博士論文（要約）

Simultaneous Body Reconfiguration and Nonholonomic Attitude Reorientation of Free-flying Space Robots

(フリーフライング宇宙ロボットの形態再構成と

非ホロノミックな姿勢変更の同時実現)

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Abstract

In recent years, a number of space robot/spacecraft missions such as on-orbit construction/repair, on-orbit refueling and debris removal have been proposed. For such advanced space missions, future free-flying robots are expected to have more versatility, adaptability, and dexterity. One promising solution to the future demand is free-flying space robots with highly redundant actuatable joints Such a redundant robot can be adaptive and versatile by morphing into various structure on-orbit and can be dexterous using multiple manipulators simultaneously. However, when a space robot reconfigures its structure, its final attitude depends on its body reconfiguration procedure due to nonholonomy. Motion planning in nonholonomic system is difficult because it is proved that any time-invariant continuous feedback control is impossible to stabilize the system. Therefore, there is no general solution to the inverse problem of obtaining the body reconfiguration procedure that achieves arbitrary body configuration and attitude simultaneously.

Many researchers investigated the attitude dynamics of the free-flying space robots, but most of them only dealt with simple systems such as planar-restricted systems and a canonical chained/power system. Therefore, little research has been done to construct a motion planning method that can handle general robot models and general three-dimensional rotation.

The purpose of this study is to construct a simultaneous body reconfiguration and attitude reorientation method that is applicable to arbitrary free-flying robots without angular momentum, which is named as *nonholonomic reorienting transformation*, or *NRT* by the author. In order to derive the widely applicable control law, a kinematics equation that preserves Lie-group structure is adopted to describe the attitude motion. Owing to its mathematical structure, the attitude motion is analytically expressed with the Magnus expansion, and some of them are approximately integrable. In particular, a *rectilinear solution* is focused on, in which joints are actuated along a rectilinear path in joint angle space. This rectilinear solution is simple but powerful tool for motion planning and induces two different types of motion planning methods: 1) rectilinear transformation planning, and 2) rectilinear invariant manifold method. The former method generates consecutive rectilinear path in joint angle space whereas the latter method asymptotically attracts the state to a fiber bundle of invariant manifold growing from the target state. Numerical simulations for both methods demonstrated that body reconfiguration and attitude reorientation is accomplished regardless of model of the robot and dimension of the motion.

In addition, singularity analysis is provided for both methods. The analysis is discussed by analogy with control moment gyros and robotic manipulators, and an effective singularity-robust steering method is imported to the proposed methods.

Finally, as an example of practical applications, an orbital station keeping with a transformable solar sail is presented. The constructed NRT maneuvers enable the solar sail to change its equilibrium attitude and to reorient to the equilibrium attitude simultaneously, which greatly enhances solar sailing ability. As a promising example of orbits, an artificial small-amplitude periodic orbit around SEL1/L2 is designed. The designed artificial orbit is an ideal platform that provides stationary thermal/geometrical environment, whereas the entire orbit and attitude maneuver do not consume any propellant.

The ability to simultaneously achieve a target body configuration and attitude means that geometry of all body components of the free-flying space robot can be arbitrarily reconfigured, which contributes to various applications. Therefore, the motion planning methods developed in this study are expected to be a fundamental technology for advanced future space robot missions.

Chapter 1

Introduction

1.1 Background

1.1.1 Growing Demand for Space Robots

In recent years, space activities have been further expanding. In particular, many space robot missions such as on-orbit satellite construction/repair, on-orbit refueling, and debris removal have been proposed recently [1]. For example, OSAM-1 mission by NASA will attempt on-orbit refueling and assembly in 2024 [2](Fig. 1.1), the Robotic Servicing of Geosynchronous Satellites (RSGS) program of DARPA will attempt on-orbit satellite repair and orbit correction in 2023 with Northrop Grumman [3] (Fig. 1.2), and ClearSpace-1 by ESA will demonstrate debris removal with robotic manipulators by no later than end of 2025 [4] (Fig. 1.3). Such missions indicate a growing demand for space robots that can perform more complicated tasks than conventional manipulation. Under this circumstance, future space robots (including spacecraft in a broad sense) will need to be more autonomous, more intelligent, more adaptive, more dexterous, and more economical.

In this context, a highly redundant free-flying space robot that has large degrees of freedom of active joints can be a promising solution in the following aspects:

- the robot can be adaptive by reconfiguring itself into multiple different morphologies,
- the robot can be dexterous by manipulating multiple instruments leveraging its redundancy, and
- the robot can be economical by saving fuel owing to its nonholonomic reorientation technique.

We refer to such robots as a *transformable free-flying space robots*. A typical example of such a transformable free-flying space robot is Transformer spacecraft under consideration mainly by JAXA since 2019. This spacecraft is designed to be able to flexibly adapt to multiple situations by reconfiguring itself into several operation modes. For example, the principal operation modes are a stowed/half-deployment mode in the launch phase, a solar sailing mode used in orbital station keeping phase, and an observation mode used in scientific observation phase

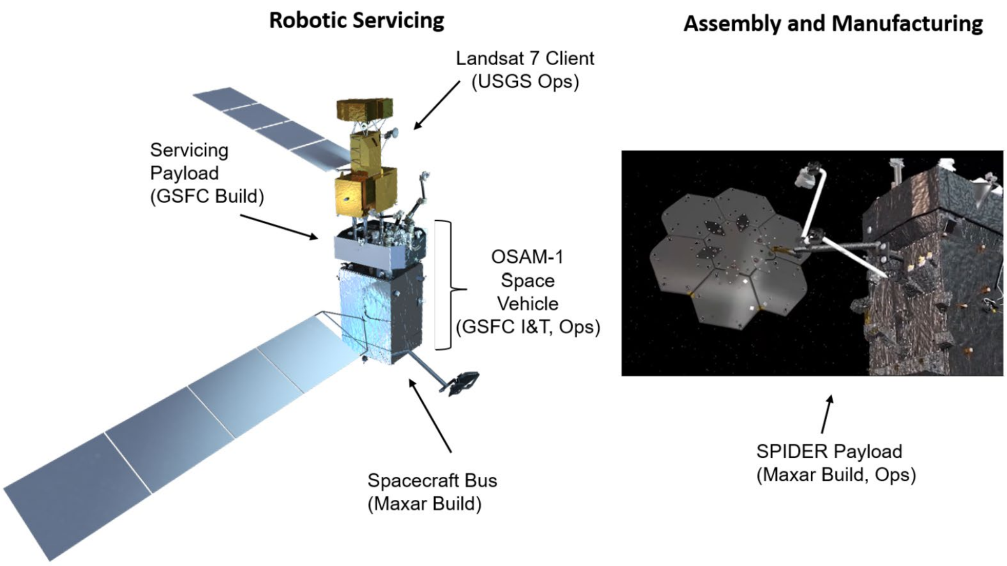

Fig. 1.1: OSAM-1 and Landsat-7, with SPIDER shown on the right [2]

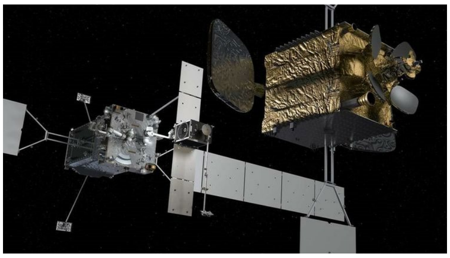

Fig. 1.2: RSGS by DARPA [5]

Fig. 1.3: ClearSpace-1 with VESPA adapter [4]

[6]. In solar sailing phase, the spacecraft can economically maintain its nominal orbit around the sun-earth L2 using solar radiation pressure [7]. Moreover, by using redundant degrees of freedom of actuatable joints, the system is expected to achieve multi-tasking, such as pointing the telescope toward the target object and simultaneously carrying out solar sailing by pointing a reflective surface toward the sun. The key technology of JAXA Transformer is simultaneous body reconfiguration and attitude reorientation, which is referred to as *nonholonomic reorienting transformation* or *NRT* for short [8], which is described in the next section.

Many science fictions have illustrated such "Transformable" fighter robots travelling in space, and some researchers have discussed its feasibility in engineering aspects [9, 10, 11]. Although it might be further behind their ideals, the JAXA Transformer can be a first step of such future advanced space robots.

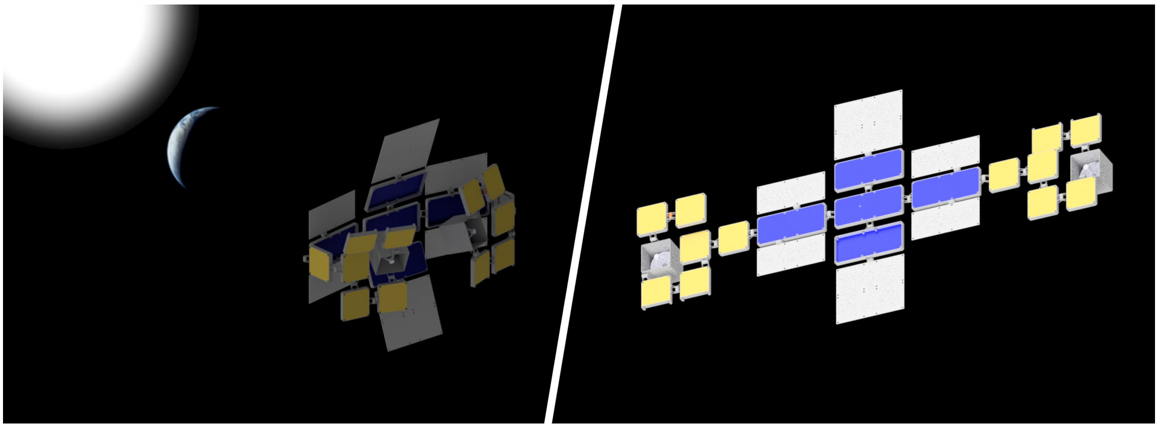

Fig. 1.4: JAXA Transformer, left) observation mode, right) solar sailing mode (credit: Transformer working group)

1.1.2 Nonholonomy of Attitude Motion

The attitude motion of a transformable free-flying robot is governed by the angular momentum conservation law if no external force/torque is exerted. Due to this dynamic constraint, when a part of the body is actuated, the attitude of the remainder of the body is reactively reoriented. The mechanics has been studied extensively for a long time, mainly regarding space manipulators [12, 13]. These studies have numerically solved a forward problem in which attitude motion is computed under the given joint actuation procedure. Such investigations have also revealed that this attitude motion is nonholonomic. Since the moment of inertia of a transformable robot is not constant when reconfiguring its body, the angular momentum conservation law becomes a non-integrable differential constraint, i.e., a nonholonomic constraint. Nonholonomy appears as an accumulation of nonlinear effects along a trajectory in state space. In particular, this effect is distinctly confirmed by an indirect, detour path in state space. For example, a vehicle cannot directly move in lateral direction, but going back and forth with proper handling can produce net displacement in the lateral direction due to nonholonomy (Fig. 1.5). In the case of a transformable free-flying robot, such a control corresponds to designing a proper indirect path in joint angle space that achieves a target final body configuration and final attitude.

Due to this nonholonomy, the attitude of the free-flying robot depends on its joint actuation procedure, and thus can be reoriented by properly actuating its body [14, 15]. This attitude reorientation maneuver is induced only by internal torque and, therefore, does not consume any propellant in principle. In addition, this maneuver does not cause momentum accumulation of reaction wheels, which leads to propellant consumption in saturation. These characteristics are preferable to elongate the life-span of the space robots. However, in order to solve the inverse problem of designing the joint actuation procedure that achieves the target body configuration and attitude, the following three issues must be resolved:

- In a non-holonomic system, state variables cannot be stabilized by time-invariant continuous feedback control (proved by the negative aspect of Brockett's theorem [16]).

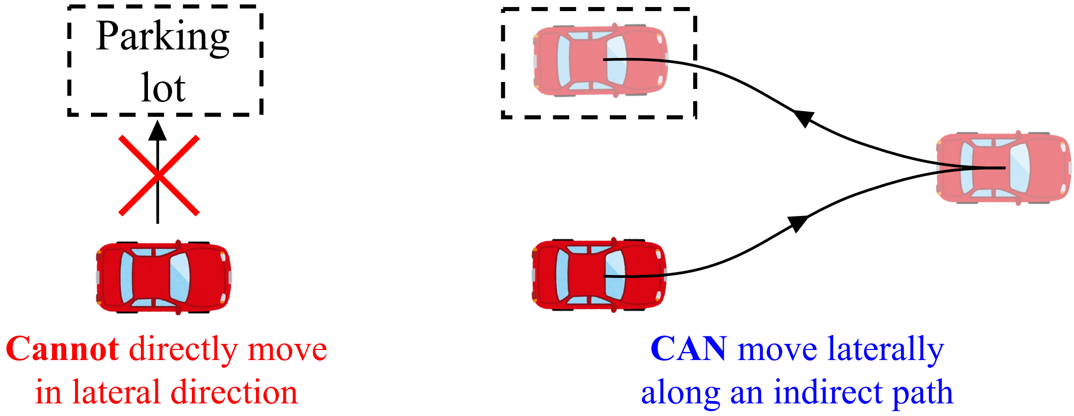

Fig. 1.5: Example of nonholonomic effect in vehicle parking

- The attitude equation of a transformable free-flying robot cannot be transformed into a canonical form, such as a chained form, and therefore cannot be controlled by common methods that are effective for canonical systems.
- In the case of general three-dimensional attitude motion, an angular velocity vector becomes non-integrable, and therefore, additional nonholonomy needs to be taken into account as well as the nonholonomy derived from the angular momentum conservation law. In this study, the former nonholonomy is referred to as *kinematic nonholonomy*, the latter nonholonomy is referred to as *momentum nonholonomy*, and the combination of the two types of nonholonomy is referred to as *hybrid nonholonomy* (see Section 2.3).

