

Mu'ayyad al-Dīn al-'Urḏī's Impact on Quṭb al-Dīn al-Shīrāzī concerning Planetary Order

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1. Introduction

Mu'ayyad Dīn 'Urḏī (ca. 1200-ca. 1266) is known to have been summoned in the 1250s by Naṣīr al-Dīn al-Ṭūsī (1201-1274) from Damascus to Marāgha and have cooperated with scholars at the Marāgha observatory.¹ Before coming to Marāgha, he wrote in Damascus *Kitāb al-hay'a*, where he criticized Ptolemy's works and proposed new planetary models. Since he was working at the Marāgha observatory, it is reasonable to think that his reformation of Ptolemaic planetary models presented in his *Kitāb al-hay'a* had some influence on the Marāgha scholars. Saliba shows that Quṭb Dīn Shīrāzī (1236-1311) adopted the main part of 'Urḏī's new planetary models in his hay'a works, though he did not denote the name of 'Urḏī.² Scholars at the Marāgha observatory except for Shīrāzī, however, did not remark 'Urḏī's new models, so it is difficult for us to notice their scholarly interaction with 'Urḏī and show how they accepted or rejected 'Urḏī's innovation.

Here, we must remark that 'Urḏī reformed, not only Ptolemaic planetary models, but also Ptolemy's planetary order in the *Kitāb al-hay'a*, where he recalculated the distances of the planets from the Earth and concluded that Venus should be above the Sun.³ As I will elucidate later, Shīrāzī concluded in his hay'a works that the planetary order offered by 'Urḏī was the correct one, although he did not mention the name of 'Urḏī. Shīrāzī's acceptance of 'Urḏī's new planetary order reminds us of a possibility that how his new planetary order caused discussions among them might reveal his actual impact on them. In this article, I will detect the Marāgha associates' opinions

concerning the planetary order from their works, and show how they responded ‘Urđī’s impact on the reformation of Ptolemaic astronomy, and finally, I will elucidate Shīrāzī’s way of incorporating ‘Urđī’s achievement into the hay’a tradition standardized by Ṭūsī’s *Tadhkira fī ‘ilm al-hay’*.

2. Abharī’s astronomical works

Tehran, Majlis shura Library MS 6195 (= MS B) contains an astronomical work entitled *Book of the Summary and Abridgment of the Almagest* (*Kitāb talkhīṣ al-majisī wa-tahdhīb-hu*; henceforth the *Talkhīṣ*) in 155 folios. As far as we know, the *Talkhīṣ* is transmitted at least by two other manuscripts, that is, Tehran, Milli Library MS 20371 and Istanbul, Ayasofya MS 2583m.⁴ Unfortunately, the information of who was the author is missing in all the three manuscripts, but, as María José Parra Pérez rightly identifies,⁵ its author was almost undoubtedly Athīr al-Dīn al-Abharī (d. 1262 or 1265).

Abharī was especially known as an author of philosophical works, but he wrote quite a few works on mathematics and astronomy,⁶ and his name was registered in the list of the Marāgha associates.⁷ He was also known as a famous teacher of philosophy, having many disciples, one of whom was Najm al-Dīn ‘Alī ibn ‘Umar al-Qazwīnī al-Kātibī (d. 1277), the author of *Ḥikmat al-‘ayn*,⁸ whose name was also contained in the list of the Marāgha associate.⁹ We must note that the name of Kātibī was mentioned in the introduction of the *Talkhīṣ*, whose beginning is as follows:¹⁰

[Q1] Since I have been associated with the most notable and closest one of my companions with my kindness towards him for a while, that is, ‘Alī ibn ‘Umar ibn ‘Alī al-Qazwīnī –may God give him success because of the fortunes–, who is like a child of mine, studying exact sciences and philosophical canons, and he has prepared for leading his study to the investigation of the art of the *Almagest*, then I want to write for him a summary of books by the

Ancients covering the secrets of it [= the art of the *Almagest*] in more plain description and arrangement. Thus, I composed this book containing parts of this art as well as its preparatory topics necessary for it besides the principles of the *Element* (MS B, f. 2b).

