}{p^i \phi^i} + \frac{1}{p^i \phi^i} (\log(M_t/M_{t+k}) + p_t^{i*}) \right\} \\
&\cdot \left[\sum_{k'=0}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} \left\{ \Psi^i + (\Psi^i \Psi^i + \Psi^{i,i})(\log(M_t/M_{t+k}) + p_t^{i*}) + (\Psi^i \Psi^{-i} + \Psi^{i,-i})(\log(M_{t+k'}/M_{t+k}) + \hat{p}_{t+k'}^{-i}) \right\} \right] \\
&+ \sum_{k=1}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left\{ 1 - \frac{1}{p^i \phi^i} + \frac{1}{p^i \phi^i} (\log(M_t/M_{t+k}) + p_t^{i*}) \right\} \Gamma^{i-i*} \\
&\cdot \left[\sum_{k'=1}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} \left\{ \Psi^{-i} + (\Psi^i \Psi^{-i} + \Psi^{i,-i})(\log(M_t/M_{t+k}) + p_t^{i*}) + (\Psi^{-i} \Psi^{-i} + \Psi^{-i,-i})(\log(M_{t+k'}/M_{t+k}) + p_{t+k'}^{*-i}) \right\} \right].
\end{aligned}
\tag{26}
$$

Note that we have

$$
\begin{aligned}
\mathbb{E}_t[\log(M_{t+k}/M_t)] &= \sum_{k'=1}^{k} \mathbb{E}_t \varepsilon_{t+k'} = \sum_{k'=1}^{k} \rho^{k'} \varepsilon_t \\
&= \rho(1-\rho^k)/(1-\rho) \cdot \varepsilon_t \text{ for } k \geq 1,
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \log(M_{t+k}/M_t) &= \sum_{k=1}^{\infty} \theta^k \beta^k \left\{ \rho(1-\rho^k)/(1-\rho) \cdot \varepsilon_t \right\} \\
&= \frac{\rho}{1-\rho} \left(\frac{\theta\beta}{1-\theta\beta} - \frac{\theta\beta\rho}{1-\theta\beta\rho} \right) \varepsilon_t,
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=0}^{\infty} \theta_i^k \theta_{-i}^{k+1} \beta^k \mathbb{E}_t \log(M_{t+k}/M_{t-1}) &= \sum_{k=0}^{\infty} \theta_i^k \theta_{-i}^{k+1} \beta^k \left\{ (1-\rho^{k+1})/(1-\rho) \cdot \varepsilon_t \right\} \\
&= \frac{1}{1-\rho} \theta_{-i} \left(\frac{1}{1-\theta_i\theta_{-i}\beta} - \frac{\rho}{1-\theta_i\theta_{-i}\beta\rho} \right) \varepsilon_t,
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k'=0}^{k} (1-\theta)\theta^{k-k'} &= (1-\theta)\theta^k \frac{1-1/\theta^{k+1}}{1-1/\theta} \\
&= -\theta^{k+1}(1-1/\theta^{k+1}) \\
&= 1-\theta^{k+1},
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k'=1}^{k} (1-\theta)\theta^{k-k'} &= (1-\theta)\theta^k \frac{1/\theta-1/\theta^{k+1}}{1-1/\theta} \\
&= -\theta^{k+1}(1/\theta-1/\theta^{k+1}) \\
&= 1-\theta^k,
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\right] &= \sum_{k=0}^{\infty}\theta_i^k\theta_{-i}^k\beta^k(1-\theta_{-i})\left[\sum_{k'=0}^{k}\theta_{-i}^{-k'}\right] \\
&= \sum_{k=0}^{\infty}\theta_i^k\theta_{-i}^k\beta^k(1-\theta_{-i})\frac{1-1/\theta_{-i}^{k+1}}{1-1/\theta_{-i}} \\
&= \frac{1-\theta_{-i}}{1-1/\theta_{-i}}\sum_{k=0}^{\infty}\theta_i^k\theta_{-i}^k\beta^k(1-1/\theta_{-i}^{k+1}) \\
&= -\theta_{-i}\left[\frac{1}{1-\theta_i\theta_{-i}\beta}-\frac{1/\theta_{-i}}{1-\theta_i\beta}\right] \\
&= \frac{-\theta_{-i}}{1-\theta_i\theta_{-i}\beta}+\frac{1}{1-\theta_i\beta},
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=1}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=1}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\right] &= \sum_{k=1}^{\infty}\theta_i^k\theta_{-i}^k\beta^k(1-\theta_{-i})\left[\sum_{k'=1}^{k}\theta_{-i}^{-k'}\right] \\
&= \sum_{k=1}^{\infty}\theta_i^k\theta_{-i}^k\beta^k(1-\theta_{-i})\frac{1/\theta_{-i}-1/\theta_{-i}^{k+1}}{1-1/\theta_{-i}} \\
&= \frac{1-\theta_{-i}}{1-1/\theta_{-i}}\sum_{k=0}^{\infty}\theta_i^k\theta_{-i}^k\beta^k(1/\theta_{-i}-1/\theta_{-i}^{k+1}) \\
&= -\theta_{-i}\left[\frac{1/\theta_{-i}}{1-\theta_i\theta_{-i}\beta}-\frac{1/\theta_{-i}}{1-\theta_i\beta}\right] \\
&= \frac{-1}{1-\theta_i\theta_{-i}\beta}+\frac{1}{1-\theta_i\beta},
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\log(M_{t+k'}/M_{t+k})\right]\\
&=\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\sum_{k''=k'+1}^{k}(-\varepsilon_{t+k''})\right]\\
&=\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\sum_{k''=k'+1}^{k}(-\rho^{k''}\varepsilon_t)\right]\\
&=-\sum_{k=0}^{\infty}\theta_i^k\beta^k\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\frac{\rho^{k'+1}-\rho^{k+1}}{1-\rho}\varepsilon_t\right]\\
&=-\sum_{k=0}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\sum_{k'=0}^{k}\{\theta_{-i}^{-k'}\rho^{k'}-\theta_{-i}^{-k'}\rho^k\}\right]\rho\varepsilon_t\\
&=-\sum_{k=0}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\frac{1-(\rho/\theta_{-i})^{k+1}}{1-\rho/\theta_{-i}}-\frac{1-(1/\theta_{-i})^{k+1}}{1-1/\theta_{-i}}\rho^k\right]\rho\varepsilon_t\\
&=-\sum_{k=0}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\frac{(1-(\rho/\theta_{-i})^{k+1})(1-1/\theta_{-i})-\{1-(1/\theta_{-i})^{k+1}\}\rho^k(1-\rho/\theta_{-i})}{(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\right]\rho\varepsilon_t\\
&=-\sum_{k=0}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\frac{1-(\rho/\theta_{-i})^{k+1}-1/\theta_{-i}+\rho^{k+1}/\theta_{-i}^{k+2}-\rho^k+\rho^k/\theta_{-i}^{k+1}+\rho^{k+1}/\theta_{-i}-\rho^{k+1}/\theta_{-i}^{k+2}}{(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\right]\rho\varepsilon_t\\
&=-\sum_{k=0}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\frac{1-(\rho/\theta_{-i})^{k+1}-1/\theta_{-i}-\rho^k+\rho^k/\theta_{-i}^{k+1}+\rho^{k+1}/\theta_{-i}}{(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\right]\rho\varepsilon_t\\