Most previous studies do not deal with the second and third problem for the sake of simplicity by assuming a system transformable into the canonical system, or assuming planar motion in which the angular velocity is integrable (See Section 1.1.3 in detail). In contrast, this study tackles all the above three problems, which adds to the value of this study.

1.1.3 Overview of Related Studies

Study on nonholonomic system has a long history. The comprehensive survey can be found in the textbook such as Bloch's [17]. The central subject of this field had been motion of rolling or sliding rigid bodies, such as a skidding knife-edge [18, 19, 20], and a wheeled vehicle moving on the ground [21, 22]. In particular, the system that can be transformable into a canonical form has been well investigated, and a lot of research on control of such canonical systems has been conducted [23, 24, 25]. For detail, please refer to Section 2.2.

As for the mechanics of free-flyers, one of the most widely known nonholonomic problem is the falling-cat phenomenon (Fig. 1.6). This is a phenomenon in which a cat can always land in an upright attitude by twisting its body in the air, even when dropped in an upside-down attitude. There have been many studies to elucidate the dynamics of this phenomenon [26, 27] and to apply the attitude maneuver of this phenomenon to free-flying robots [28, 29, 30, 31, 32, 33].

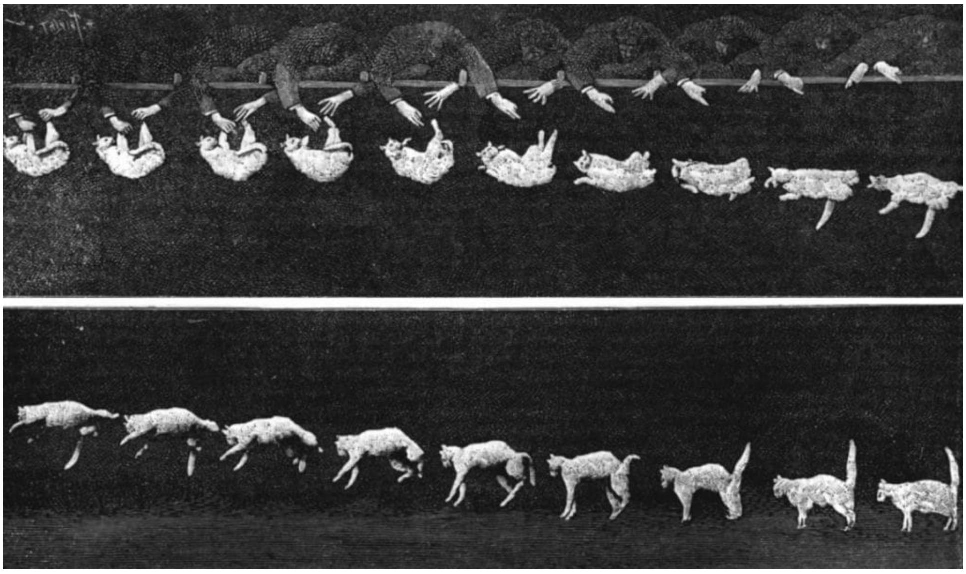

Fig. 1.6: Falling cat photographed by Etienne-Jules Marey in 1984 (Public domain) [34]

In the 1980s, since numerous space robots were proposed to construct space stations [35, 36], more studies focused on the dynamics of free-flying robots. The related formulations are roughly classified into two types: Euler-Lagrange formulation and Kane's formulation. The Euler-Lagrange formulation is energy-based and systematic but has difficulty in handling three-dimensional attitude motion because a generalized coordinate cannot be defined for angular velocity due to its non-integrability. On the other hand, Kane's formulation is based on the Newton-Euler formulation. The Newton-Euler formulation generally has difficulty in handling the constraint forces, but Kane solved this problem by performing a mathematical operation that prevents the constraint forces from appearing in the equations. Therefore, in general, Kane's method is suitable for dealing with the three-dimensional attitude motion of free-flying robots. The fundamental equations in the present study are based on Kane's formulation.

The principal task of a space manipulator is to control the position and attitude of the end-effector, and the reaction of attitude is canceled by thrusters or reaction wheels conventionally [37, 38, 39]. However, some researchers proposed actively using the attitude reaction when actuating its manipulator. In the case of planar motion, several attitude reorientation methods have been proposed that leverage the nonholonomy of the attitude motion [40, 41, 42, 43, 44]. However, since the angular velocity is integrable in planar motion (i.e., the angular velocity can be expressed as time derivative of the azimuthal angle), the kinematic nonholonomy is not taken into account in these studies. Therefore, these methods cannot be directly applied to general three-dimensional attitude control. On the other hand, some research focused on the system with only kinematic nonholonomy. The typical example is a rigid spacecraft equipped with two reaction wheels [45, 46, 47]. Such systems with either momentum nonholonomy or kinematic nonholonomy can be handled by simple geometrical strategies.

A combined system, i.e., a three-dimensional nonholonomic attitude control system of free-flying robot is one of the most difficult fields, and thus few studies actively use the nonholonomic properties. The hierarchical Lyapunov method proposed by Nakamura and Mukherjee[15] is one of the few examples by which to achieve the desired end-effector trajectory under some joint constraints using its nonholonomy. In this method, the first-priority Lyapunov function is constructed to obtain the target end-effector state and the second-priority Lyapunov function is constructed to satisfy the constraints on the state space. However, it is known that the constraints of the second Lyapunov function are not always satisfied, and convergence to the target is not always guaranteed in this method. Another example is the method of using sinusoidal input in joint actuation [48, 49, 50]. This method can achieve arbitrary attitude reorientation by repeating sinusoidal joint actuation. However, the applicability of this method is limited because the input is restricted to sinusoids. In addition, this method is not agile because the attitude is gradually shifted to the target by repeating sinusoidal actuation. In addition, none of these methods cover transfiguration of robots that dramatically changes its structure and even its function. Thus, although the dynamics of transformable free-flying robots has been studied for a long time, a method by which to achieve a target morphology and attitude simultaneously has not been sufficiently studied.

Research closely related to this study was conducted by Ohashi et al. [51, 52]. Their method was motion-primitive based and took a comprehensive combination of joint actuation patterns. They stored all results of the total attitude changes of the corresponding joint actuation patterns. Figure 1.7 shows an example of their motion primitive databases plotting rotational direction and its rotational efficiency for 3-panel model. In this method, the number of combinations increases exponentially with the increase in the number of joints. Therefore, their control method cannot fully make use of the attitude reorientation ability of the highly redundant free-flying robot. In addition, they solved the attitude motion of the robot by using numerical integration, which cannot extract any differential information to solve the inverse problem, which limited their method to a brute-force search. Moreover, Ohashi et al. used a heuristic or meta-heuristic algorithm, such as a genetic algorithm to solve the inverse problem, but this requires a huge calculation for each attitude reorientation planning task. Figure 1.8 summarizes the relationship among the related studies.

1.2 Purpose and Contributions

The purpose of this study is to construct motion planning methods for free-flying space robots that simultaneously performs body reconfiguration and nonholonomic attitude reorientation (or NRT), and to propose its applicability in space missions. In particular, the methods have two noticeable characteristics: 1) they deal with multi-degree-of-freedom transfiguration that dramatically change the robot structure, and 2) they assume general non-planar attitude motion. To this end, contributions of this research are as follows:

Derivation of an approximate analytical solution In order to solve the inverse problem in a nonholonomic system, it is necessary to systematically understand the motion of the system and design an appropriate indirect detour path in state space (see Fig. 1.5 for reference). To this end, we derive an approximate analytical solution of the attitude motion

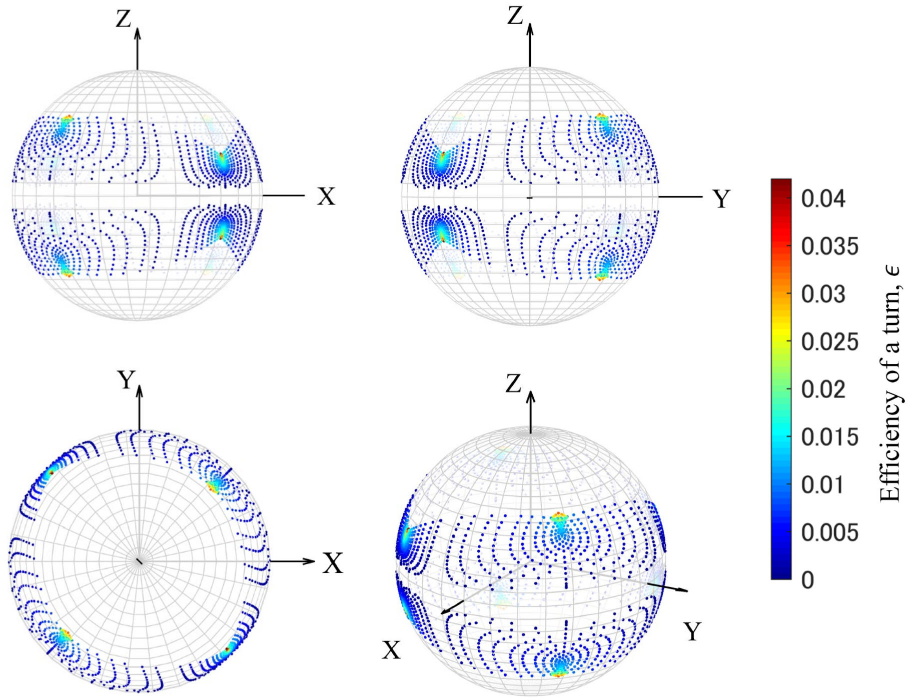

Fig. 1.7: Ohashi's motion primitive database of rotational direction and its rotational efficiency for 3-panel model [52]

during body reconfiguration. The solution analytically provides information on sensitivity of final attitude with respect to the detour of body reconfiguration. Conventional numerical integration method for solving attitude motion cannot extract information on such attitude sensitivity. Therefore it requires brute force computations and cannot handle large degrees-of-freedom system. Compared to the conventional method, the proposed method is able to handle highly redundant, large degrees-of-freedom system. Among integrable analytical solutions, the rectilinear solution is used throughout this thesis, in which joints are actuated along a line segment in joint angle space.

Design of motion planning methods Using the derived rectilinear solution, we can design a motion planning method that simultaneously achieves body reconfiguration and attitude reorientation. There are two main methods proposed in this study. One is *rectilinear transformation planning*, which achieves the target morphology and attitude by a consecutive line segment paths in joint angle space. In this method, the joint actuation law is simple and the singularity can be easily avoided by increasing the number of waypoints. The second method is *rectilinear invariant manifold method*, which designs a bundle of manifolds growing from a target state. The motion planning is completed in two phases: 1) asymptotic approaching to one manifold in the bundle, and 2) sliding on the manifold to the target state. Figure 1.9 shows illustrations of the above two methods and corresponding analogy with car parking problem. These two distinct methods are complementary and both of them exhibit different characteristics depending on the situations.