[Q1] فلما صحبني مدة من الزمان أعزّ رفقائي عليّ وأولاهم بالإحسان إليّ ولديّ وهو علي بن عمر بن علي القزويني وفقه الله للخيرات الذي هو بمثابة ولد منّي باحثاً عن العلوم الحقيقية والقوانين الحكيمة وأدى منتهى بحثه إلى النظر في صناعة المجسطي، فأردت أن أعمل له ملخصاً من كتب الأقدمين يحتوي على أسرارها بأوضح بيان وترتيب. فجمعت هذا الكتاب مشتملاً على أجزاء هذه الصناعة ومقدماتها المحتاج إليها بعد ما في الأسطوسات من المبادئ.

This introduction shows that Abharī aims to summarize the *Almagest* for Kātibī.

Here, we must note that in Book 10, Section 1 “How to divide the heaven into orbs”, he explains the planetary order with recourse to occultations and parallaxes, as follows:

[Q2] Since the Moon occults Mercury and Mercury occults Venus, it is known that Mercury's orb is above the Moon's orb and under Venus's orb. And it is found that the Sun's parallax is smaller than Venus' parallax, while it is not found that Mars has any parallax, so it is known that the Sun's orb is above Venus's orb and under Mars' orb. And it is found that Mars occults Jupiter, and Jupiter occults Saturn, which occults the fixed stars, so it is known that Jupiter's orb is above Mars' orb and under Saturn's orb and Saturn's orb is under the orb of the fixed stars (MS B, f. 114a).¹¹

[Q2] ولما كان القمر يكسف عطارد وعطارد يكسف الزهرة فعلم أنّ فلك عطارد فوق فلك القمر وتحت فلك الزهرة. وقد وجد اختلاف منظر الشمس أقلّ من اختلاف منظر الزهرة ولم يوجد للمريخ اختلاف المنظر، فعلم أنّ فلك الشمس فوق فلك الزهرة وتحت فلك المريخ. ووجد المريخ يكسف المشتري والمشتري يكسف زحل وهو يكسف الثوابت، فعلم أنّ فلك المشتري فوق فلك المريخ وتحت فلك زحل وفلك زحل تحت فلك الثوابت.

¹¹ فعلم [علم: ب. 2 فعلم] علم: ب.

Besides the *Talkhīṣ*, we find another astronomical work composed by Abharī containing a discussion of the planetary order extant in Istanbul, Carullah MS 1499 (= MS C), ff. 11b-81a without title and introduction. Its front page (f. 11a) has a note added by the copyist as a kind of its title: “that is followed by what the master Athīr al-Dīn al-Mufaḍḍal ibn ‘Umar al-Abharī summarizes (ikhtaṣara) concerning ‘ilm al-hay’a from Kūshyār’s hay’a and Ibn Aflaḥ al-Ishbīlī’s hay’a” (henceforth, I will call this work as the *Ikhtaṣara*), suggesting that this work is on ‘ilm al-hay’a with recourse to Kūshyār ibn Labbān’s (fl. the early eleventh-century C.E) *zīj* as well as Jābir ibn Aflaḥ’s (fl. The twelfth-century C.E) *Iṣlāḥ al-majisī*. Remarkably, the *Ikhtaṣara* ends with the explanation about the planetary order, as follows:

[Q3]

Since the Moon occults the rest of the planets, it is known that its orb is under all orbs. Likewise, Mercury occults Venus, and Venus occults Mars, and Mars occults Jupiter, and Jupiter occults Saturn, and Saturn occults the fixed stars, so their orbs are places according to this order. As for the Sun, it is known that its orb is above the Moon’s orb; however, its place in relation to the rest of the orbs is not certain (MS C, f. 88a).¹²

[Q3] لَمَّا كَانَ الْقَمَرُ يَكْسِفُ مَا سِوَاهُ، عَلِمَ أَنَّ فَلَكَهُ تَحْتَ جَمِيعِ الْأَفْلَاقِ. وَكَذَلِكَ عَطَارِدُ يَكْسِفُ الزُّهْرَةَ وَالزُّهْرَةُ تَكْسِفُ الْمَرْيَخَ وَالْمَرْيَخُ يَكْسِفُ الْمَشْتَرِيَّ وَالْمَشْتَرِيُّ يَكْسِفُ زُحْلًا وَزُحْلًا يَكْسِفُ النَّوَابِتَ، فَأَفْلَاقُهَا مَوْضُوعَةٌ عَلَى هَذَا التَّرْتِيبِ. وَأَمَّا الشَّمْسُ فَقَدْ عَلِمَ أَنَّ فَلَكَهَا فَوْقَ فَلَكِ الْقَمَرِ، وَأَمَّا وَضْعُهُ بِالنِّسْبَةِ إِلَى سَائِرِ الْأَفْلَاقِ فَعِغْرٌ مَتَّفِقِينَ.
¹ هَذَا] هَذَا: ج.