&=-\sum_{k=0}^{\infty}\theta^k\theta_{-i}^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\left[\frac{\rho^k/\theta_{-i}^{k+1}(1-\rho)-\rho^k(1-\rho/\theta_{-i})+1-1/\theta_{-i}}{(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\right]\rho\varepsilon_t\\
&=-\frac{1-\theta_{-i}}{(1-\rho)(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\left[\frac{(1-\rho)/\theta_{-i}}{1-\theta_i\beta\rho}-\frac{1-\rho/\theta_{-i}}{1-\theta_i\theta_{-i}\beta\rho}+\frac{1-1/\theta_{-i}}{1-\theta_i\theta_{-i}\beta}\right]\rho\varepsilon_t\\
&=\frac{1}{(1-\rho)(1-\rho/\theta_{-i})}\left[\frac{1-\rho}{1-\theta_i\beta\rho}-\frac{\theta_{-i}-\rho}{1-\theta_i\theta_{-i}\beta\rho}-\frac{1-\theta_{-i}}{1-\theta_i\theta_{-i}\beta}\right]\rho\varepsilon_t,
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{k=1}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=1}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\log(M_{t+k'}/M_{t+k})\right]\\
&=-\sum_{k=1}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\sum_{k'=1}^{k}\{\theta_{-i}^{-k'}\rho^{k'}-\theta_{-i}^{-k'}\rho^k\}\right]\rho\varepsilon_t\\
&=-\sum_{k=1}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\frac{\rho/\theta_{-i}-(\rho/\theta_{-i})^{k+1}}{1-\rho/\theta_{-i}}-\frac{1/\theta_{-i}-(1/\theta_{-i})^{k+1}}{1-1/\theta_{-i}}\rho^k\right]\rho\varepsilon_t\\
&=-\sum_{k=1}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\frac{(\rho/\theta_{-i}-(\rho/\theta_{-i})^{k+1})(1-1/\theta_{-i})-\{1/\theta_{-i}-(1/\theta_{-i})^{k+1}\}\rho^k(1-\rho/\theta_{-i})}{(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\right]\rho\varepsilon_t\\
&=-\sum_{k=1}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\frac{\rho/\theta_{-i}-(\rho/\theta_{-i})^{k+1}-\rho/\theta_{-i}^2+\rho^{k+1}/\theta_{-i}^{k+2}-\rho^k/\theta_{-i}+\rho^k/\theta_{-i}^{k+1}+\rho^{k+1}/\theta_{-i}^2-\rho^{k+1}/\theta_{-i}^{k+2}}{(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\right]\rho\varepsilon_t\\
&=-\sum_{k=1}^{\infty}\theta_i^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\theta_{-i}^k\left[\frac{\rho/\theta_{-i}-(\rho/\theta_{-i})^{k+1}-\rho/\theta_{-i}^2-\rho^k/\theta_{-i}+\rho^k/\theta_{-i}^{k+1}+\rho^{k+1}/\theta_{-i}^2}{(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\right]\rho\varepsilon_t\\
&=-\sum_{k=1}^{\infty}\theta_i^k\theta_{-i}^k\beta^k\frac{1-\theta_{-i}}{1-\rho}\left[\frac{\rho^k/\theta_{-i}^{k+1}(1-\rho)-\rho^k/\theta_{-i}(1-\rho/\theta_{-i})+\rho/\theta_{-i}(1-1/\theta_{-i})}{(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\right]\rho\varepsilon_t\\
&=-\frac{1-\theta_{-i}}{(1-\rho)(1-\rho/\theta_{-i})(1-1/\theta_{-i})}\left[\frac{(1-\rho)\theta_i\beta\rho/\theta_{-i}}{1-\theta_i\beta\rho}-\frac{(1-\rho/\theta_{-i})\theta_i\beta\rho}{1-\theta_i\theta_{-i}\beta\rho}+\frac{\theta_i\beta\rho(1-1/\theta_{-i})}{1-\theta_i\theta_{-i}\beta}\right]\rho\varepsilon_t\\
&=\frac{\theta_i\beta\rho}{(1-\rho)(1-\rho/\theta_{-i})}\left[\frac{1-\rho}{1-\theta_i\beta\rho}-\frac{\theta_{-i}-\rho}{1-\theta_i\theta_{-i}\beta\rho}-\frac{1-\theta_{-i}}{1-\theta_i\theta_{-i}\beta}\right]\rho\varepsilon_t,
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}_t \begin{pmatrix} p^{*i}_{t+k'} \\ p^{*-i}_{t+k'} \\ \varepsilon_{t+k'+1} \end{pmatrix} &= \mathbb{E}_t \begin{pmatrix} \Gamma^{ii} & \Gamma^{i-i} & \Gamma^{\varepsilon} \\ \Gamma^{i-i*} & \Gamma^{ii*} & \Gamma^{\varepsilon *} \\ 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} p^{*i}_{t+k'-1} \\ p^{*-i}_{t+k'-1} \\ \varepsilon_{t+k'} \end{pmatrix} \\
&= \begin{pmatrix} \Gamma^{ii} & \Gamma^{i-i} & \Gamma^{\varepsilon} \\ \Gamma^{i-i*} & \Gamma^{ii*} & \Gamma^{\varepsilon *} \\ 0 & 0 & \rho \end{pmatrix}^{k'+1} \begin{pmatrix} \hat{p}^{i}_{t-1} \\ \hat{p}^{-i}_{t-1} \\ \varepsilon_{t} \end{pmatrix} \\
&\equiv \mathbb{\Gamma}^{k'+1} \begin{pmatrix} \hat{p}^{i}_{t-1} \\ \hat{p}^{-i}_{t-1} \\ \varepsilon_{t} \end{pmatrix}
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
&\sum_{k=0}^{\infty} \theta_i^k \beta^k \left[\sum_{k'=0}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} \mathbb{\Gamma}^{k'+1} \right] \\
=&\sum_{k=0}^{\infty} \theta_i^k \beta^k (1-\theta_{-i}) \theta_{-i}^{k} \left[\sum_{k'=0}^{k} \theta_{-i}^{-k'} \mathbb{\Gamma}^{k'+1} \right] \\
=&\sum_{k=0}^{\infty} \theta_i^k \beta^k (1-\theta_{-i}) \theta_{-i}^{k} \mathbb{\Gamma} \left[I - (\mathbb{\Gamma}/\theta_{-i}) \right]^{-1} \left[I - (\mathbb{\Gamma}/\theta_{-i})^{k+1} \right] \\
=&(1-\theta_{-i}) \mathbb{\Gamma} \left[I - (\mathbb{\Gamma}/\theta_{-i}) \right]^{-1} \sum_{k=0}^{\infty} \theta_i^k \theta_{-i}^k \beta^k \left[I - (\mathbb{\Gamma}/\theta_{-i})^{k+1} \right] \\
=&(1-\theta_{-i}) \mathbb{\Gamma} \left[I - (\mathbb{\Gamma}/\theta_{-i}) \right]^{-1} \left[\frac{1}{1-\theta_i \theta_{-i} \beta} I - \mathbb{\Gamma}/\theta_{-i} \left[I - (\theta_i \beta \mathbb{\Gamma}) \right]^{-1} \right],
\end{aligned}
$$