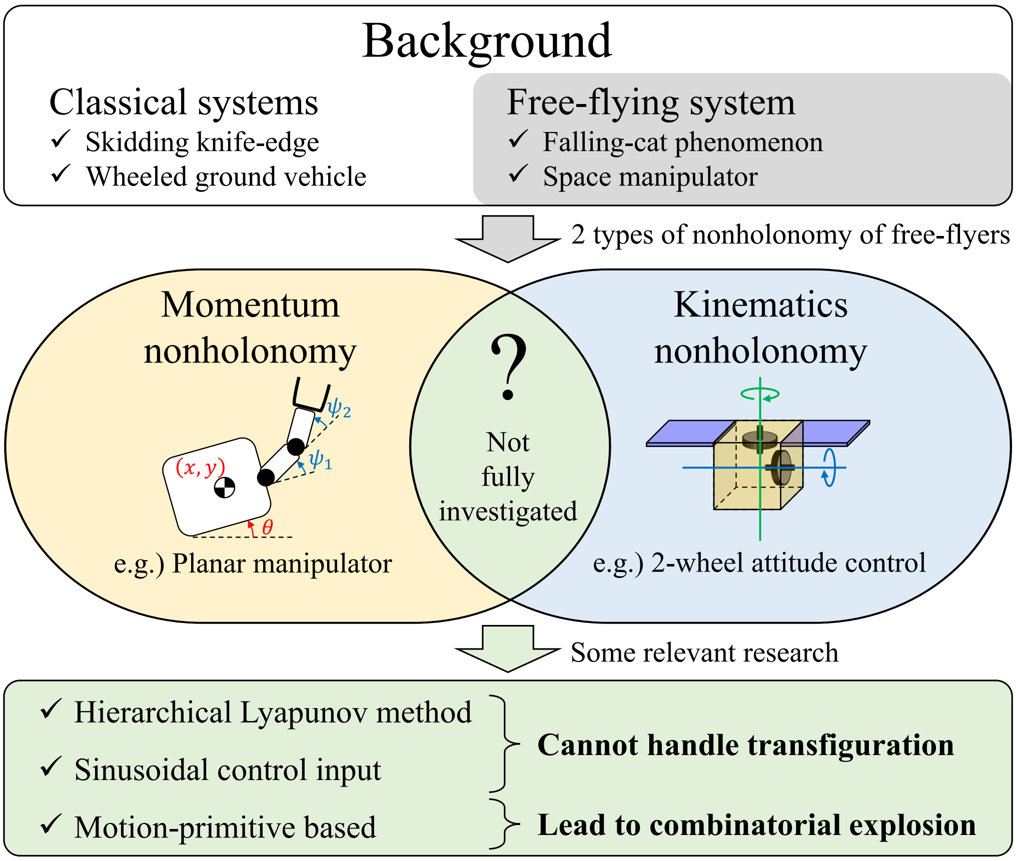

Fig. 1.8: Summary of related studies

Application to space missions In order to demonstrate the effectiveness of the proposed methods, an application to a space mission is presented. As a typical application, a propellant-free solar sailing method is introduced through a numerical example. The simultaneous body reconfiguration and attitude reorientation allows a transformable solar sail to change its equilibrium attitude under solar radiation pressure while orienting its attitude to that equilibrium attitude simultaneously. This maneuver will greatly enhance capability of solar sailing.

1.3 Organization

This thesis begins with overviewing preliminaries on dynamics of free-flying multi rigid bodies (Section 2.1), fundamental knowledge about control of nonholonomic system (Section 2.2), and introduction of kinematic nonholonomy and momentum nonholonomy (Section 2.3). These preliminaries are bases of the discussion in the following chapters as well as supporting the introductions mentioned in Chapter 1.

Chapter 3 describes the analytical solutions to the fundamental kinematics equation in Section 2.1. The kinematics equation for direction cosine matrix preserves Lie-group structure of

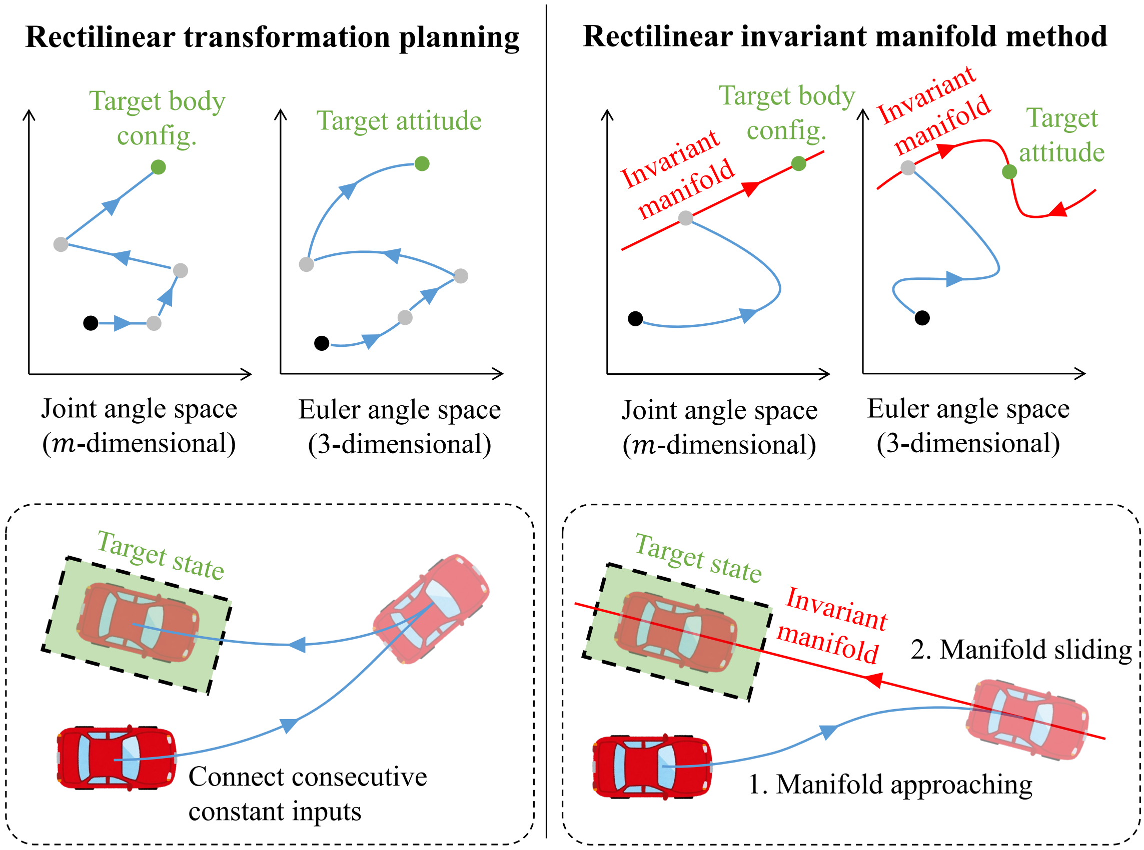

Fig. 1.9: Illustration of the proposed methods and corresponding analogy with car parking problem; left) rectilinear transformation planning, right) rectilinear invariant manifold method

three-dimensional rotation, and hence can be approximately solved by the Magnus expansion. In addition, polynomial/rectilinear solution and sinusoidal solution are concretely displayed as examples of analytically integrable solutions. Moreover, computational performance and approximation accuracy of both solutions are investigated.

Chapter 4 describes one of the proposed motion planning methods, the rectilinear transformation planning. To derive the motion planning scheme, nonholonomic sensitivity tensor is first derived. This tensor provides information on sensitivity of final attitude with respect to intermediate joint trajectories, and hence is the important indicator for motion planning in nonholonomic system. Next several sections are dedicated to describing initial guess search, a path modification scheme with and without constraints. Finally, numerical simulations validate effectiveness of the method.

Chapter 5 describes another proposed motion planning method, the rectilinear invariant manifold method. In order to describe the geometrical structure of state space, the nonholonomic system is reformulated in the context of sub-Riemannian geometry. In addition, we show all possible rectilinear solution growing from a certain state constitutes a fiber bundle in multi-dimensional space. Moreover, a control law to asymptotically attract state variables onto the fiber bundle is derived and its sufficient condition of convergence is discussed with the Lyapunov control theory. Numerical examples show the effectiveness of the method and some failure case due to singularity.

Chapter 6 focuses on singularity analysis of the proposed two motion planning methods. The singularity analysis is largely based on the singularity analysis of control moment gyro (CMG) and robotic manipulator following the result of previous studies. We show that escapable null motion and singularity robust steering law successfully works for our motion planning methods.

Chapter 7 introduces solar sailing technique using NRT maneuver as one practical application of the proposed method. With the aid of simultaneous body reconfiguration and attitude reorientation, the equilibrium attitude and reorientation to it arbitrarily accomplished, which enhances solar sailing ability. Since the NRT neither consumes any propellant nor accumulate momentum of reaction wheels, the transformable solar sail can achieve completely propellant-free orbital maneuver, which can be a promising astrodynamics technique in future space missions.

Chapter 2

Preliminaries

2.1 Dynamics of Free-flying Space Robot

In this section, attitude motion of a transformable free-flying robot is formulated. The nomenclature of body components and their positions are shown in Fig. 2.1. In this representation, an entire spacecraft is divided into a main body, assigned to index $k = 0$, and some other branches growing from the main body. Here, for convenience of formulation, we assume that each branch has no loop structure. This assumption assures one-directional index assignment from the main body (innermost, root) to the tip of each branch (outermost, tip). Body indices in each branch are allotted sequentially from an inner body to an outer body. The hinge joint component connecting the k-th body and its inside neighbor is labeled as a k-th joint, and a set of all bodies outside of the k-th joint are labeled as $\hat{k}$, which is referred to as a k-th outer group. (From the above definitions, all indices in the outer group $\hat{k}$ are larger than k.) This $\hat{k}$ grouping contributes to simple formulations because all bodies in the k-th outer group move in the same manner with respect to the main body when the k-th joint is actuated. The rotational degrees of freedom of each hinge joint is set to be 1, and the rotational angle of the k-th joint is described as θ^k. Any joint rotation can be expressed by a combination of 1-dimenional rotations, and hence this assumption does not lose generality. The body frame is fixed to the main body ($k = 0$) and its origin is at the center of mass (CoM) of the main body. Thus, attitude of the entire multi body system is represented by the attitude of the main body hereafter. Note that the position of the CoM of an entire robot is not constant in the body-fixed coordinate as the robot changes its body configuration.

We use Kane's formulation in the present paper, which is, in general, suitable for handling three-dimensional attitude motion [12, 13]. A general form of the dynamics equation can be described as follows:

$$\begin{bmatrix} M_{vv} & M_{v\omega} & M_{v\theta} \\ M_{\omega v} & M_{\omega\omega} & M_{\omega\theta} \\ M_{\theta v} & M_{\theta\omega} & M_{\theta\theta} \end{bmatrix} \begin{bmatrix} w_v \\ w_\omega \\ w_\theta \end{bmatrix} + \begin{bmatrix} d_v \\ d_\omega \\ d_\theta \end{bmatrix} = \begin{bmatrix} \tau_v \\ \tau_\omega \\ \tau_\theta \end{bmatrix} \tag{2.1}$$

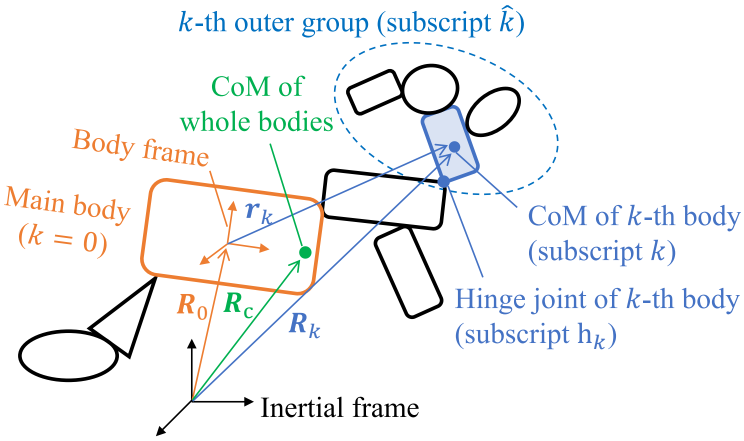

Fig. 2.1: Nomenclature of body components and their positions

or for short,

$$Mw + d = \tau \tag{2.2}$$

The corresponding generalized velocities are the translational velocity of the CoM of the entire spacecraft v_v, the angular velocity of the main body v_ω, and the joint actuation speed v_θ, which are all expressed in the body frame:

$$v_v = v_\mathrm{c} = \begin{bmatrix} v_{\mathrm{c},x} \\ v_{\mathrm{c},y} \\ v_{\mathrm{c},z} \end{bmatrix} \in \mathbb{R}^3 \qquad v_\omega = \omega_0 = \begin{bmatrix} \omega_{0,x} \\ \omega_{0,y} \\ \omega_{0,z} \end{bmatrix} \in \mathbb{R}^3 \qquad v_\theta = \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_m \end{bmatrix} \in \mathbb{R}^m \tag{2.3}$$

An explicit description of the components in Equation (2.1) is given in Appendix A. In particular, $M_{\omega\omega}$ and $M_{\omega\theta}$ are important in the following discussion because these values provide the relationship between joint actuation and attitude motion:

$$\begin{aligned} M_{\omega\omega} &= I_\mathrm{c} \\ M_{\omega\theta,j} &= \left(I_{\hat{j}} - m_{\hat{j}} r_{\hat{j}\mathrm{c}}^\times r_{\hat{j}\mathrm{h}_j}^\times \right)^\mathsf{T} \lambda_j \end{aligned} \tag{2.4}$$

where $M_{\omega\theta,j}$ indicates the j-th column of $M_{\omega\theta}$. With these expressions, total angular momentum h_c around the centroid of the entire spacecraft can be represented as follows:

$$h_\mathrm{c} = M_{\omega\omega} v_\omega + M_{\omega\theta} v_\theta \tag{2.5}$$

In the present paper, we focus on attitude motion in which the total angular momentum is always zero. Applying this assumption to Equation (2.5) yields the following kinematic relationship:

$$v_\omega = \omega_0 = -M_{\omega\omega}^{-1} M_{\omega\theta} v_\theta = -M_{\omega\omega}^{-1} M_{\omega\theta} \dot{\theta} \tag{2.6}$$

Now, we use a rotational matrix (i.e., direction cosine matrix) C to express the attitude of the main body. This is because the rotational matrix is the most natural expression of the Lie group structure of three-dimensional rotation, which helps to analytically solve the differential equation. Substituting Equation (2.6) into the equation of a rotational matrix, we obtain

$$\dot{C} = -\omega_0^\times C = \left(M_{\omega\omega}^{-1} M_{\omega\theta} \dot{\theta} \right)^\times C = (g(\theta) u)^\times C \tag{2.7}$$

where $g(\theta) = M_{\omega\omega}^{-1} M_{\omega\theta} \in \mathbb{R}^{3 \times m}$, and control input $u = \dot{\theta} \in \mathbb{R}^m$. In the present research, the joint actuation speed $\dot{\theta}$ is regarded as control input, rather than joint actuation torque. This is because the angular momentum conservation law is expressed in integral form, and, therefore, the kinematics level equation can appropriately grasp the characteristics of the dynamics when there is no external force or torque. This equation expresses the relationship between attitude change and hinge joint actuation and has two important features:

1. Equation (2.7) is a nonholonomic constraint (or non-integrable differential constraint). Although the system is underactuated because the attitude (3 DoF) + body configuration (m DoF) is controlled by m-dimensional control input $u = \dot{\theta} \in \mathbb{R}^m$, the nonholonomy practically provides extra degrees of freedom in control space and therefore enables the underactuated system to be controllable (locally, at least).
2. The function $g(\theta)$ is independent of attitude C, and therefore, in this equation, we can independently handle kinematic nonholonomy (induced by the non-integrability of a three-dimensional angular velocity vector) and momentum nonholonomy (induced by the change in the moment of inertia).