Interestingly, an astronomical section of the *Ḥikmat al-‘ayn* written by Kātibī, a student of Abharī, contains a discussion about the planetary order.

3. Kātibī's attitude towards the planetary order

The *Hikmat al-'ayn* consists of two parts: Part 1 on metaphysics, and Part 2 on natural philosophy, whose Section 3 is on the orbs. In this section, Kātibī first elucidates the characteristics of the simple body, the compound body, linear motion, and circular motion, and then he presents the planetary motion. At the end of this section, he explains the planetary order, as follows:

[Q4]

Since the Moon occults Mercury, Mercury [occults] Venus, Mars [occults] Jupiter, and Jupiter [occults] Saturn, and Saturn [occults] the fixed stars, it is known that the orb of the occulting [planet] is under the orb of the occulted [planet]. Since it is found that in some conjunctions with the Sun, Venus is like a stain (*shāma*) on its [i.e., the Sun's] face, not [on the face of] Mercury, it is known that the Sun's orb is above Venus's orb and under Mars' orb. In this way, the Master said it. I notice, however, that some mathematicians deny that [order] and believe that Venus' orb is above the Sun's orb.¹³

This part illustrates that, after he presents the standard planetary order with recourse to occultations, he adds a witness by “the Master” on a Venus transit. Of course, the person whom he calls the Master is Ibn Sīnā (ca. 980-1037), who reported a Venus transit in the *Shifā'*: “I say that I saw Venus as a spot (*khāl*) or a stain (*shāma*) on the Sun's surface.”¹⁴ Kātibī's keeping the specific wording “a stain (*shāma*)” illustrates his reference to the *Shifā'*.

Even more remarkably, Kātibī ends his note about the planetary order ([Q4]) with a comment on the existence of scholars who think the order of the planets differently: “I notice, however, that some mathematicians deny that [order] and believe that Venus' orb is above the Sun's orb”. Although he does not deny this new order explicitly, he

thinks that a Venus transit is strong evidence rejecting it. His remark shows that in his days, this new order becomes an important thesis, so he should mention it. In fact, this order is the one proposed by ‘Urđī, a colleague of his in the Marāgha observatory.

4. ‘Urđī’s new thesis on the planetary order and his impact on the Marāgha associates

As mentioned beforehand, ‘Urđī declared a new planetary order in his *Kitāb al-hay’ā*, where recalculating the distances of the planets from the Earth, he concluded that Venus’ orb should be above the Sun’s orb. Thus, the fact that Kātibī mentioned this discovery even in this short section makes it clear that ‘Urđī’s argument gave a great impact to scholars affiliated with the Marāgha observatory.

On the other hand, we notice that the teacher Abharī did not mention Urđī’s new order in the *Talkhīṣ* and the *Ikhtaṣara*. In the *Talkhīṣ* ([Q2]), he explained the standard order, i.e., Moon-Mercury-Venus and Mars-Jupiter-Saturn, with recourse to occultations, and as to the order of Venus-the Sun-Mars, he proved the correctness of this order by utilizing their parallaxes (“the Sun’s parallax is smaller than Venus’ parallax, while it is not found that Mars has any parallax”). In the *Ikhtaṣara* ([Q3]), however, he only utilized occultations without mentioning parallaxes, and did not give an opinion about the Sun’s place, just noting “however, its [= the Sun’s] place in relation to the rest of the orbs is not certain”. Then, we have a question: Why did his confidence in the parallaxes disappear in the *Ikhtaṣara*? One of the possible reasons is that he recognized Jābir ibn Aflaḥ’s argument on the place of Venus and Mercury.

Jābir ibn Aflaḥ, a scholar active in 12th century Andalusia, was the author of the *Iṣlāḥ al-majisī* (henceforth, the *Iṣlāḥ*), where he pointed out several defects of the *Almagest*. His most severe attack was on the places of Venus and Mercury.¹⁵ In this critique, he complained about Ptolemy’s baseless argument about their places, so he calculated the parallaxes of Venus and Mercury, and found that both have smaller par-

allaxes than the Sun. So, he concluded that their orbs should be above the Sun's orb. Thus, if Abharī read the *Iṣlāḥ*, he should withdraw his argument about the order of Venus-the Sun-Mars based on their parallaxes.