thus

$$
\begin{aligned}
&\sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left[\sum_{k'=0}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} \begin{pmatrix} p^{*i}_{t+k'} \\ p^{*-i}_{t+k'} \\ \varepsilon_{t+k'+1} \end{pmatrix} \right] \\
=&(1-\theta_{-i}) \Gamma \left[I - (\mathbb{\Gamma}/\theta_{-i}) \right]^{-1} \left[\frac{1}{1-\theta_i \theta_{-i} \beta} I - \mathbb{\Gamma}/\theta_{-i} \left[I - (\theta_i \beta \mathbb{\Gamma}) \right]^{-1} \right] \begin{pmatrix} \hat{p}^{i}_{t-1} \\ \hat{p}^{-i}_{t-1} \\ \varepsilon_{t} \end{pmatrix}
\end{aligned} \tag{28}
$$

$$
\sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left[\sum_{k'=0}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} p^{*-i}_{t+k'} \right] \equiv \mathbb{\Gamma}^{i}_{k0} \hat{p}^{i}_{t-1} + \mathbb{\Gamma}^{-i}_{k0} \hat{p}^{-i}_{t-1} + \mathbb{\Gamma}^{\varepsilon}_{k0} \varepsilon_t. \tag{29}
$$

$$
\begin{aligned}
&\sum_{k=1}^{\infty} \theta_i^k \beta^k \left[\sum_{k'=1}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} \mathbb{\Gamma}^{k'+1} \right] \\
=&\sum_{k=1}^{\infty} \theta_i^k \beta^k (1-\theta_{-i}) \theta_{-i}^{k} \mathbb{\Gamma} \left[\sum_{k'=1}^{k} \theta_{-i}^{-k'} \mathbb{\Gamma}^{k'} \right] \\
=&\sum_{k=1}^{\infty} \theta_i^k \beta^k (1-\theta_{-i}) \theta_{-i}^{k} \mathbb{\Gamma} \left[I - (\mathbb{\Gamma}/\theta_{-i}) \right]^{-1} \left[\mathbb{\Gamma}/\theta_{-i} - (\mathbb{\Gamma}/\theta_{-i})^{k+1} \right] \\
=&(1-\theta_{-i}) \mathbb{\Gamma} \left[I - (\mathbb{\Gamma}/\theta_{-i}) \right]^{-1} \mathbb{\Gamma}/\theta_{-i} \sum_{k=1}^{\infty} \theta_i^k \theta_{-i}^k \beta^k \left[I - (\mathbb{\Gamma}/\theta_{-i})^{k} \right] \\
=&(1-\theta_{-i}) \mathbb{\Gamma} \left[I - (\mathbb{\Gamma}/\theta_{-i}) \right]^{-1} \mathbb{\Gamma}/\theta_{-i} \left[\frac{\theta_i \theta_{-i} \beta}{1-\theta_i \theta_{-i} \beta} I - \theta_i \beta \mathbb{\Gamma} \left[I - (\theta_i \beta \mathbb{\Gamma}) \right]^{-1} \right],
\end{aligned}
$$

thus

$$\sum_{k=1}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=1}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\begin{pmatrix}p_{t+k'}^{*i}\\ p_{t+k'}^{*-i}\\ \varepsilon_{t+k'+1}\end{pmatrix}\right]$$
$$=(1-\theta_{-i})\Gamma\left[I-(\mathbb{\Gamma}/\theta_{-i})\right]^{-1}\mathbb{\Gamma}/\theta_{-i}\left[\frac{\theta_i\theta_{-i}\beta}{1-\theta_i\theta_{-i}\beta}I-\theta_i\beta\mathbb{\Gamma}\left[I-(\theta_i\beta\mathbb{\Gamma})\right]^{-1}\right]\begin{pmatrix}\hat{p}_{t-1}^i\\ \hat{p}_{t-1}^{-i}\\ \varepsilon_t\end{pmatrix} \tag{30}$$

$$\sum_{k=1}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=1}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}p_{t+k'}^{*-i}\right]\equiv\mathbb{\Gamma}_{k1}^{i}\hat{p}_{t-1}^{i}+\mathbb{\Gamma}_{k1}^{-i}\hat{p}_{t-1}^{-i}+\mathbb{\Gamma}_{k1}^{\varepsilon}\varepsilon_t. \tag{31}$$

Thus, equation (26) is rearranged as

$$\begin{aligned}
0=&\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[1+\Psi^i(\log(M_t/M_{t+k})+p_t^{i*})\right]\\
&+\sum_{k=0}^{\infty}\theta_i^k\theta_{-i}^{k+1}\beta^k\mathbb{E}_t\left[\Psi^{-i}(\log(M_{t-1}/M_{t+k})+\hat{p}_{t-1}^{-i})\right]\\
&+\sum_{k=0}^{\infty}\theta_i^k\beta^k\mathbb{E}_t\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\left\{\Psi^{-i}(\log(M_{t+k'}/M_{t+k})+p_{t+k'}^{*-i})\right\}\right]\\
&+\sum_{k=0}^{\infty}\theta_i^k\theta_{-i}^{k+1}\beta^k\\
&\quad\cdot\mathbb{E}_t\left[(1-\frac{1}{p^i\phi^i})\Psi^i+\left(\frac{\Psi^i}{p^i\phi^i}+(1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^i+\Psi^{i,i})\right)(\log(M_t/M_{t+k})+p_t^{i*})\right.\\
&\quad\left.+(1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^{-i}+\Psi^{i,-i})(\log(M_{t-1}/M_{t+k})+\hat{p}_{t-1}^{-i})\right]\\
&+\sum_{k=0}^{\infty}\theta_i^k\beta^k(1-\frac{1}{p^i\phi^i})\Psi^i\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\\
&+\sum_{k=0}^{\infty}\theta_i^k\beta^k\left(\frac{\Psi^i}{p^i\phi^i}+(1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^i+\Psi^{i,i})\right)\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}(\log(M_t/M_{t+k})+p_t^{i*})\\
&+\sum_{k=0}^{\infty}\theta_i^k\beta^k(1-\frac{1}{p^i\phi^i})\left[\sum_{k'=0}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}(\Psi^i\Psi^{-i}+\Psi^{i,-i})(\log(M_{t+k'}/M_{t+k})+\hat{p}_{t+k'}^{-i})\right]\\
&+(1-\frac{1}{p^i\phi^i})\left(\frac{-1}{1-\theta_i\theta_{-i}\beta}+\frac{1}{1-\theta_i\beta}\right)\Psi^{-i}\Gamma^{i-i*}\\
&+\sum_{k=1}^{\infty}\theta_i^k\beta^k(1-\theta_{-i}^k)\mathbb{E}_t\left\{\left(\frac{1}{p^i\phi^i}\Psi^{-i}+(1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^{-i}+\Psi^{i,-i})\right)\{\log(M_t/M_{t+k})+p_t^{i*}\}\right\}\Gamma^{i-i*}\\
&+\sum_{k=1}^{\infty}\theta_i^k\beta^k\mathbb{E}_t(1-\frac{1}{p^i\phi^i})\Gamma^{i-i*}\left[\sum_{k'=1}^{k}(1-\theta_{-i})\theta_{-i}^{k-k'}\left\{(\Psi^{-i}\Psi^{-i}+\Psi^{-i,-i})(\log(M_{t+k'}/M_{t+k})+p_{t+k'}^{*-i})\right\}\right]
\end{aligned}$$

$$
\begin{aligned}
0 =& \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left[1 + \Psi^i (\log(M_t/M_{t+k}) + p_t^{i*}) \right] \\
&+ \sum_{k=0}^{\infty} \theta_i^k \theta_{-i}^{k+1} \beta^k \mathbb{E}_t \left[\Psi^{-i} (\log(M_{t-1}/M_{t+k}) + \hat{p}_{t-1}^{-i}) \right] \\
&+ \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left[\sum_{k'=0}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} \left\{ \Psi^{-i} (\log(M_{t+k'}/M_{t+k}) + p_{t+k'}^{*-i}) \right\} \right] \\
&+ \sum_{k=0}^{\infty} \theta_i^k \beta^k \mathbb{E}_t \left\{ (1 - \frac{1}{p^i \phi^i}) \Psi^i + \left(\frac{\Psi^i}{p^i \phi^i} + (1 - \frac{1}{p^i \phi^i}) (\Psi^i \Psi^i + \Psi^{i,i}) \right) (\log(M_t/M_{t+k}) + p_t^{i*}) \right\} \\
&+ \sum_{k=0}^{\infty} \theta_i^k \theta_{-i}^{k+1} \beta^k \mathbb{E}_t (1 - \frac{1}{p^i \phi^i}) (\Psi^i \Psi^{-i} + \Psi^{i,-i}) (\log(M_{t-1}/M_{t+k}) + \hat{p}_{t-1}^{-i}) \\
&+ \sum_{k=0}^{\infty} \theta_i^k \beta^k (1 - \frac{1}{p^i \phi^i}) \left[\sum_{k'=0}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} (\Psi^i \Psi^{-i} + \Psi^{i,-i}) (\log(M_{t+k'}/M_{t+k}) + \hat{p}_{t+k'}^{-i}) \right] \\
&+ (1 - \frac{1}{p^i \phi^i}) \left(\frac{-1}{1 - \theta_i \theta_{-i} \beta} + \frac{1}{1 - \theta_i \beta} \right) \Psi^{-i} \Gamma^{i-i*} \\
&+ \sum_{k=1}^{\infty} \theta_i^k \beta^k (1 - \theta_{-i}^k) \mathbb{E}_t \left\{ \left(\frac{1}{p^i \phi^i} \Psi^{-i} + (1 - \frac{1}{p^i \phi^i}) (\Psi^i \Psi^{-i} + \Psi^{i,-i}) \right) \{ \log(M_t/M_{t+k}) + p_t^{i*} \} \right\} \Gamma^{i-i*} \\
&+ \sum_{k=1}^{\infty} \theta_i^k \beta^k \mathbb{E}_t (1 - \frac{1}{p^i \phi^i}) \Gamma^{i-i*} \left[\sum_{k'=1}^{k} (1-\theta_{-i}) \theta_{-i}^{k-k'} \left\{ (\Psi^{-i} \Psi^{-i} + \Psi^{-i,-i}) (\log(M_{t+k'}/M_{t+k}) + p_{t+k'}^{*-i}) \right\} \right]
\end{aligned}
$$