In Chapter 3, Equation (2.7) is further investigated in order to obtain an approximate analytical attitude solution.

2.2 Fundamentals of Nonholonomic System

2.2.1 Controllability of Nonholonomic System

A nonholonomic system is a system in which non-integrable differential constraints are imposed. Comprehensive survey can be found in the textbook such as Bloch's [17]. The theory of the nonholonomic system has a long history and especially in the field of mechanics, motion of a rolling or sliding rigid body has been extensively studied as a typical subject [53]. Prevalent problems are motion of a knife-edge skidding on a plane, known as a Chaplygin sleigh [18, 19, 20], or a vehicle moving on the ground with non-sliding rolling wheels [21, 22]. Since non-integrable differential constraints cannot be expressed as constraints on the state variables, the constraints do not reduce some dimensions of the system. It means that the reachable space of the state variables is not directly constrained in the nonholonomic system, whereas the instantaneous velocity is constrained. In the field of control theory, this fact means that we can control more state variables than degrees of freedom of control input, i.e., it has a potential to control an underactuated system.

Unlike linear systems, we cannot generally guarantee the controllability of nonholonomic systems. This is because in a nonlinear system $\dot{x} = f(x) + g(x)u$, distribution of singular points is determined by the specific form of $f(x)$ and $g(x)$, which makes it difficult to discuss its controllability in general situation [54]. However, for a symmetrically affine system $\dot{x} = g(x)u$, local controllability can be examined by *Chow's theorem* [55, 56]. In order to state the Chow's theorem, we define a distribution $\mathcal{C}(x)$ as follows:

Definition. *For nonholonomic system $\dot{x} = \sum_{k=1}^{m} g_k(x)u_k$, $(x, g_k \in \mathbb{R}^n)$, the distribution $\mathcal{C}(x)$ at $x = x_0$ is defined as:*

$$\mathcal{C}(x_0) = \mathrm{span}(\Delta_\kappa) \tag{2.8}$$

where κ is a degree of nonholonomy and the Δ_i is recursively defined as follows:

$$\begin{aligned} \Delta_1 &= \mathrm{span}\{g_1(x_0), g_2(x_0), \cdots, g_m(x_0)\} \\ \Delta_{i+1} &= \Delta_i + [\Delta_1, \Delta_i] \\ \textit{where} \quad [\Delta_1, \Delta_i] &= \mathrm{span}\{[g, h] : g \in \Delta_1, h \in \Delta_i\} \end{aligned} \tag{2.9}$$

Degree of nonholonomy κ is defined as the smallest integer such that the rank Δ_k is equal to that of Δ_{k+1} [56]. $[f, g]$ is a Lie bracket or a commutator, which is defined as:

$$[g, h] = g \circ h - h \circ g \tag{2.10}$$

where $\circ$ signifies a product defined for X such that $g, h \in X$. In particular, for vector fields, this is written as follows:

$$[g, h] = \frac{\partial h}{\partial x} g - \frac{\partial g}{\partial x} h \tag{2.11}$$

Based on this definition, Chow's theorem is stated as follows:

Theorem (Chow's theorem)**.** *The symmetrically affine system $\dot{x} = \sum_{k=1}^{m} g_k(x)u_k$ is locally controllable at $x = x_0$ if and only if* rank $\mathcal{C}(x_0) = n$

For the problem of simultaneously controlling body configuration and attitude, the system dimension is $n = m + 3$ where m is the degree of freedom of joint control input and 3 is the dimension of three-dimensional attitude motion. If rank $\mathcal{C}(x_0) = m + 3$, a free-flying robot can achieve arbitrary body configuration and attitude at least in the neighborhood of $x = x_0$.

However, even for the locally controllable nonholonomic system, the state cannot be asymptotically stabilized by a time-invariant continuous feedback control. This fact is proved by the negative aspect of Brockett's theorem [16]. The Brockett's theorem states the necessary condition of asymptotic stabilizability for a nonlinear system, but the underactuated nonholonomic system does not satisfy the necessary condition. Therefore, in order to achieve a target state in the nonholonomic system, time-variant feedback or discontinuous feedback control must be designed. This is the background reason the proposed motion planning method in this study uses multiple-stage, discontinuous control scheme.

2.2.2 Chained System and Power System

Nonholonomic constraints appear in a wide variety of mechanical systems. Among the systems, a *chained system* and a *power system* are known as the most mathematically structured (canonical) system and therefore, a lot of effective control laws are proposed. The chained system is the system defined as follows [57, 54]:

$$\begin{aligned}
\dot{z}_1 &= v_1 \\
\dot{z}_2 &= v_2 \\
\dot{z}_3 &= z_2 v_1 \\
\dot{z}_4 &= z_3 v_1 \\
&\vdots \\
\dot{z}_n &= z_{n-1} v_1
\end{aligned} \tag{2.12}$$

And the power system is the system defined as:

$$\begin{aligned}
\dot{z}_1 &= v_1 \\
\dot{z}_2 &= v_2 \\
\dot{z}_3 &= z_1 v_2 \\
\dot{z}_4 &= \frac{1}{2} z_1^2 v_2 \\
&\vdots \\
\dot{z}_n &= \frac{1}{(n-2)!} z_1^{n-2} v_2
\end{aligned} \tag{2.13}$$

Khennouf proposed a method to use an invariant manifold around a target state for the chained system with $n = 3$ [23], and Luo extended the Khennouf's method to n-dimensional power system [24]. The another important method is a feedforward based method in which sinusoidal input is applied for a chained system [25]. Other methods can be found in [58].

Some control system can be converted into the chained system or the power system. Murray proved that all symmetrically affine system with state dimension $n = 3$ or 4 and control dimension $m = 2$ can be converted into the chained system [59]. For example, the planar three-link free-flying robot is a typical system that can be converted into the chained form (Fig. 2.2). This system has three-dimensional state variables $[x\ y\ \theta]^\mathsf{T}$ and two-dimensional control input $u = [\psi_1\ \psi_2]^\mathsf{T}$. However, a general three-dimensionally rotating free-flying robot cannot be converted into the chained system, which is the control target in this study.

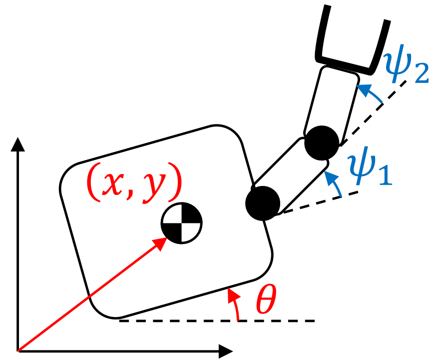

Fig. 2.2: Example of a planar three-link free-flying robot

2.3 Kinematic Nonholonomy and Momentum Nonholonomy

Kinematic nonholonomy and momentum nonholonomy are terminologies the author defined. They indicate distinct nonholonomic effects concerned with three-dimensional rotation of a free-flying space robot. The kinematic nonholonomy is derived from non-integrability of three-dimensional angular velocity, whereas momentum nonholonomy is derived from non-integrability of the angular momentum conservation law when the moment of inertia of the robot is not constant. These two types of nonholonomy are distinct and can be solely induced. For example, in Fig. 2.3, a person seated in a rotary chair alternately bends and stretches his legs and twists his upper body, resulting in net attitude rotation due to the intermediate difference in the moment of inertia. In this example, the effect of kinematic nonholonomy is zero because the axis of rotation is always in the same direction, and therefore the effect of momentum nonholonomy is purely produced. On the other hand, Fig. 2.4 shows an example of a spacecraft with only two reaction wheels. Owing to the kinematic nonholonomy, this spacecraft can rotate around the third axis by alternating rotations with two wheels. In this example, the effect of momentum nonholonomy is zero because the moment of inertia of the spacecraft is fixed, and the effect of kinematic nonholonomy is purely produced.

Most previous research dealt with each nonholonomy solely; attitude control of planar free-flyers is the typical example for the pure momentum nonholonomy [40, 41, 42, 43, 44], and two-wheel control of a rigid spacecraft is the typical example for the pure kinematic nonholonomy [45, 46, 47]. Such systems with single nonholonomy let the nonholonomic effect be expressed with global geometrical relationship. For example, the linear and angular momentum conservation law of a planar free-flying space robot shown in Fig. 2.5 are expressed as [60]:

$$\begin{aligned} &m_\mathrm{s} x_\mathrm{s} + m_\mathrm{h} x_\mathrm{h} = 0, \quad m_\mathrm{s} y_\mathrm{s} + m_\mathrm{h} y_\mathrm{h} = 0 \\ &I_\mathrm{s}\dot{\theta}_\mathrm{s} + m_\mathrm{s}(x_\mathrm{s}\dot{y}_\mathrm{s} - y_\mathrm{s}\dot{x}_\mathrm{s}) + m_\mathrm{h}(x_\mathrm{h}\dot{y}_\mathrm{h} - y_\mathrm{h}\dot{x}_\mathrm{h}) = 0 \end{aligned} \tag{2.14}$$

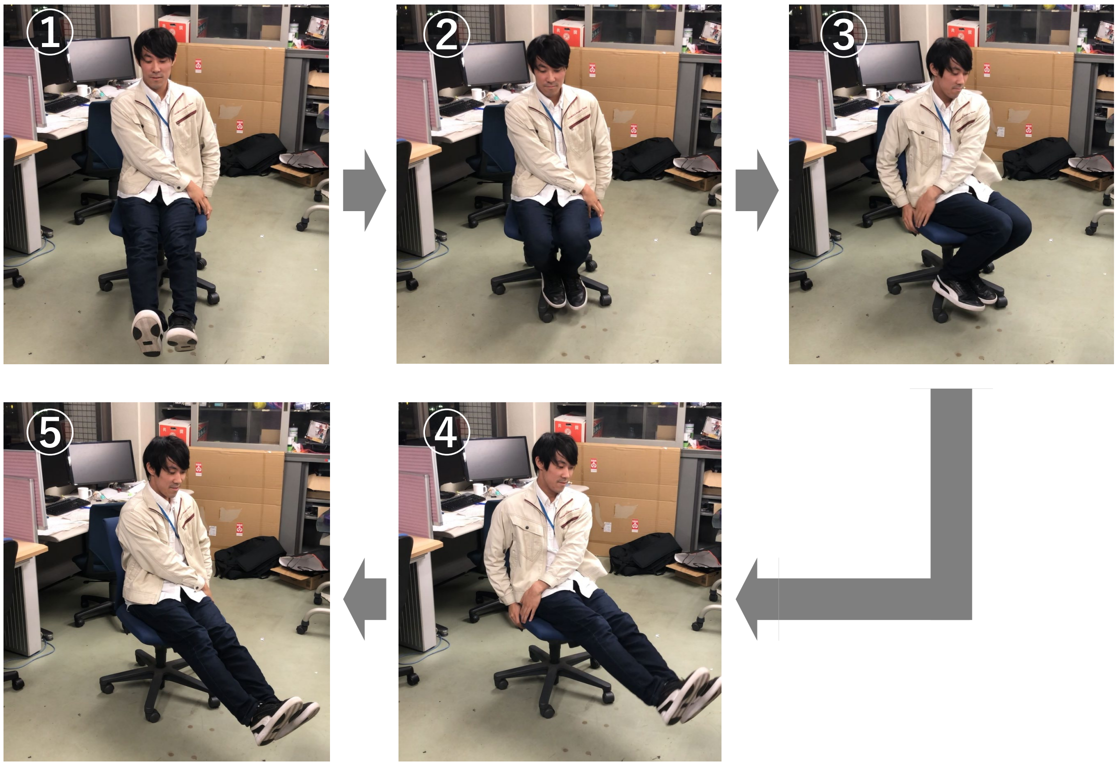

Fig. 2.3: Example of momentum nonholonomy

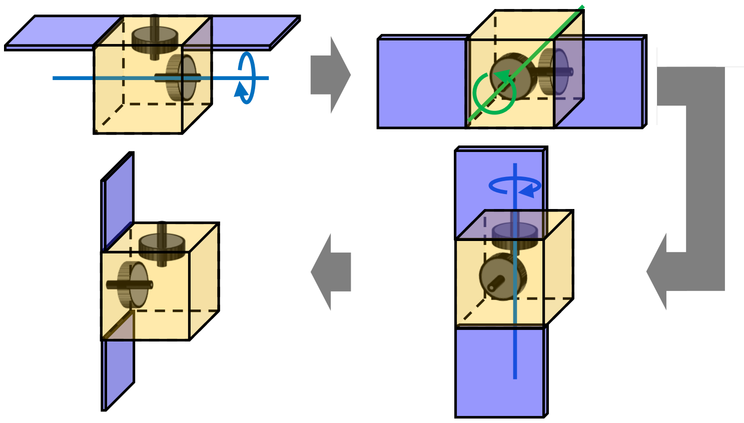

Fig. 2.4: Example of kinematic nonholonomy

where I, m, (x, y) are respectively moment of inertia, mass, and position of CoM, and subscript s and h express the space robot main body and the manipulator hand. Cancelling x_s and y_s results in the following relationship:

$$\dot{\theta}_\mathrm{s} = a(x_\mathrm{h}\dot{y}_\mathrm{h} - y_\mathrm{h}\dot{x}_\mathrm{h}), \quad a = -\frac{m_\mathrm{h}(m_\mathrm{s} + m_\mathrm{h})}{m_\mathrm{s} I_\mathrm{s}} \tag{2.15}$$