$$
\begin{aligned}
0 =& \frac{1}{1-\theta_i\beta} \\
&- \Psi^i \frac{\rho}{1-\rho}\left(\frac{\theta_i\beta}{1-\theta_i\beta} - \frac{\theta_i\beta\rho}{1-\theta_i\beta\rho}\right)\varepsilon_t + \Psi^i \frac{1}{1-\theta_i\beta}\left(\Gamma^{ii}\hat{p}^i_{t-1} + \Gamma^{i-i}\hat{p}^{-i}_{t-1} + \Gamma^{\varepsilon}\varepsilon_t\right) \\
&- \Psi^{-i}\frac{1}{1-\rho}\theta_{-i}\left(\frac{1}{1-\theta_i\theta_{-i}\beta} - \frac{\rho}{1-\theta_i\theta_{-i}\beta\rho}\right)\varepsilon_t \\
&+ \Psi^{-i}\frac{\theta_{-i}}{1-\theta_i\theta_{-i}\beta}\hat{p}^{-i}_{t-1} \\
&+ \Psi^{-i}\frac{1}{(1-\rho)(1-\rho/\theta_{-i})}\left[\frac{1-\rho}{1-\theta_i\beta\rho} - \frac{\theta_{-i}-\rho}{1-\theta_i\theta_{-i}\beta\rho} - \frac{1-\theta_{-i}}{1-\theta_i\theta_{-i}\beta}\right]\rho\varepsilon_t \\
&+ \Psi^{-i}\left(\mathbb{\Gamma}^i_{k0}\hat{p}^i_{t-1} + \mathbb{\Gamma}^{-i}_{k0}\hat{p}^{-i}_{t-1} + \mathbb{\Gamma}^{\varepsilon}_{k0}\varepsilon_t\right) \\
&+ (1-\frac{1}{p^i\phi^i})\Psi^i\frac{1}{1-\theta_i\beta} \\
&- \left(\frac{\Psi^i}{p^i\phi^i} + (1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^i + \Psi^{i,i})\right)\frac{\rho}{1-\rho}\left(\frac{\theta_i\beta}{1-\theta_i\beta} - \frac{\theta_i\beta\rho}{1-\theta_i\beta\rho}\right)\varepsilon_t \\
&+ \left(\frac{\Psi^i}{p^i\phi^i} + (1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^i + \Psi^{i,i})\right)\frac{1}{1-\theta_i\beta}\left(\Gamma^{ii}\hat{p}^i_{t-1} + \Gamma^{i-i}\hat{p}^{-i}_{t-1} + \Gamma^{\varepsilon}\varepsilon_t\right) \\
&- (1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^{-i} + \Psi^{i,-i})\frac{1}{1-\rho}\theta_{-i}\left(\frac{1}{1-\theta_i\theta_{-i}\beta} - \frac{\rho}{1-\theta_i\theta_{-i}\beta\rho}\right)\varepsilon_t \\
&+ (1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^{-i} + \Psi^{i,-i})\frac{\theta_{-i}}{1-\theta_i\theta_{-i}\beta}\hat{p}^{-i}_{t-1} \\
&+ (1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^{-i} + \Psi^{i,-i})\frac{1}{(1-\rho)(1-\rho/\theta_{-i})}\left[\frac{1-\rho}{1-\theta_i\beta\rho} - \frac{\theta_{-i}-\rho}{1-\theta_i\theta_{-i}\beta\rho} - \frac{1-\theta_{-i}}{1-\theta_i\theta_{-i}\beta}\right]\rho\varepsilon_t \\
&+ (1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^{-i} + \Psi^{i,-i})(\mathbb{\Gamma}^i_{k0}\hat{p}^i_{t-1} + \mathbb{\Gamma}^{-i}_{k0}\hat{p}^{-i}_{t-1} + \mathbb{\Gamma}^{\varepsilon}_{k0}\varepsilon_t) \\
&+ (1-\frac{1}{p^i\phi^i})\left(\frac{-1}{1-\theta_i\theta_{-i}\beta} + \frac{1}{1-\theta_i\beta}\right)\Psi^{-i}\Gamma^{i-i*} \\
&- \left(\frac{1}{p^i\phi^i}\Psi^{-i} + (1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^{-i} + \Psi^{i,-i})\right)\Gamma^{i-i*}\left\{\frac{\rho}{1-\rho}\left(\frac{\theta_i\beta}{1-\theta_i\beta} - \frac{\theta_i\beta\rho}{1-\theta_i\beta\rho}\right)\varepsilon_t - \frac{\rho}{1-\rho}\left(\frac{\theta_i\theta_{-i}\beta}{1-\theta_i\theta_{-i}\beta} - \frac{\theta_i\theta_{-i}\beta\rho}{1-\theta_i\theta_{-i}\beta\rho}\right)\varepsilon_t\right\} \\
&+ \left(\frac{1}{p^i\phi^i}\Psi^{-i} + (1-\frac{1}{p^i\phi^i})(\Psi^i\Psi^{-i} + \Psi^{i,-i})\right)\left(\frac{\theta_i\beta}{1-\theta_i\beta} - \frac{\theta_i\theta_{-i}\beta}{1-\theta_i\theta_{-i}\beta}\right)\Gamma^{i-i*}\left(\Gamma^{ii}\hat{p}^i_{t-1} + \Gamma^{i-i}\hat{p}^{-i}_{t-1} + \Gamma^{\varepsilon}\varepsilon_t\right) \\
&+ (1-\frac{1}{p^i\phi^i})\Gamma^{i-i*}(\Psi^{-i}\Psi^{-i} + \Psi^{-i,-i})\frac{\theta_i\beta\rho}{(1-\rho)(1-\rho/\theta_{-i})}\left[\frac{1-\rho}{1-\theta_i\beta\rho} - \frac{\theta_{-i}-\rho}{1-\theta_i\theta_{-i}\beta\rho} - \frac{1-\theta_{-i}}{1-\theta_i\theta_{-i}\beta}\right]\rho\varepsilon_t \\
&+ (1-\frac{1}{p^i\phi^i})\Gamma^{i-i*}(\Psi^{-i}\Psi^{-i} + \Psi^{-i,-i})(\mathbb{\Gamma}^i_{k1}\hat{p}^i_{t-1} + \mathbb{\Gamma}^{-i}_{k1}\hat{p}^{-i}_{t-1} + \mathbb{\Gamma}^{\varepsilon}_{k1}\varepsilon_t).
\end{aligned}
$$

When firms A and B exist ($n = 2$) in each product line, we express the above equation for firm $i = A$ (its competitor $-i$ is denoted by B) as

$$
\begin{aligned}
0 =& \frac{1}{1-\theta_A\beta} \\
&- \Psi^A \frac{\rho}{1-\rho}\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\beta\rho}{1-\theta_A\beta\rho}\right)\varepsilon_t + \Psi^A \frac{1}{1-\theta_A\beta}\left(\Gamma^{AA}\hat{p}^A_{t-1} + \Gamma^{AB}\hat{p}^B_{t-1} + \Gamma^{A\varepsilon}\varepsilon_t\right) \\
&- \Psi^B \frac{1}{1-\rho}\theta_B\left(\frac{1}{1-\theta_A\theta_B\beta} - \frac{\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t + \Psi^B \frac{\theta_B}{1-\theta_A\theta_B\beta}\hat{p}^B_{t-1} \\
&+ \Psi^B \frac{1}{(1-\rho)(1-\rho/\theta_B)}\left[\frac{1-\rho}{1-\theta_A\beta\rho} - \frac{\theta_B-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_B}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
&+ \Psi^B\left(\mathbb{\Gamma}^{BA}_{k0}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{BB}_{k0}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{B\varepsilon}_{k0}\varepsilon_t\right) \\
&+ (1-\frac{1}{p^A\phi^A})\Psi^A\frac{1}{1-\theta_A\beta} \\
&- \left(\frac{\Psi^A}{p^A\phi^A} + (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^A + \Psi^{AA})\right)\frac{\rho}{1-\rho}\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\beta\rho}{1-\theta_A\beta\rho}\right)\varepsilon_t \\
&+ \left(\frac{\Psi^A}{p^A\phi^A} + (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^A + \Psi^{AA})\right)\frac{1}{1-\theta_A\beta}\left(\Gamma^{AA}\hat{p}^A_{t-1} + \Gamma^{AB}\hat{p}^B_{t-1} + \Gamma^{A\varepsilon}\varepsilon_t\right) \\
&- (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\frac{1}{1-\rho}\theta_B\left(\frac{1}{1-\theta_A\theta_B\beta} - \frac{\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t \\
&+ (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\frac{\theta_B}{1-\theta_A\theta_B\beta}\hat{p}^B_{t-1} \\
&+ (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\frac{1}{(1-\rho)(1-\rho/\theta_B)}\left[\frac{1-\rho}{1-\theta_A\beta\rho} - \frac{\theta_B-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_B}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
&+ (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})(\mathbb{\Gamma}^{BA}_{k0}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{BB}_{k0}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{B\varepsilon}_{k0}\varepsilon_t) \\
&+ (1-\frac{1}{p^A\phi^A})\left(\frac{-1}{1-\theta_A\theta_B\beta} + \frac{1}{1-\theta_A\beta}\right)\Psi^B\Gamma^{BA} \\
&- \left(\frac{1}{p^A\phi^A}\Psi^B + (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\right)\Gamma^{BA}\left\{\frac{\rho}{1-\rho}\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\beta\rho}{1-\theta_A\beta\rho}\right)\varepsilon_t - \frac{\rho}{1-\rho}\left(\frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta} - \frac{\theta_A\theta_B\beta\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t\right. \\
&+ \left(\frac{1}{p^A\phi^A}\Psi^B + (1-\frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\right)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\left(\Gamma^{AA}\hat{p}^A_{t-1} + \Gamma^{AB}\hat{p}^B_{t-1} + \Gamma^{A\varepsilon}\varepsilon_t\right) \\
&+ (1-\frac{1}{p^A\phi^A})\Gamma^{BA}(\Psi^B\Psi^B + \Psi^{BB})\frac{\theta_A\beta\rho}{(1-\rho)(1-\rho/\theta_B)}\left[\frac{1-\rho}{1-\theta_A\beta\rho} - \frac{\theta_B-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_B}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
&+ (1-\frac{1}{p^A\phi^A})\Gamma^{BA}(\Psi^B\Psi^B + \Psi^{BB})(\mathbb{\Gamma}^{BA}_{k1}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{BB}_{k1}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{B\varepsilon}_{k1}\varepsilon_t). \qquad (32)
\end{aligned}
$$

Similarly, firm B optimizes its reset price as

$$
\begin{aligned}
0 = & \frac{1}{1-\theta_B\beta} \\
& - \Psi^{B*}\frac{\rho}{1-\rho}\left(\frac{\theta_B\beta}{1-\theta_B\beta} - \frac{\theta_B\beta\rho}{1-\theta_B\beta\rho}\right)\varepsilon_t + \Psi^{B*}\frac{1}{1-\theta_B\beta}\left(\Gamma^{BB}\hat{p}^B_{t-1} + \Gamma^{BA}\hat{p}^A_{t-1} + \Gamma^{B\varepsilon}\varepsilon_t\right) \\
& - \Psi^{A*}\frac{1}{1-\rho}\theta_A\left(\frac{1}{1-\theta_A\theta_B\beta} - \frac{\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t + \Psi^{A*}\frac{\theta_A}{1-\theta_A\theta_B\beta}\hat{p}^A_{t-1} \\
& + \Psi^{A*}\frac{1}{(1-\rho)(1-\rho/\theta_A)}\left[\frac{1-\rho}{1-\theta_B\beta\rho} - \frac{\theta_A-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_A}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
& + \Psi^{A*}\left(\mathbb{\Gamma}^{AB}_{k0}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{AA}_{k0}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{A\varepsilon}_{k0}\varepsilon_t\right) \\
& + (1-\frac{1}{p^B\phi^B})\Psi^{B*}\frac{1}{1-\theta_B\beta} \\
& - \left(\frac{\Psi^{B*}}{p^B\phi^B} + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{B*}+\Psi^{BB*})\right)\frac{\rho}{1-\rho}\left(\frac{\theta_B\beta}{1-\theta_B\beta} - \frac{\theta_B\beta\rho}{1-\theta_B\beta\rho}\right)\varepsilon_t \\
& + \left(\frac{\Psi^{B*}}{p^B\phi^B} + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{B*}+\Psi^{BB*})\right)\frac{1}{1-\theta_B\beta}\left(\Gamma^{BB}\hat{p}^B_{t-1} + \Gamma^{BA}\hat{p}^A_{t-1} + \Gamma^{B\varepsilon}\varepsilon_t\right) \\
& - (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*}+\Psi^{BA*})\frac{1}{1-\rho}\theta_A\left(\frac{1}{1-\theta_A\theta_B\beta} - \frac{\rho}{1-\theta_A\theta_B\beta\rho}\right)\varepsilon_t \\
& + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*}+\Psi^{BA*})\frac{\theta_A}{1-\theta_A\theta_B\beta}\hat{p}^A_{t-1} \\
& + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*}+\Psi^{BA*})\frac{1}{(1-\rho)(1-\rho/\theta_A)}\left[\frac{1-\rho}{1-\theta_B\beta\rho} - \frac{\theta_A-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_A}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
& + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*}+\Psi^{BA*})(\mathbb{\Gamma}^{AB}_{k0}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{AA}_{k0}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{A\varepsilon}_{k0}\varepsilon_t) \\
& + (1-\frac{1}{p^B\phi^B})\left(\frac{-1}{1-\theta_A\theta_B\beta} + \frac{1}{1-\theta_B\beta}\right)\Psi^{A*}\Gamma^{AB} \\
& - \left(\frac{1}{p^B\phi^B}\Psi^{A*} + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*}+\Psi^{BA*})\right)\Gamma^{AB}\left\{\frac{\rho}{1-\rho}\left(\frac{\theta_B\beta}{1-\theta_B\beta} - \frac{\theta_B\beta\rho}{1-\theta_B\beta\rho}\right)\varepsilon_t - \frac{\rho}{1-\rho}\left(\frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta} - \frac{\theta_A\theta_B\beta\rho}{1-\theta_A\theta_B\beta\rho}\right.\right. \\
& + \left(\frac{1}{p^B\phi^B}\Psi^{A*} + (1-\frac{1}{p^B\phi^B})(\Psi^{B*}\Psi^{A*}+\Psi^{BA*})\right)\left(\frac{\theta_B\beta}{1-\theta_B\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{AB}\left(\Gamma^{BB}\hat{p}^B_{t-1} + \Gamma^{BA}\hat{p}^A_{t-1} + \Gamma^{B\varepsilon}\varepsilon_t\right) \\
& + (1-\frac{1}{p^B\phi^B})\Gamma^{AB}(\Psi^{A*}\Psi^{A*}+\Psi^{AA*})\frac{\theta_B\beta\rho}{(1-\rho)(1-\rho/\theta_A)}\left[\frac{1-\rho}{1-\theta_B\beta\rho} - \frac{\theta_A-\rho}{1-\theta_A\theta_B\beta\rho} - \frac{1-\theta_A}{1-\theta_A\theta_B\beta}\right]\rho\varepsilon_t \\
& + (1-\frac{1}{p^B\phi^B})\Gamma^{AB}(\Psi^{A*}\Psi^{A*}+\Psi^{AA*})(\mathbb{\Gamma}^{AB}_{k1}\hat{p}^B_{t-1} + \mathbb{\Gamma}^{AA}_{k1}\hat{p}^A_{t-1} + \mathbb{\Gamma}^{A\varepsilon}_{k1}\varepsilon_t).
\end{aligned}
\tag{33}
$$

■

Steady State

Lemma 1 *Firm A's steady-state price under price stickiness equals*

$$p^A\phi^A = 1 - \left\{1 + \Psi^A + \frac{\theta_A\beta(1-\theta_B)}{1-\theta_A\theta_B\beta}\Psi^B\Gamma^{BA}\right\}^{-1}. \tag{34}$$