By using Stokes' theorem, attitude difference $\Delta\theta_{\mathrm{s}}$ when the hand is actuated along a closed trajectory ∂D can be described as follows:

$$\Delta\theta_{\mathrm{s}} = \int_{\partial D} a(x_{\mathrm{h}}\mathrm{d}y_{\mathrm{h}} - y_{\mathrm{h}}\mathrm{d}x_{\mathrm{h}}) = 2a\int_{D} \mathrm{d}x_{\mathrm{h}} \wedge \mathrm{d}y_{\mathrm{h}} \tag{2.16}$$

where $\wedge$ is a wedge product. Thus, the effect of momentum nonholonomy is globally associated with area of the closed trajectory of the hand for the planar free-flyer. As for the kinematic nonholonomy, the amount of third-axis rotation in two-wheel control is similarly expressed as area on unit sphere swept by the third axis (generally known as the *coning effect* [61]). Therefore, their motion planning can be handled with purely geometrical operations.

In a general three-dimensional attitude motion of the free-flying space robot, however, these two nonholonomic effects are combined, resulting in a complex attitude behavior. Its nonholonomic effect can no longer be associated with a simple global geometry. This study aims to generally handle the case with which two types of nonholonomy are involved, which we refer to as a *hybrid nonholonomic system*. Mathematical analysis of the combined nonholonomy is provided based on parallelogram actuation.

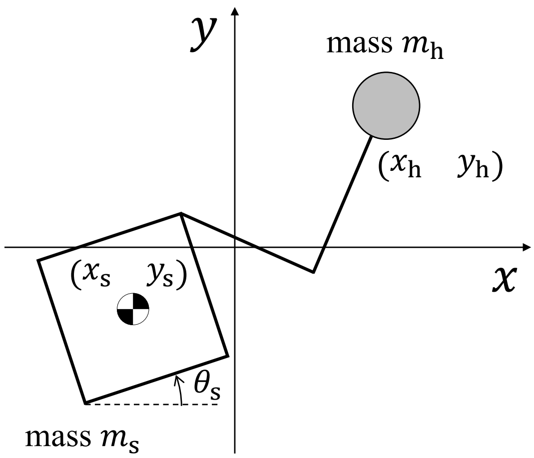

Fig. 2.5: Example of a planar space robot model (replication of a figure in [60] by the author)

2.4 Model Definition of JAXA Transformer

As mentioned in Section 1, the proposed methods are effective for a robot with highly redundant, large degrees-of-freedom joints. As a typical example of such a robot, this study uses a model of JAXA Transformer in most numerical examples [6]. As shown in the left of Fig. 2.6, the spacecraft consists of 27 bodies and 18 rotational actuators. This figure shows the fully deployed body configuration, in which all joint angles are set to 0 degree. The local body-fixed coordinate attached to each body is defined such that they correspond to the *x*-*y*-*z* coordinates in this fully deployed body configuration. As mentioned in Section 2.1, attitude of the robot is represented by the attitude of the body-fixed coordinate attached to the body #0. The non-actuatable joints (#4,5,14,15,23,24,25,26) are rigidly fixed at 0 degree, and thus the model is equivalently regarded as 19 rigid bodies and 18 rotational actuators. Thus the joint angle vector $\theta \in \mathbb{R}^m$ contains angles of the 18 actuatable joints. The mass and dimensions of each body are according to Table 2.1. All bodies are defined as cuboids and the corresponding side lengths are L_x, L_y, and L_z. The definitions of $p_{\underline{k}}$, p_k, and λ_k in Table 2.1 are shown in the right of Fig. 2.6, where the subscript $\underline{k}$ indicates the inside neighbor of the k-th body (definition of the term *inside* here follows Section 2.1). The values of $p_{\underline{k}}$ in Table 2.1 is expressed in the local body-fixed coordinates of the $\underline{k}$-th body, and p_k and λ_k are expressed in the local body-fixed coordinates of the k-th body. The angle limit of each joint is $-\pi/2 \leq \theta^k \leq \pi/2$. The maximum joint speed is set to be 10 deg/s, and the joint actuation speed in each stroke is normalized such that the fastest joint is actuated at 10 deg/s. However, joint actuation speed does not affect the following result as far as the total angular momentum is zero.

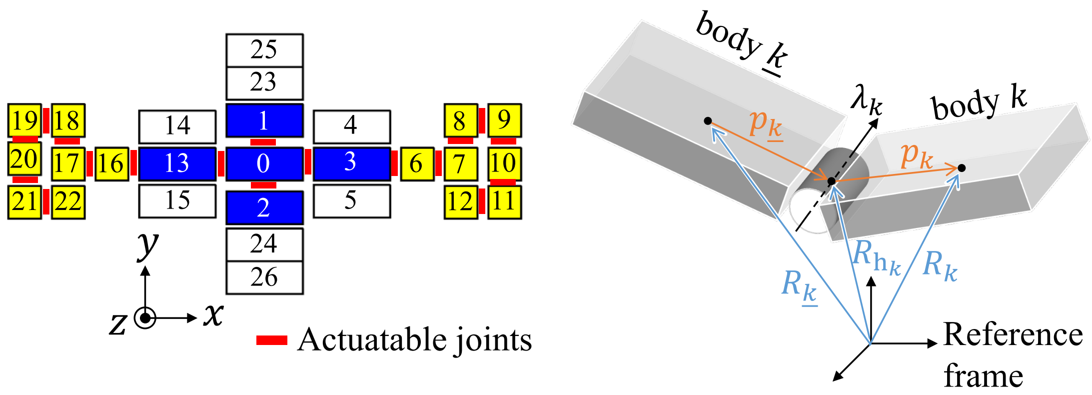

Fig. 2.6: Model definitions: left) Body connection map of JAXA Transformer, right) Definitions of some parameters

Figure 2.7 and 2.8 show the deployed configuration and the observation configuration respectively. Joint angles of the deployed configuration are all zero, whereas joint angles of the observation configuration are listed in Table 2.2. These configurations are frequently used in simulations throughout this thesis.

Table 2.1: Mass and dimension properties of JAXA Transformer

Body #	Mass [kg]	$L_x \times L_y \times L_z$ [mm]	$p_{\underline{k}}$ [mm]	p_k [mm]	λ_k
0	7.0	$1400 \times 600 \times 90$	-	-	-
1	7.0	$1400 \times 600 \times 90$	[0, 400, -50]	[0, 400, 50]	[1, 0, 0]
2	7.0	$1400 \times 600 \times 90$	[0, -400, 50]	[0, -400, -50]	[1, 0, 0]
3	7.0	$1400 \times 600 \times 90$	[800, 0, 70]	[800, 0, -70]	[0, 1, 0]
4	5.0	$1400 \times 600 \times 10$	[0, 340, 60]	[0, 340, -15]	[1, 0, 0]
5	5.0	$1400 \times 600 \times 10$	[0, -340, 50]	[0, -340, -5]	[1, 0, 0]
6	10.0	$600 \times 600 \times 90$	[800, 0, 50]	[400, 0, -50]	[0, 1, 0]
7	10.0	$600 \times 600 \times 90$	[400, 0, -50]	[400, 0, 50]	[0, 1, 0]
8	10.0	$600 \times 600 \times 90$	[0, 400, 50]	[0, 400, -50]	[1, 0, 0]
9	10.0	$600 \times 600 \times 90$	[400, 0, -50]	[400, 0, 50]	[0, 1, 0]
10	30.0	$600 \times 600 \times 600$	[0, -400, 50]	[0, -400, -300]	[1, 0, 0]
11	10.0	$600 \times 600 \times 90$	[0, -300, 400]	[0, -400, 50]	[1, 0, 0]
12	10.0	$600 \times 600 \times 90$	[-400, 0, 50]	[-400, 0, -50]	[0, 1, 0]
13	7.0	$1400 \times 600 \times 90$	[-800, 0, -50]	[-800, 0, 50]	[0, 1, 0]
14	5.0	$1400 \times 600 \times 10$	[0, 340, 60]	[0, 340, -15]	[1, 0, 0]
15	5.0	$1400 \times 600 \times 10$	[0, -340, 50]	[0, -340, -5]	[1, 0, 0]
16	10.0	$600 \times 600 \times 90$	[-800, 0, 70]	[-400, 0, -70]	[0, 1, 0]
17	10.0	$600 \times 600 \times 90$	[-400, 0, -50]	[-400, 0, 50]	[0, 1, 0]
18	10.0	$600 \times 600 \times 90$	[0, 400, -50]	[0, 400, 50]	[1, 0, 0]
19	10.0	$600 \times 600 \times 90$	[-400, 0, 50]	[-400, 0, -50]	[0, 1, 0]
20	30.0	$600 \times 600 \times 600$	[0, -400, -50]	[0, -300, -400]	[1, 0, 0]
21	10.0	$600 \times 600 \times 90$	[0, -400, 305]	[0, -400, -50]	[1, 0, 0]
22	10.0	$600 \times 600 \times 90$	[400, 0, -50]	[400, 0, 50]	[0, 1, 0]
23	5.0	$1400 \times 600 \times 10$	[0, 340, -50]	[0, 340, 5]	[1, 0, 0]
24	5.0	$1400 \times 600 \times 10$	[0, -340, 50]	[0, -340, -5]	[1, 0, 0]
25	5.0	$1400 \times 600 \times 10$	[0, 300, 5]	[0, 300, -5]	[1, 0, 0]
26	5.0	$1400 \times 600 \times 10$	[0, -300, -5]	[0, -300, 5]	[1, 0, 0]

Table 2.2: List of joint angles in observation configuration (non-actuatable joints are embraced by a bracket)

Joint 1–13		Joint 14–26	
i	θ^i	i	θ^i
1	-20	(14)	0
2	20	(15)	0
3	10	16	-80
(4)	0	17	-80
(5)	0	18	0
6	80	19	10
7	80	20	-30
8	0	21	0
9	-10	22	0
10	20	(23)	0
11	0	(24)	0
12	0	(25)	0
13	-10	(26)	0