From equation (24), in the steady state, we should have

$$
\begin{aligned}
0 = & \frac{1}{1-\theta_A\beta} \\
& + (1-\frac{1}{p^A\phi^A})\Psi^A\frac{1}{1-\theta_A\beta} \\
& + (1-\frac{1}{p^A\phi^A})\left(\frac{-1}{1-\theta_A\theta_B\beta} + \frac{1}{1-\theta_A\beta}\right)\Psi^B\Gamma^{BA}
\end{aligned}
$$

$$0 = 1 + (1 - \frac{1}{p^A\phi^A})\Psi^A + (1 - \frac{1}{p^A\phi^A})\frac{\theta_A\beta(1-\theta_B)}{1-\theta_A\theta_B\beta}\Psi^B\Gamma^{BA}$$

$$-1 = p^A\phi^A - 1 + (p^A\phi^A - 1)\Psi^A + (p^A\phi^A - 1)\frac{\theta_A\beta(1-\theta_B)}{1-\theta_A\theta_B\beta}\Psi^B\Gamma^{BA}$$

$$-1 = (p^A\phi^A - 1)\left\{1 + \Psi^A + \frac{\theta_A\beta(1-\theta_B)}{1-\theta_A\theta_B\beta}\Psi^B\Gamma^{BA}\right\}$$

$$p^A\phi^A = 1 - \left\{1 + \Psi^A + \frac{\theta_A\beta(1-\theta_B)}{1-\theta_A\theta_B\beta}\Psi^B\Gamma^{BA}\right\}^{-1}. \tag{35}$$

When $\Gamma^{BA} = 0$, then

$$p^A = \frac{\Psi^A}{\Psi^A + 1}\frac{1}{\phi^A}.$$

■

Log-linearization around the Steady State

Lemma 2 *When $p^A \simeq \frac{\Psi^A}{\Psi^A+1}\frac{1}{\phi^A}$, the degree of dynamic strategic complementarity Γ^{AB} satisfies*

$$\begin{aligned}\Gamma^{AB} = &\left\{(\Psi^B\Psi^B + \Psi^{BB})\mathbb{\Gamma}^{BB}_{k1}\Gamma^{BA} + \Psi^{AB}\left(\frac{\theta_B}{1-\theta_A\theta_B\beta} + \mathbb{\Gamma}^{BB}_{k0}\right)\right\} \\ &\cdot\left\{\frac{1}{1-\theta_A\beta}\left(\Psi^A(\Psi^A+1) - \Psi^{AA}\right) - (\Psi^{AB} - \Psi^B)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\right\}^{-1},\end{aligned} \tag{36}$$

where $\mathbb{\Gamma}^{BB}_{k1}$ and $\mathbb{\Gamma}^{BB}_{k0}$ are given in Proposition 1. When $|\mathbb{\Gamma}^{BB}_{k0}|$ and $|\mathbb{\Gamma}^{BB}_{k1}| \ll 1$, Γ^{AB} is approximated as

$$\Gamma^{AB} = \frac{\theta_B}{1-\theta_A\theta_B\beta}\Psi^{AB}\cdot\left\{\frac{1}{1-\theta_A\beta}\left(\Psi^A(\Psi^A+1) - \Psi^{AA}\right) - (\Psi^{AB} - \Psi^B)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\right\}^{-1}. \tag{37}$$

In equation (24), the term of $\hat{p}^B_{t-1}$ equals

$$\begin{aligned}0 =& \Psi^A\frac{1}{1-\theta_A\beta}\Gamma^{AB}\hat{p}^B_{t-1} \\ &+ \Psi^B\frac{\theta_B}{1-\theta_A\theta_B\beta}\hat{p}^B_{t-1} \\ &+ \Psi^B\mathbb{\Gamma}^{BB}_{k0}\hat{p}^B_{t-1} \\ &+ \left(\frac{\Psi^A}{p^A\phi^A} + (1 - \frac{1}{p^A\phi^A})(\Psi^A\Psi^A + \Psi^{AA})\right)\frac{1}{1-\theta_A\beta}\Gamma^{AB}\hat{p}^B_{t-1} \\ &+ (1 - \frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\frac{\theta_B}{1-\theta_A\theta_B\beta}\hat{p}^B_{t-1} \\ &+ (1 - \frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\mathbb{\Gamma}^{BB}_{k0}\hat{p}^B_{t-1} \\ &+ \left(\frac{1}{p^A\phi^A}\Psi^B + (1 - \frac{1}{p^A\phi^A})(\Psi^A\Psi^B + \Psi^{AB})\right)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\Gamma^{AB}\hat{p}^B_{t-1} \\ &+ (1 - \frac{1}{p^A\phi^A})\Gamma^{BA}(\Psi^B\Psi^B + \Psi^{BB})\mathbb{\Gamma}^{BB}_{k1}\hat{p}^B_{t-1},\end{aligned}$$

Suppose $\Psi^B = 0$. Then, the term of $\hat{p}^{-i}_{t-1}$ equals $0 = \Gamma^{AB}\cdot const$, which leads to $\Gamma^{AB} = 0$.

By approximating as $p^A \simeq \frac{\Psi^A}{\Psi^A+1}\frac{1}{\phi^A}$, we have the condition for Γ^{AB} to satisfy:

$$\begin{aligned}
0 =&\Psi^A \frac{1}{1-\theta_A\beta}\Gamma^{AB} \\
&+ \Psi^B \frac{\theta_B}{1-\theta_A\theta_B\beta} \\
&+ \Psi^B \mathbb{\Gamma}_{k0}^{BB} \\
&+ \left(\Psi^A + 1 - \frac{1}{\Psi^A}(\Psi^A\Psi^A + \Psi^{AA})\right)\frac{1}{1-\theta_A\beta}\Gamma^{AB} \\
&- \frac{1}{\Psi^A}(\Psi^A\Psi^B + \Psi^{AB})\frac{\theta_B}{1-\theta_A\theta_B\beta} \\
&- \frac{1}{\Psi^A}(\Psi^A\Psi^B + \Psi^{AB})\mathbb{\Gamma}_{k0}^{BB} \\
&+ \left(\frac{\Psi^A+1}{\Psi^A}\Psi^B - \frac{1}{\Psi^A}(\Psi^A\Psi^B + \Psi^{AB})\right)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\Gamma^{AB} \\
&- \frac{1}{\Psi^A}\Gamma^{BA}(\Psi^B\Psi^B + \Psi^{BB})\mathbb{\Gamma}_{k1}^{BB},
\end{aligned}$$

$$\begin{aligned}
0 =&\Psi^A \frac{1}{1-\theta_A\beta}\Gamma^{AB} \\
&+ \Psi^B \frac{\theta_B}{1-\theta_A\theta_B\beta} \\
&+ \Psi^B \mathbb{\Gamma}_{k0}^{BB} \\
&+ \left(1 - \frac{\Psi^{AA}}{\Psi^A}\right)\frac{1}{1-\theta_A\beta}\Gamma^{AB} \\
&- \frac{1}{\Psi^A}(\Psi^A\Psi^B + \Psi^{AB})\frac{\theta_B}{1-\theta_A\theta_B\beta} \\
&- \frac{1}{\Psi^A}(\Psi^A\Psi^B + \Psi^{AB})\mathbb{\Gamma}_{k0}^{BB} \\
&+ \left(\frac{\Psi^B}{\Psi^A} - \frac{\Psi^{AB}}{\Psi^A}\right)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\Gamma^{AB} \\
&- \frac{1}{\Psi^A}\Gamma^{BA}(\Psi^B\Psi^B + \Psi^{BB})\mathbb{\Gamma}_{k1}^{BB},
\end{aligned}$$

$$\begin{aligned}
(\Psi^{AB} - \Psi^B)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\Gamma^{AB} + (\Psi^B\Psi^B + \Psi^{BB})\mathbb{\Gamma}_{k1}^{BB}\Gamma^{BA} + \Psi^{AB}\left(\frac{\theta_B}{1-\theta_A\theta_B\beta} + \mathbb{\Gamma}_{k0}^{BB}\right) \\
= \frac{1}{1-\theta_A\beta}\left(\Psi^A(\Psi^A+1) - \Psi^{AA}\right)\Gamma^{AB}.
\end{aligned}$$

$$\begin{aligned}
\Gamma^{AB} =& \left\{(\Psi^B\Psi^B + \Psi^{BB})\mathbb{\Gamma}_{k1}^{BB}\Gamma^{BA} + \Psi^{AB}\left(\frac{\theta_B}{1-\theta_A\theta_B\beta} + \mathbb{\Gamma}_{k0}^{BB}\right)\right\} \\
&\cdot \left\{\frac{1}{1-\theta_A\beta}\left(\Psi^A(\Psi^A+1) - \Psi^{AA}\right) - (\Psi^{AB} - \Psi^B)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\right\}^{-1}. \quad (38)
\end{aligned}$$