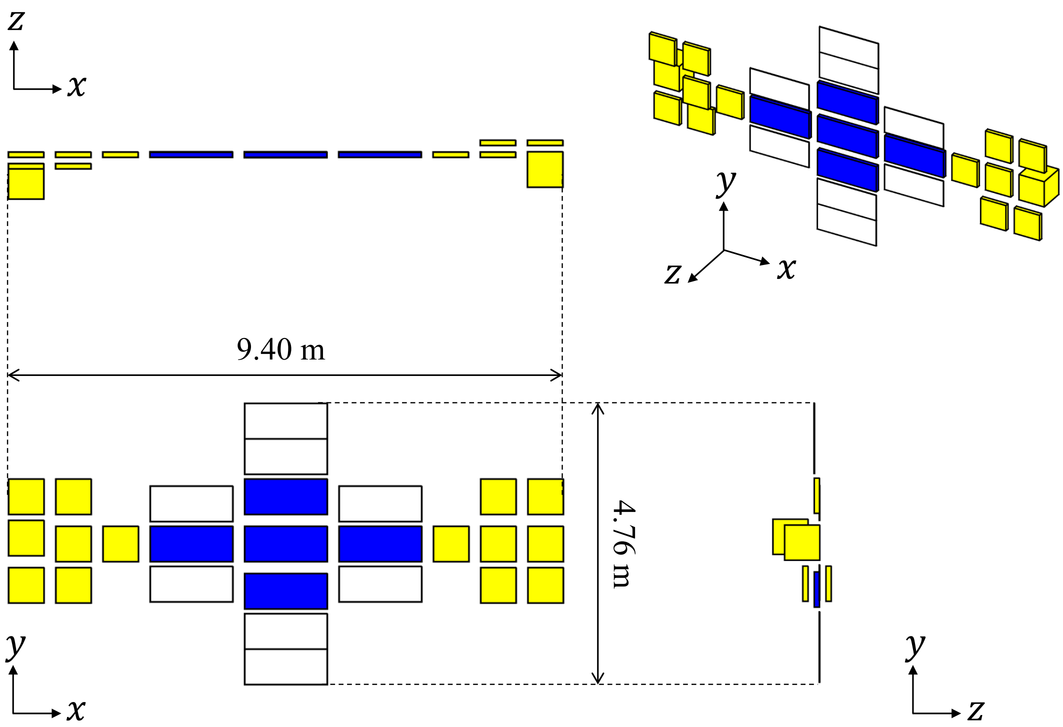

Fig. 2.7: Deployed configuration of JAXA Transformer

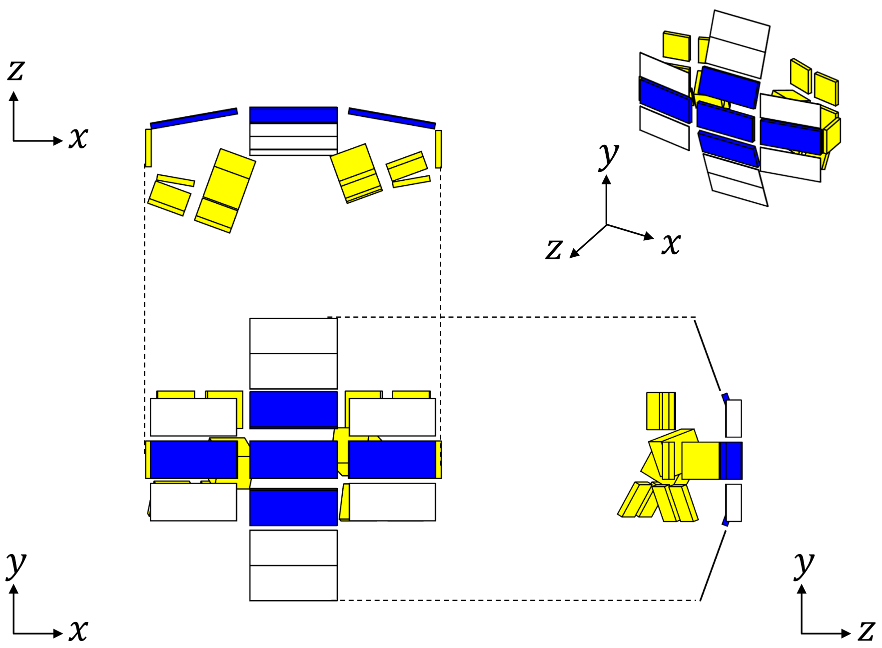

Fig. 2.8: Observation configuration of JAXA Transformer

Chapter 3

Analysis of Attitude Kinematics Equation

The contents in this chapter is omitted in this version for future publications.

Chapter 4

Rectilinear Transformation Planning

In the previous chapter, we showed that the rectilinear solution can handle body reconfiguration with less computational cost among the derived approximate analytical solutions. In this chapter, we address the main issue of this research: motion planning of the simultaneous body reconfiguration and nonholonomic attitude reorientation, i.e., nonholonomic reorienting transformation (NRT). This problem can be interpreted as the problem of searching for a trajectory in joint angle space joining the initial point and the target point, such that the final attitude reaches the target. In particular, this chapter constructs a method to use consecutive line segments to connect the two points in joint angle space with the rectilinear solution. We refer to the method as *rectilinear transformation planning* in this research. In this motion planning, we first give an initial solution of the path in joint angle space, and then modify the path so that the target attitude is achieved. Using line segments for the trajectory reduces the problem to distribution of intermediate waypoints of the trajectory. The rectilinear transformation planning constructed in this chapter has the following characteristics:

- The method can handle a complex attitude motion with succession of simple control inputs.
- The method can handle arbitrary number of waypoints, which enhances performance of singularity avoidance.
- Violation of joint angle limits can be easily detected only by checking terminal conditions.
- The control law provides a long-range trajectory in feedforward form.

Chapter 5

Rectilinear Invariant Manifold Method

In this chapter, we derive a two-stage control using an invariant manifold which achieves simultaneous body reconfiguration and attitude reorientation. There have been various previous studies on discontinuous control in nonholonomic systems. In particular, a number of control methods have been proposed for canonical forms such as the chained system and power system described in Section 2.2.2. For example, Khennouf proposed a method for constructing an invariant manifold for $n = 3$ chained system [23]. Luo extended the Khennouf's method that is effective for n-dimensional power system [24]. Ikeda applied variable constraint control to n-dimensional $(n-1)$-input system such as planar free-flying space robot [62]. The advantage of the Ikeda's method is that the system does not need to be transformed into a chained system, thus avoiding singularities due to the transformation of equations. Another example of discontinuous control in nonholonomic systems is sliding mode control. For example, sliding mode control is applied to trajectory tracking of wheeled mobile vehicle and stabilization of nonholonomic systems [63, 64, 65, 66]. Sliding mode control is equivalent to generating an artificial manifold by providing high-frequency control input, but it potentially has the chattering problem.

In this section, we focus on an invariant manifold generated by the rectilinear solution derived. If a trajectory in joint angle space is restricted in the rectilinear form, motion of its state is constrained on 1-dimensional manifold. We refer to this manifold as a *rectilinear invariant manifold*, and propose a two-stage control method using it: asymptotic approach to the manifold and control constrained on the manifold. The characteristics of this method are as follows:

- In the asymptotic control to the invariant manifold, the control law is described in feedback form. Therefore, robustness against several types of disturbance can be expected.
- Unlike most manifold-based methods for chained/power systems, the proposed control law is valid for a general free-flying robot, that is, it can control a general n-dimensional m-input non-canonical system.
- The control law can be simply formulated since it is based on the rectilinear solution.

In particular, the first characteristic is a major difference from the rectilinear transformation planning, which only provides a feedforward control law. The robustness is a great advantage of the invariant manifold method.

Chapter 6

Singularity Analysis

This chapter mainly discusses controllability of the system and singularity of the proposed two control method: *rectilinear transformation planning* and *rectilinear invariant manifold method*. Similar singularity analysis is often discussed in other fields of control. One common research subject is control moment gyro (CMG) of spacecrafts. CMG is an internal-force attitude control device which exchanges angular momentum with a spacecraft. In particular, single-gimbal CMG has significant advantages of mechanical simplicity and high torque amplification, but has intrinsic singularity issue. Margulies had provided general framework of mathematical formulation and singularity analysis for single-gimbal CMG [67]. In particular, intensive investigations are performed on a pyramid-type CMG composed of 4 single-gimbal CMG distributed in a pyramid configuration [68, 69, 70, 71, 72, 73, 74, 75]. These works reported that the singular states are classified into two types, saturation singularity and internal singularity. In addition, the internal singularity is further classified into *elliptic singularity* and *hyperbolic singularity*. The hyperbolic singularity is possible to be escaped; the singular CMG configuration can be reconfigured into non-singular configuration by null motion. The elliptic singularity is the most difficult singularity because the null motion is trapped around the singular state, and thus impossible to be escaped by null motion. Another field in which the singularity is intensively discussed is control of robotic manipulators [76, 77, 78, 79, 80, 81, 82]. A lot of researchers investigated the singularity of robotic manipulators. Some control methods are shared with the CMG control.

The control methods proposed in Chapter 4 and 5 have common kinematic structures with the control of CMG and robotic manipulators. The analogies among those four systems are shown in Figure 6.1. In this figure, control inputs of the system are colored in blue, intermediate trajectories are colored in green, and target variables expressed by the sum of the green-colored trajectories are colored in magenta. The kinematic differential equation in these systems are commonly described as (derivatives of target variables) = (Jacobian) $\times$ (control inputs), and the singularity of the system is evaluated by the Jacobian; the system is singular if rank of the Jacobian becomes less than dimension of the target variables.

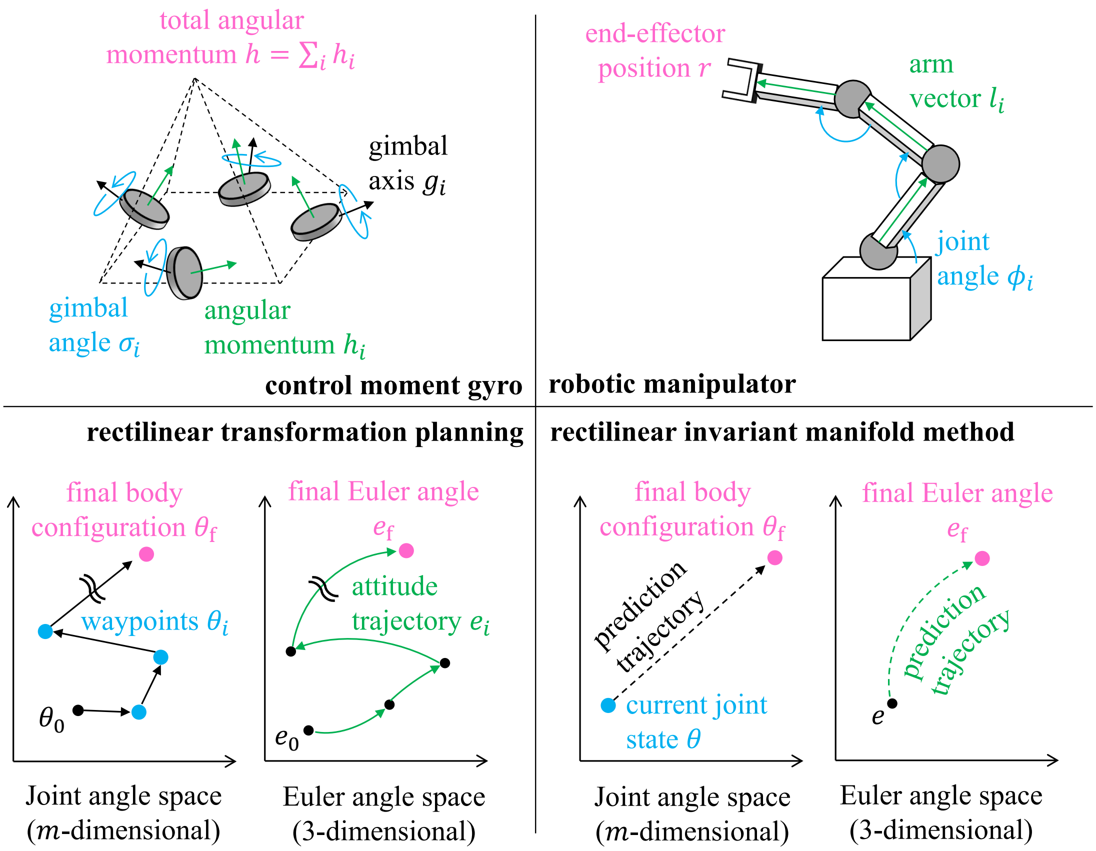

Fig. 6.1: Analogies among CMG, manipulator, and two proposed methods

Chapter 7

Application to Solar Sailing

This section provides an example of applications of the proposed NRT methods. NRT achieves arbitrary body reconfiguration and attitude reorientation, and thus a free-flying robot can arbitrarily reconfigure the geometry of all body components with respect to inertial frame as far as the order of body connection is fixed. This generic interpretation opens up many possibilities of applications. Figure 7.1 shows possible applications of the proposed methods and corresponding missions planned by JAXA Transformer [6]. Each category in the applications is described as follows:

- *Reorienting transformation*: Changing entire structure of a robot while enabling arbitrary reorientation of the terminal attitude. Robots can be more *adaptive* by mutating into multiple different space robots.
- *Transforming flight control*: Controlling external force exerted on a robot by changing its structure. The external force includes solar radiation pressure (SRP), air drag, magnetic torque, gravity gradient torque etc. Active use of external force contributes to saving fuel consumption, and hence robots can be more *sustainable*.
- *Multiple manipulation*: Manipulating multiple instruments using redundant degree of freedom of joint angles. Attitude requirements imposed on instruments such as cameras, antennas, and end-effectors are simultaneously satisfied. Robots can be more *versatile* and *dexterous* by operating multiple instruments simultaneously.

The rectilinear transformation planning and the rectilinear invariant manifold method both plays an essential role for the above all functions. In addition, these methods are formulated in generic form and hence widely applicable to a variety of free-flying robots.

In particular, this chapter provides an example of solar sailing applications. Solar sailing is an orbital maneuver which leverages solar radiation pressure (SRP) to propel a spacecraft. The concept had been proposed for a long time [83, 84], and recently several solar sail missions have been successfully accomplished. IKAROS (Interplanetary Kite-craft Accelerated by Radiation Of the Sun) by JAXA first successfully accomplished all nominal missions in 2010, in which acceleration by solar radiation pressure is confirmed on orbit [85]. A CubeSat solar sail, LightSail 2 by the Planetary Society was successfully launched in 2019 and reoriented its attitude with respect to the sun, finally confirming desired photonic acceleration [86].