In addition, suppose $|\mathbb{\Gamma}_{k0}^{BB}|$ and $|\mathbb{\Gamma}_{k1}^{BB}| \ll 1$. Then, equation (38) is approximated as

$$\Gamma^{AB} = \frac{\theta_B}{1-\theta_A\theta_B\beta}\Psi^{AB} \cdot \left\{\frac{1}{1-\theta_A\beta}\left(\Psi^A(\Psi^A+1) - \Psi^{AA}\right) - (\Psi^{AB} - \Psi^B)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA}\right\}^{-1}. \quad (39)$$

■

Proof of Corollary 1 Suppose $\Psi^{AB} > 0$, $\Psi^A(\Psi^A + 1) - \Psi^{AA} > 0$, $\Psi^{AB} - \Psi^B > 0$, Γ^{BA} is positive and not too large, $|\partial\Gamma^{BA}/\partial\theta_A| \ll 1$ and $|\partial\Gamma^{BA}/\partial\theta_B| \ll 1$. Then, equation (37) shows

$$\begin{aligned} \Gamma^{AB} &> 0 \\ \partial\Gamma^{AB}/\partial\Gamma^{BA} &> 0 \\ \partial\Gamma^{AB}/\partial\Psi^{AA} &> 0 \\ \partial\Gamma^{AB}/\partial\Psi^{AB} &> 0 \\ \partial\Gamma^{AB}/\partial\theta_A &> 0 \\ \partial\Gamma^{AB}/\partial\theta_B &> 0. \end{aligned}$$

Slutsky symmetry and constant returns to scale imply $\Psi^A + \Psi^B = -1$ when the number of firms is two. Thus, we have

$$\Gamma^{AB} = \frac{\theta_B}{1-\theta_A\theta_B\beta}(\Psi^{AB}/\Psi^A) \cdot \left\{ \frac{1}{1-\theta_A\beta}\left(\Psi^A + 1 - \Psi^{AA}/\Psi^A\right) - (\Psi^{AB}/\Psi^A + 1 + 1/\Psi^A)\left(\frac{\theta_A\beta}{1-\theta_A\beta} - \frac{\theta_A\theta_B\beta}{1-\theta_A\theta_B\beta}\right)\Gamma^{BA} \right\}^{-1}.$$

Assuming that the superelasticity such as Ψ^{AA}/Ψ^A and Ψ^{AB}/Ψ^A is given, it follows

$$\partial\Gamma^{AB}/\partial\Psi^A < 0.$$

D.4 Inflation Dynamics

When we assume CES duopolistic competition, the aggregate price index is given by

$$\log P_t = \frac{1}{1-\sigma}\log\left[\frac{b}{2}\left(p_t^A\right)^{1-\sigma} + \frac{2-b}{2}\left(p_t^B\right)^{1-\sigma}\right].$$

Thus, the log-linearized aggregated price is given by

$$\begin{aligned} \log P + \hat{P}_t &= \frac{1}{1-\sigma}\log\left[\frac{b}{2}\left(p^A e^{\hat{p}_t^A}\right)^{1-\sigma} + \frac{2-b}{2}\left(p^B e^{\hat{p}_t^B}\right)^{1-\sigma}\right] \\ &= \frac{1}{1-\sigma}\log\left[\frac{b}{2}\left(p^A(1+\hat{p}_t^A)\right)^{1-\sigma} + \frac{2-b}{2}\left(p^B(1+\hat{p}_t^B)\right)^{1-\sigma}\right] \\ &= \frac{1}{1-\sigma}\log\left[\frac{b}{2}\left(p^A\right)^{1-\sigma} + \frac{2-b}{2}\left(p^B\right)^{1-\sigma} + \frac{b}{2}\left(p^A\right)^{1-\sigma}(1-\sigma)\hat{p}_t^A + \frac{2-b}{2}\left(p^B\right)^{1-\sigma}(1-\sigma)\hat{p}_t^B\right] \\ &= \frac{1}{1-\sigma}\log\left[\frac{b}{2}\left(p^A\right)^{1-\sigma} + \frac{2-b}{2}\left(p^B\right)^{1-\sigma}\right] \\ &+ \frac{\frac{b}{2}\left(p^A\right)^{1-\sigma}\hat{p}_t^A + \frac{2-b}{2}\left(p^B\right)^{1-\sigma}\hat{p}_t^B}{\frac{b}{2}\left(p^A\right)^{1-\sigma} + \frac{2-b}{2}\left(p^B\right)^{1-\sigma}}. \end{aligned} \tag{40}$$

When we assume Hotelling duopolistic competition, the aggregate price index is given by

$$\log P_t = \int_0^1 \log p_t^j dj$$

for product line j. For each j, suppose the log-linearized prices set by firms A and B are given by $\hat{p}_t^A$ and $\hat{p}_t^B$, respectively. Then, $x = \frac{\delta - \frac{\log p^A - \log p^B}{\tau}}{2} - \frac{\hat{p}_t^A - \hat{p}_t^B}{2\tau}$ consumers buy from firm A at $\hat{p}_t^A$ and $1-x$ consumers buy from firm B at $\hat{p}_t^B$. Thus, the log-linearized price aggregated at the level of product line j is given by

$$
\begin{aligned}
& x\hat{p}_t^A + (1-x)\hat{p}_t^B \\
&= \left(\frac{\delta - \frac{\log p^A - \log p^B}{\tau}}{2} - \frac{\hat{p}_t^A - \hat{p}_t^B}{2\tau} \right) \left(\log p^A + \hat{p}_t^A \right) \\
&\quad + \left(\frac{2 - \delta - \frac{\log p^B - \log p^A}{\tau}}{2} - \frac{\hat{p}_t^B - \hat{p}_t^A}{2\tau} \right) \left(\log p^B + \hat{p}_t^B \right) \\
&\simeq \left(\frac{\delta - \frac{\log p^A - \log p^B}{\tau}}{2} \right) \log p^A + \left(\frac{2 - \delta - \frac{\log p^B - \log p^A}{\tau}}{2} \right) \log p^B \\
&\quad + \left(\frac{\delta - \frac{\log p^A - \log p^B}{\tau}}{2} \right) \hat{p}_t^A + \left(\frac{2 - \delta - \frac{\log p^B - \log p^A}{\tau}}{2} \right) \hat{p}_t^B \\
&\quad - \frac{\log p^A - \log p^B}{2\tau} (\hat{p}_t^A - \hat{p}_t^B) \\
&= \left(\frac{\delta - \frac{\log p^A - \log p^B}{\tau}}{2} \right) \log p^A + \left(\frac{2 - \delta - \frac{\log p^B - \log p^A}{\tau}}{2} \right) \log p^B \\
&\quad + \frac{\delta}{2}\hat{p}_t^A + \frac{2-\delta}{2}\hat{p}_t^B - \frac{\log p^A - \log p^B}{\tau} (\hat{p}_t^A - \hat{p}_t^B). \qquad (41)
\end{aligned}
$$

Note that

$$
\begin{aligned}
\hat{p}_t^A &= \int_0^1 \hat{p}_t^A dj \\
&= \theta_A(\hat{p}_{t-1}^A - \varepsilon_t) + (1-\theta_A)\hat{p}_t^{A*} \\
&= \theta_A(\hat{p}_{t-1}^A - \varepsilon_t) + (1-\theta_A)\left(\Gamma^{AA}\hat{p}_{t-1}^A + \Gamma^{AB}\hat{p}_{t-1}^B + \Gamma^{A\varepsilon}\varepsilon_t \right), \qquad (42)
\end{aligned}
$$

$$
\begin{aligned}
\hat{p}_t^B &= \int_0^1 \hat{p}_t^B dj \\
&= \theta_B(\hat{p}_{t-1}^B - \varepsilon_t) + (1-\theta_B)\hat{p}_t^{B*} \\
&= \theta_B(\hat{p}_{t-1}^B - \varepsilon_t) + (1-\theta_B)\left(\Gamma^{BB}\hat{p}_{t-1}^B + \Gamma^{BA}\hat{p}_{t-1}^A + \Gamma^{B\varepsilon}\varepsilon_t \right). \qquad (43)
\end{aligned}
$$

Aggregate Output Aggregate output is given by $Y_t = M_t/P_t$. The log-linearization yields

$$\hat{Y}_t = -\hat{P}_t. \qquad (44)$$

E Comparison with a CES Monopolistic Competition Model

Consumption is aggregated following the CES form of aggregation:

$$C_t = \left\{ \int_0^1 C_t(j)^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}}. \qquad (45)$$

This yields demand and the price index given by $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\sigma} Y_t$ and $P_t = \left\{ \int_0^1 P_t(j)^{1-\sigma} dj \right\}^{\frac{1}{1-\sigma}}$, respectively, where $C_t(j) = Y_t(j)$.

Pricing under Price Stickiness We assume that half of the firms have price stickiness given by θ^A, while the remaining half have price stickiness given by θ^B, where $\theta_0 = (\theta_A + \theta_B)/2$. For θ being either θ_A or θ_B, firm j sets $\bar{p}_t$ to maximize

$$\max \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \beta^k \frac{\Lambda_{t+k}}{\Lambda_t} \frac{P_t}{P_{t+k}} \left(\bar{p}_t Y_{t+k}(j) - W_{t+k} Y_{t+k}(j)\right).$$

The first-order condition leads to

$$0 = \sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \frac{\Lambda_{t+k}}{\Lambda_t} \frac{P_t}{P_{t+k}} Y_{t+k} \bar{p}_t^{-\sigma-1} P_{t+k}^{\sigma} \left[(1-\sigma)\bar{p}_t + \sigma M_{t+k}\right].$$

In log-linearization, denoting $\bar{p}_t \equiv \frac{\sigma}{\sigma-1} M_t e^{p_t^*}$, we have

$$0 = \sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \left[M_{t+k}/M_t - 1 - p_t^*\right].$$

$$\begin{aligned}
p_t^* &= (1-\theta\beta) \sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \left[M_{t+k}/M_t - 1\right] \\
&= (1-\theta\beta) \sum_{k=0}^{\infty} \theta^k \beta^k \sum_{k'=1}^{k} \mathbb{E}_t \varepsilon_{t+k'} \\
&= (1-\theta\beta) \sum_{k=0}^{\infty} \theta^k \beta^k \sum_{k'=1}^{k} \rho^{k'} \varepsilon_t \\
&= (1-\theta\beta) \sum_{k=0}^{\infty} \theta^k \beta^k \frac{\rho(1-\rho^k)}{1-\rho} \varepsilon_t \\
&= \frac{\rho\theta\beta}{1-\rho\theta\beta} \varepsilon_t.
\end{aligned}$$

The aggregate price is given by

$$(P_t)^{1-\varepsilon} = \left(P_t^A\right)^{1-\sigma} + \left(P_t^B\right)^{1-\sigma},$$

where

$$\begin{aligned}
\left(P_t^{A,B}\right)^{1-\sigma} &= \int_0^{1/2} P_t^{A,B}(j)^{1-\sigma} dj \\
&= \frac{1-\theta_{A,B}}{2} \left(p_t^*\right)^{1-\sigma} + \frac{\theta_A}{2} \left(P_{t-1}^{A,B}\right)^{1-\sigma}.
\end{aligned}$$

Log-linearization yields

$$\begin{aligned}
\hat{P}_t &= \hat{P}_t^A + \hat{P}_t^B \\
\hat{P}_t^{A,B} &= \frac{1-\theta_{A,B}}{2} p_t^* + \frac{\theta_{A,B}}{2} \hat{P}_{t-1}^{A,B} - \frac{\theta_{A,B}}{2} \varepsilon_t.
\end{aligned}$$

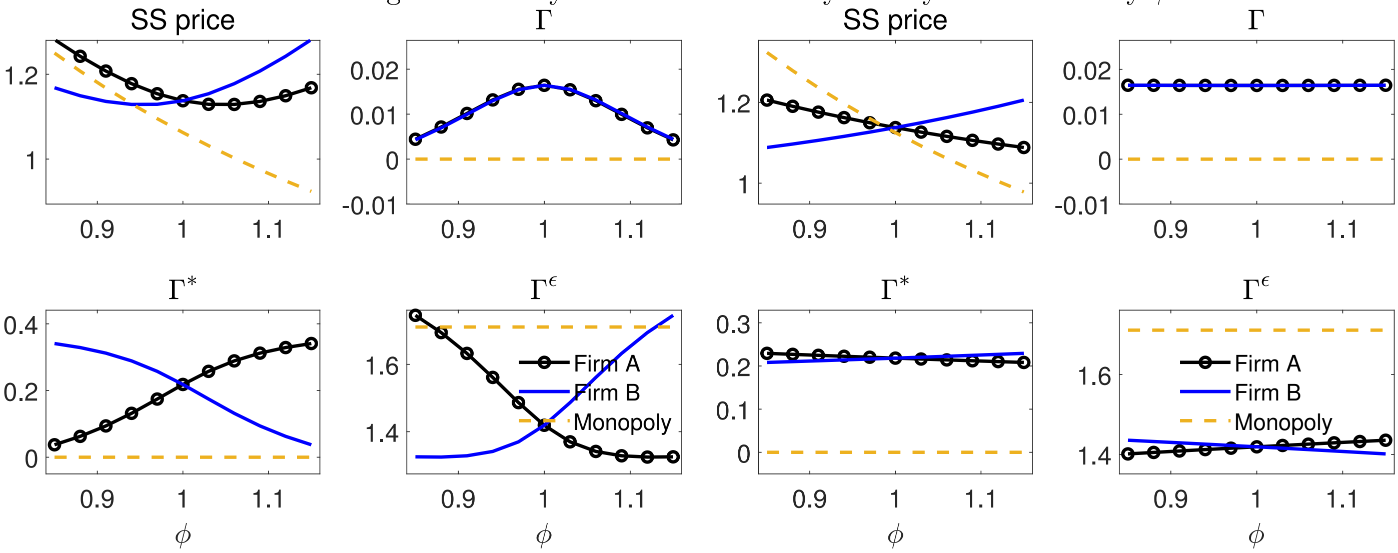
Thus, we have

$$\begin{aligned}
\hat{P}_t &= \left(\frac{1-\theta_A}{2} + \frac{1-\theta_B}{2}\right) p_t^* + \frac{\theta_A + \theta_B}{2} (\hat{P}_{t-1}^A + \hat{P}_{t-1}^B) - \left(\frac{\theta_A}{2} + \frac{\theta_B}{2}\right) \varepsilon_t \\
&= (1-\theta_0) p_t^* + \theta_0 \hat{P}_{t-1} - \theta_0 \varepsilon_t. \qquad (46)
\end{aligned}$$

This equation shows that mean-preserving heterogeneity in price stickiness does not change aggregate inflation dynamics in the log-linearized model.

F Further Numerical Results

Figure 5: Policy Functions under Asymmetry in Productivity ϕ



Notes: The figure shows the steady-state (SS) prices and coefficients of the policy functions for the optimal reset price. Denoting optimal pricing as $\hat{p}_t^i = \Gamma^{ii}\hat{p}_{t-1}^i + \Gamma^{i-i}\hat{p}_{t-1}^{-i} + \Gamma^{i\varepsilon}\varepsilon_t$ for $i = A, B$, we show Γ^{AA} and Γ^{BB} for firms A and B, respectively as Γ in each top right-hand panel. Similarly, Γ^{AB} and Γ^{BA} are shown as Γ^* in each bottom left-hand panel, while $\Gamma^{A\varepsilon}$ and $\Gamma^{B\varepsilon}$ are shown as Γ^ε in each bottom right-hand panel. Left- and right-hand figures are based on the CES preferences and Hotelling's models, respectively. The horizontal axis represents productivity ϕ^A for firm A. The two firms are equally competitive when $\phi = 1$.