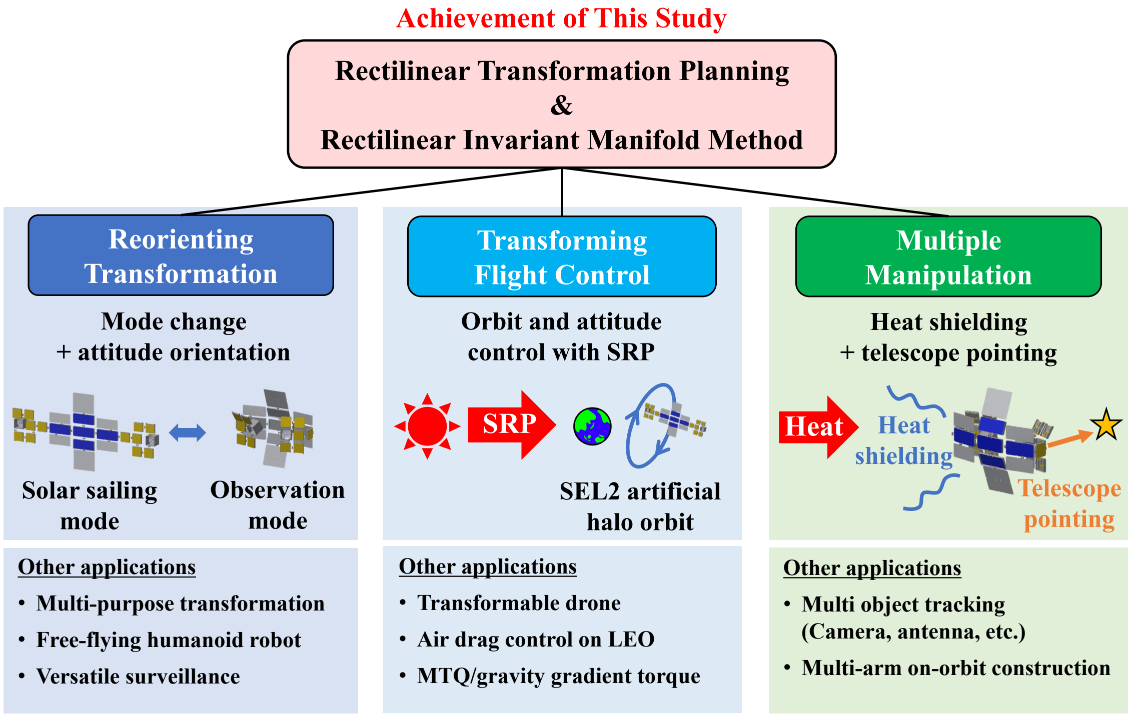

Fig. 7.1: Possible applications of the proposed motion planning methods and corresponding missions of JAXA Transformer

Theoretically, solar sailing does not require any expendable propellant as long as the attitude against the sun is properly controlled. However, it even consumes propellant to control attitude with respect to the sun. If a solar sail can perform NRT, the problem is expected to be overcome. Figure 7.2 shows dynamics relationship among SRP, body configuration, attitude motion and orbital motion. Supposing that a free-flying robot can control its body configuration and attitude, it can arbitrarily change the equilibrium attitude under SRP and simultaneously can be reoriented to the equilibrium attitude. Moreover, if the attitude is reoriented so that SRP propels the robot in a desired direction, the robot can also control orbital motion by photonic acceleration. The entire maneuver does not consume any propellant including attitude control, which can greatly enhance the ability of solar sailing. The concept of this *transformable solar sail* has been proposed by JAXA Transformer working group [6, 7].

The contents in this chapter are largely based on the master thesis by the author [87].

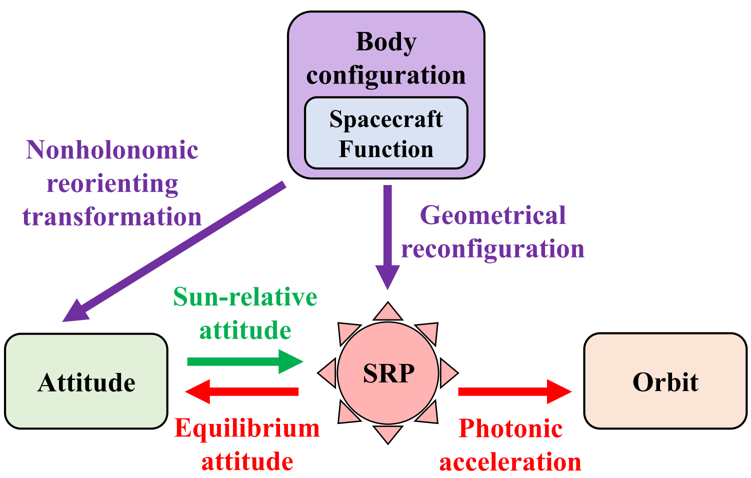

Fig. 7.2: Dynamics relationship of SRP, body configuration, attitude motion and orbital motion

Chapter 8

Conclusions

This thesis has addressed simultaneous body reconfiguration and nonholonomic attitude reorientation of a free-flying space robot, which is referred to as nonholonomic reorienting transformation (NRT) by the author. In Chapter 2, some preliminaries are formulated and the author has pointed out that the attitude motion of free-flying space robots exhibit two distinct types of nonholonomy: kinematic nonholonomy and momentum nonholonomy. In Chapter 3, the approximate analytical solution was derived for kinematics equation of a direction cosine matrix. The Lie-group structure preserved in the kinematics equation helped to solve the equation with the Magnus expansion. Among the integrable solutions, the rectilinear solution was shown to be most useful tool for NRT in our interest, and Chapter 4 and 5 used the solution to construct two NRT methods: the rectilinear transformation planning and the rectilinear invariant manifold method. The rectilinear transformation planning provides consecutive rectilinear joint actuations to achieve the target body configuration and attitude simultaneously. The rectilinear invariant manifold method provides two-stage control law with feedback of the current state. The characteristics and effectiveness of each methods were revealed through some numerical examples. The final chapter provided one example of practical applications: solar sailing using NRT. It demonstrated that completely propellant-free station-keeping using solar radiation pressure owing to the proposed NRT methods. Remarkable contributions of this study are:

1. Unlike most related studies on nonholonomic motion planning, the proposed method can handle general three-dimensional attitude motion with arbitrary number of body components.
2. The newly derived analytical solution enables to handle large-degree-of-freedom system, which was impossible with a brute-force numerical search by previous research.
3. Simultaneous body reconfiguration and attitude reorientation indicates that geometries of all body components can be arbitrarily reconfigured. It can greatly enhance performance of free-flying space robots in terms of adaptability, versatility, dexterity, and sustainability.

Bibliography

[1] Xue Zhihui et al. “Review of in-space assembly technologies”. In: *Chinese Journal of Aeronautics* 34.11 (2021), pp. 21–47.

[2] Michael A Shoemaker et al. “OSAM-1 DECOMMISSIONING ORBIT DESIGN”. In: *Proceedings of AAS/AIAA Astrodynamics Specialist Conference*. AAS 20-460. Lake Tahoe, USA, Aug. Aug. 2020.

[3] Jennifer S Hudson and Daniel Kolosa. “Versatile on-orbit servicing mission design in geosynchronous earth orbit”. In: *Journal of Spacecraft and Rockets* 57.4 (2020), pp. 844–850. DOI: `10.2514/1.A34701`.

[4] Robin Biesbroek et al. “the Clearspace-1 Mission: Esa and Clearspace Team Up To Remove Debris”. In: *Proc. 8th Eur. Conf. Sp. Debris*. 2021, pp. 1–3.

[5] Northrop Grumman. *Public-private partnership is pioneering robotic servicing of satellites*. 2020. URL: `https://news.northropgrumman.com/news/releases/northrop-grummans-wholly-owned-subsidiary-spacelogistics-selected-by-darpa-as-commercial-partner-for-robotic-servicing-mission` (visited on 11/30/2021).

[6] Yoshiki Sugawara et al. “Transformable spacecraft: Feasibility study and conceptual design”. In: *Proceedings of the International Astronautical Congress*. IAC-20-D1.2.8.x59887. Virtual, Oct. 2020.

[7] Yuki Kubo and Junichiro Kawaguchi. “Propellant Free Station Keeping around Sun Earth L2 Using Solar Radiation Pressure for a Transformable Spacecraft”. In: *Proceedings of International Symposium on Space Technology and Science*. 2019-d-116s. Fukui, Japan, June 2019.

[8] Yuki Kubo et al. “Current Status on Nonholonomic Attitude Maneuver for Transformer Project”. In: *Proceedings of 21st Space Science Symposium*. g17-1. Sagamihara, Japan, Jan. 2021.

[9] Toshiaki Iwata. “AMBAC [Active Mass Balance Auto Control (System)] - Attitude Control Function Using Limbs Motion (in Japanese)”. In: *Journal of the Society of Instrument and Control Engineers* 43.1 (2004), pp. 8–9.

[10] Katsuya Kanaoka. “Nevertheless, that super robot can move”. In: *Journal of the Robotics Society of Japan* 29.2 (2011), pp. 140–143.

[11] Katsuya Kanaoka. “How does the super robot move (in Japanese)”. In: The Nikkan Kogyo Shimbun, 2010. Chap. 2, 3.

[12] Thomas R Kane and David A Levinson. "The use of Kane's dynamical equations in robotics". In: *The International Journal of Robotics Research* 2.3 (1983), pp. 3–21. DOI: 10.1177/027836498300200301.

[13] Thomas R Kane and David A Levinson. *Dynamics, theory and applications.* McGraw Hill, 1985.

[14] Evangelos G Papadopoulos. "Nonholonomic behavior in free-floating space manipulators and its utilization". In: *Nonholonomic Motion Planning.* Springer, 1993, pp. 423–445. DOI: 10.1007/978-1-4615-3176-0_11.

[15] Yoshihiko Nakamura and Ranjan Mukherjee. "Exploiting nonholonomic redundancy of free-flying space robots". In: *IEEE Transactions on Robotics and Automation* 9.4 (1993), pp. 499–506. DOI: 10.1109/70.246062.

[16] Roger W Brockett et al. "Asymptotic stability and feedback stabilization". In: *Differential geometric control theory* 27.1 (1983), pp. 181–191.

[17] A. M. Bloch. *Nonholonomic Mechanics and Control.* New York: Springer, 2003.

[18] Sergei Alekseevich Chaplygin. "On the motion of a heavy body of revolutionon a horizontal plane (Russian)". In: *Physiscs Section of the Imperial Society of Friends of Physics, Anthropology and Ethnographics* 9 (1897), pp. 10–16.

[19] Sergei Alekseevich Chaplygin. "On the theory of motion of nonholonomic systems. The reducing-multiplier theorem". In: *Regular and Chaotic Dynamics* 13.4 (2008), pp. 369–376.

[20] A.V. Borisov and I.S. Mamayev. "The dynamics of a Chaplygin sleigh". In: *Journal of Applied Mathematics and Mechanics* 73.2 (2009), pp. 156–161. ISSN: 0021-8928. DOI: https://doi.org/10.1016/j.jappmathmech.2009.04.005. URL: https://www.sciencedirect.com/science/article/pii/S0021892809000653.

[21] James Reeds and Lawrence Shepp. "Optimal paths for a car that goes both forwards and backwards". In: *Pacific journal of mathematics* 145.2 (1990), pp. 367–393.

[22] Ryszard Andrzejewski and Jan Awrejcewicz. *Nonlinear dynamics of a wheeled vehicle.* Vol. 10. Springer Science & Business Media, 2006.

[23] Hayate Khennouf and C Canudas De Wit. "On the construction of stabilizing discontinuous controllers for nonholonomic systems". In: *IFAC Proceedings Volumes* 28.14 (1995), pp. 667–672.

[24] Jihao Luo and Panagiotis Tsiotras. "Exponentially convergent control laws for nonholonomic systems in power form". In: *Systems & control letters* 35.2 (1998), pp. 87–95.

[25] Richard M Murray and Sosale Shankara Sastry. "Nonholonomic motion planning: Steering using sinusoids". In: *IEEE transactions on Automatic Control* 38.5 (1993), pp. 700–716.

[26] Henry R Muller and Lewis H Weed. "Notes on the falling reflex of cats". In: *American Journal of Physiology-Legacy Content* 40.3 (1916), pp. 373–379. DOI: 10.1152/ajplegacy.1916.40.3.373.

[27] TR Kane and MP Scher. "A dynamical explanation of the falling cat phenomenon". In: *International journal of solids and structures* 5.7 (1969), pp. 663–670. DOI: 10.1016/0020-7683(69)90086-9.

[28] Kazuo Yamafuji, Tsuyoshi Kobayashi, and Takashi Kawamura. "Elucidation of twisting motion of a falling cat and its realization by a robot". In: *Journal of the Robotics Society of Japan* 10.5 (1992), pp. 648–654. DOI: 10.7210/jrsj.10.648.

[29] Masaki Yamakita and Yasuhito Omagari. "Motion control of cat-like robot (in Japanese)". In: *Journal of the Society of Instrument and Control Engineers* 42.9 (2003), pp. 735–741.

[30] Jun'ichiro Kawaguchi, Osamu Mori, and Koki Minamikawa. "Non-holonomic turn and its application to spacecraft attitude maneuvers". In: *Proceedings of 56th International Astronautical Congress*. IAC-05-C1.2.08. Fukuoka, Japan, Oct. 2005.

[31] Lu Li, Jiaxuan Zhao, and Yinfeng Xia. "Landing posture adjustment and buffer performance analysis of a cat robot". In: *2018 2nd IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC)*. IEEE. 2018, pp. 357–363. DOI: 10.1109/IMCEC.2018.8469620.

[32] Xavier Garant and Clement Gosselin. "Design and Experimental Validation of Reorientation Manoeuvres for a Free Falling Robot Inspired From the Cat Righting Reflex". In: *IEEE Transactions on Robotics* (2020). DOI: 10.1109/TRO.2020.3031241.

[33] Nikita Rudin et al. "Cat-Like Jumping and Landing of Legged Robots in Low Gravity Using Deep Reinforcement Learning". In: *IEEE Transactions on Robotics* (2021), pp. 1–12. DOI: 10.1109/TRO.2021.3084374.

[34] Etienne-Jules Marey. *Falling Cat*. 1894. URL: https://en.wikipedia.org/wiki/Falling_Cat (visited on 11/30/2021).

[35] David L Akin et al. "Space applications of automation, robotics and machine intelligence systems (ARAMIS) phase II". In: *NASA Contract Report* 3734 (1983).

[36] M Bronez, M Clarke, and Alberta Quinn. "Requirements development for a free-flying robot–The" Robin"". In: *Proceedings. 1986 IEEE International Conference on Robotics and Automation*. Vol. 3. IEEE. Feb. 1987, pp. 667–672. DOI: 10.1109/ROBOT.1986.1087628.

[37] Miguel A Torres and Steven Dubowsky. "Minimizing spacecraft attitude disturbances in space manipulator systems". In: *Journal of guidance, control, and dynamics* 15.4 (1992), pp. 1010–1017. DOI: 10.2514/3.20936.

[38] Andrea Antonello, Alfredo Valverde, and Panagiotis Tsiotras. "Dynamics and control of spacecraft manipulators with thrusters and momentum exchange devices". In: *Journal of Guidance, Control, and Dynamics* 42.1 (2019), pp. 15–29. DOI: 10.2514/1.G003601.

[39] Lingling Shi, Jayantha Katupitiya, and Nathan Kinkaid. "A robust attitude controller for a spacecraft equipped with a robotic manipulator". In: *2016 American Control Conference (ACC)*. IEEE. July 2016, pp. 4966–4971. DOI: 10.1109/ACC.2016.7526140.

[40] Ranjan Mukherjee and Masayuki Kamon. "Almost smooth time-invariant control of planar space multibody systems". In: *IEEE Transactions on Robotics and Automation* 15.2 (1999), pp. 268–280. DOI: 10.1109/70.760348.

[41] Shinji Hokamoto and Toshirou Funasako. "Feedback control of a planar space robot using a moving manifold". In: *Journal of the Robotics Society of Japan* 25.5 (2007), pp. 745–751. DOI: 10.7210/jrsj.25.745.

[42] Hirohisa Kojima and Shinji Kasahara. "An adaptive invariant manifold-based switching control for a planar two-link space robot". In: *Transactions of the Japan Society for Aeronautical and Space Sciences* 54.184 (2011), pp. 144–152. DOI: 10.2322/jjsass.58.233.

[43] Ka Wai Lee, Hirohisa Kojima, and Pavel M Trivailo. "Applications of Optimal Trajectory Planning and Invariant Manifold Based Control for Robotic Systems in Space". In: *Advances in Spacecraft Technologies* (2011), p. 497. DOI: 10.5772/13455.

[44] Federico Trovarelli et al. "Attitude Control of an Underactuated Planar Multibody System Using Momentum Preserving Internal Torques". In: *AIAA Scitech 2020 Forum*. Jan. 2021, p. 1686. DOI: 10.2514/6.2020-1686.

[45] Sungpil Kim and Youdan Kim. "Spin-axis stabilization of a rigid spacecraft using two reaction wheels". In: *Journal of Guidance, Control, and Dynamics* 24.5 (2001), pp. 1046–1049.

[46] Takateru Urakubo, Kazuo Tsuchiya, and Katsuyoshi Tsujita. "Attitude control of a spacecraft with two reaction wheels". In: *Journal of Vibration and Control* 10.9 (2004), pp. 1291–1311.

[47] Shingo Shimoda, Takashi Kubota, and Ichiro Nakatani. "Attitude control of satellite with two wheels considering maneuver path (in Japanese)". In: *Transactions of the Society of Instrument and Control Engineers* 41.10 (2005), pp. 813–820.

[48] Katsuhiko Yamada and Shoji Yoshikawa. "Feedback control of space robot attitude by cyclic arm motion". In: *Journal of guidance, control, and dynamics* 20.4 (1997), pp. 715–720. DOI: 10.2514/2.4102.

[49] Takahiro Suzuki and Yoshihiko Nakamura. "Planning spiral motion of nonholonomic space robots". In: *Proceedings of IEEE International Conference on Robotics and Automation*. Vol. 1. IEEE. 1996, pp. 718–725.

[50] William Todd Cerven and Victoria L Coverstone. "Optimal reorientation of a multibody spacecraft through joint motion using averaging theory". In: *Journal of Guidance, Control, and Dynamics* 24.4 (2001), pp. 788–795. DOI: 10.2514/2.4779.

[51] Kaoru Ohashi, Toshihiro Chujo, and Junichiro Kawaguchi. “Optimal Motion Planning in Attitude Maneuver Using Non Holonomic Turns for a Transformable Spacecraft”. In: *Proceedings of AAS/AIAA Astrodynamics Specialist Conference*. AAS-18-359. Snowbird, USA, Aug. 2018.

[52] Kaoru Ohashi. “Research on Attitude Maneuver Using Nonholonomic Turn of Transformable Spacecraft (in Japanese)”. MA thesis. Tokyo, Japan: The University of Tokyo, 2019.

[53] Zexiang Li and John Canny. “Motion of two rigid bodies with rolling constraint”. In: *IEEE Transactions on Robotics and Automation* 6.1 (1990), pp. 62–72.

[54] Tsutomu Mita. *Introduction to Nonlinear Control -Skill Control of Underactuated Robots- (Japanese)*. Syokodo Pub. Co., 2000.

[55] Henk Nijmeijer and Arjan J Van der Schaft. *Nonlinear dynamical control systems*. Vol. 175. Springer, 1990.

[56] Richard M Murray, Zexiang Li, and S Shankar Sastry. *A mathematical introduction to robotic manipulation*. CRC press, 1994.

[57] Ilya Kolmanovsky and N Harris McClamroch. “Developments in nonholonomic control problems”. In: *IEEE Control systems magazine* 15.6 (1995), pp. 20–36.

[58] Zexiang Li and John F Canny. *Nonholonomic motion planning*. Vol. 192. Springer Science & Business Media, 2012.

[59] Richard M Murray. “Nilpotent bases for a class of nonintegrable distributions with applications to trajectory generation for nonholonomic systems”. In: *Mathematics of Control, Signals and Systems* 7.1 (1994), pp. 58–75.

[60] Katsuhiko Yamada. “Trajectory generation in nonholonomic system (in Japanese)”. In: *Journal of the Society of Instrument and Control Engineers* 36.6 (1997), pp. 390–395.

[61] L.E. Goodman. “Effects of Finite Rotation on Gyroscopic Devices”. In: *Journal of Applied Mechanics* 210 (1958).

[62] Takayuki Ikeda et al. “Variable constraint control of underactuated free flying robots-mechanical design and convergence”. In: *Proceedings of the 38th IEEE Conference on Decision and Control (Cat. No. 99CH36304)*. Vol. 3. IEEE. 1999, pp. 2539–2544.

[63] Jong-Min Yang and Jong-Hwan Kim. “Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots”. In: *IEEE Transactions on robotics and automation* 15.3 (1999), pp. 578–587.

[64] Dongkyoung Chwa. “Sliding-mode tracking control of nonholonomic wheeled mobile robots in polar coordinates”. In: *IEEE transactions on control systems technology* 12.4 (2004), pp. 637–644.

[65] Anthony Bloch and Sergey Drakunov. “Stabilization of a nonholonomic system via sliding modes”. In: *Proceedings of 1994 33rd IEEE Conference on Decision and Control*. Vol. 3. IEEE. 1994, pp. 2961–2963.

[66] Thierry Floquet, Jean-Pierre Barbot, and Wilfrid Perruquetti. "Higher-order sliding mode stabilization for a class of nonholonomic perturbed systems". In: *Automatica* 39.6 (2003), pp. 1077–1083.

[67] G Margulies and Aubrun J.N. "Geometric theory of single-gimbal control moment gyro system". In: *J. Astronaut. Sci.* 26 (1978), pp. 159–191.

[68] Nazareth S Bedrossian et al. "Redundant single gimbal control moment gyroscope singularity analysis". In: *Journal of Guidance, Control, and Dynamics* 13.6 (1990), pp. 1096–1101.

[69] Bong Wie. "Singularity analysis and visualization for single-gimbal control moment gyro systems". In: *Journal of Guidance, Control, and Dynamics* 27.2 (2004), pp. 271–282.

[70] Bong Wie. "Singularity escape/avoidance steering logic for control moment gyro systems". In: *Journal of guidance, control, and dynamics* 28.5 (2005), pp. 948–956.

[71] H Kurokawa, N Yajima, and S Usui. "A new steering law of a single-gimbal cmg system of pyramid configuration". In: *IFAC Proceedings Volumes* 18.4 (1985), pp. 251–257.

[72] Nazareth S Bedrossian et al. "Steering law design for redundant single-gimbal control moment gyroscopes". In: *Journal of Guidance, Control, and Dynamics* 13.6 (1990), pp. 1083–1089.

[73] Haruhisa Kurokawa. "Constrained steering law of pyramid-type control moment gyros and ground tests". In: *Journal of Guidance, Control, and Dynamics* 20.3 (1997), pp. 445–449.

[74] Kohei Takada, Hirohisa Kojima, and Naoki Matsuda. "Control moment gyro singularity-avoidance steering control based on singular-surface cost function". In: *Journal of guidance, control, and dynamics* 33.5 (2010), pp. 1442–1450.

[75] Katsuhiko Yamada, Takashi Asai, and Ichiro Jikuya. "Inverse kinematics in pyramid-type single-gimbal control moment gyro system". In: *Journal of Guidance, Control, and Dynamics* 39.8 (2016), pp. 1893–1903.

[76] K Sugimoto, J Duffy, and KH Hunt. "Special configurations of spatial mechanisms and robot arms". In: *Mechanism and Machine Theory* 17.2 (1982), pp. 119–132.

[77] John Baillieul, John Hollerbach, and Roger Brockett. "Programming and control of kinematically redundant manipulators". In: *The 23rd IEEE Conference on Decision and Control*. IEEE. 1984, pp. 768–774.

[78] KJ Waldron, Shih-Liang Wang, and SJ Bolin. "A study of the Jacobian matrix of serial manipulators". In: (1985).

[79] Tsuneo Yoshikawa. "Manipulability of robotic mechanisms". In: *The international journal of Robotics Research* 4.2 (1985), pp. 3–9.

[80] Nazareth S Bedrossian. "Classification of singular configurations for redundant manipulators". In: *Proceedings., IEEE International Conference on Robotics and Automation*. IEEE. 1990, pp. 818–823.

[81] Stefano Chiaverini. “Singularity-robust task-priority redundancy resolution for real-time kinematic control of robot manipulators”. In: *IEEE Transactions on Robotics and Automation* 13.3 (1997), pp. 398–410.

[82] Giacomo Marani et al. “A real-time approach for singularity avoidance in resolved motion rate control of robotic manipulators”. In: *Proceedings 2002 IEEE International Conference on Robotics and Automation (Cat. No. 02CH37292)*. Vol. 2. IEEE. 2002, pp. 1973–1978.

[83] C Sauer Jr. “Optimum solar-sail interplanetary trajectories”. In: *Astrodynamics Conference*. 1976, p. 792.

[84] M Leipold and O Wagner. “’Solar Photonic Assist’Trajectory Design for Solar Sail Missions to the Outer Solar System and Beyond”. In: *Spaceflight Dynamics 1998* 100 (1998).

[85] Yuichi Tsuda et al. “Flight status of IKAROS deep space solar sail demonstrator”. In: *Acta astronautica* 69.9-10 (2011), pp. 833–840.

[86] David A Spencer et al. “The LightSail 2 solar sailing technology demonstration”. In: *Advances in Space Research* 67.9 (2021), pp. 2878–2889.

[87] Yuki Kubo. “Station-Keeping Method of a Small Circular Halo Orbit Using Solar Radiation Pressure for a Transformable Spacecraft around Sun-Earth L2 Point”. MA thesis. Tokyo, Japan: The University of Tokyo, 2019.