Indeed, Andalusia was far from Marāgha, so the access to Jābir's work might be restricted to scholars active around Marāgha like Abharī. The contents of the *Talkhīs* ([Q2]) indicate that when Abharī composed it, the *Iṣlāḥ* was unavailable to him. The fact that, however, Quṭb Dīn Shīrāzī, a younger contemporary of Abharī and a disciple of Kātibī, wrote a commentary on the *Iṣlāḥ*,¹⁶ shows that it arrived among the Marāgha associates possibly at some point of Abharī's lifetime. Its abrupt circulation among them after almost one hundred years-long silence on Jābir's achievement in this region is significant. Why did they suddenly recognize the importance of the *Iṣlāḥ*? Here we can point out 'Urḍī's contact with them in Marāgha as an important factor agitating their interest in Jābir's work; for 'Urḍī's *Kitāb al-hay'a* explicitly mentioned Jābir as an important predecessor criticizing the *Almagest*, so he might advocate the importance of the *Iṣlāḥ* among them.

The impact of 'Urḍī's new thesis on the planetary order accompanied with Jābir's criticism on the standard order can be detected more explicitly in Kātibī's short description in [Q4]: while he stopped using parallaxes in explaining the reason of the order of Venus-the Sun-Mars, he provided a witness of Venus transit from Ibn Sīnā to show the place of the Venus together with a mention to 'Urḍī's thesis. Kātibī's defence of the standard order in [Q4] reflects his awareness of 'Urḍī's argument concerning the planetary order as well as Jābir's, which indicates that Kātibī acknowledged Jābir's criticism through 'Urḍī.

The above analysis illustrates that the change of Abharī's attitude towards the planetary order from the *Talkhīs* to the *Ikhtaṣara* as well as the strategy of Kātibī's defence of the standard order show the existence of vivid discussion concerning the place of the Sun among scholars around Marāgha, which occurred abruptly in their days probably at the time when 'Urḍī contacted them. Thus, their opinions concerning the planetary

order in their works sometimes give criteria determining the order of their composition dates.

For example, we are now sure that Abharī composed the *Talkhīṣ*, and after he learned other arguments about the order of Venus-Mercury-the Sun from the *Iṣlāḥ* as well as the *Kitāb al-hay'a*, he wrote the *Ikhtaṣara* in which he added a comment about the uncertainty of the Sun's place. As for the case of Kātībī, he certainly wrote the *Ḥikmat al-'ayn* after he recognized 'Urḏī's new thesis at the time of 'Urḏī's arrival to Marāgha.

Moreover, since 'Urḏī was invited to Marāgha by Ṭūsī, we can expect that 'Urḏī's new planetary order also gave some impact on Ṭūsī at some point. Thus, I will overview Ṭūsī's opinions about the planetary order in his works.

First, in the *Essential Understanding of the Configuration of the Orbs* (*Zubdat al-idrak fi hay'at al-aftak*; henceforth, the *Zubda*), Ṭūsī explained the standard order with recourse to occultations and parallaxes just as Abharī did in the *Talkhīṣ*.¹⁷ Given that Ṭūsī and Abharī learned astronomy under the same teacher Kamāl Dīn ibn Yūnus (1156-1242),¹⁸ their using parallaxes for proving the order of Venus-the Sun-Mars may be influenced by Kamāl Dīn's teaching. Next, in the *Risālah-i Mu'iniyya dar hay'a* (henceforth, the *Mu'iniyya*), Ṭūsī repeated almost the same argument as that in the *Zubda*.¹⁹

The above survey shows that 'Urḏī's argument, as well as Jābir's, did not influence the *Zubda* and the *Mu'iniyya*. But, in *al-Tadhkira fi 'ilm al-hay'a* (henceforth, the *Tadhkira*), he drastically reshaped his arguments on the planetary order.

First, in Book 2, Chapter 2 "On the Arrangement and Order of the Bodies", he explained the standard planetary order only by using occultations without mentioning parallaxes,²⁰ and in Book 4, Chapters 2-7, he calculated the distances and bodies of the planets afresh and proved the standard order.²¹ Ṭūsī's refinement of the way of explaining the standard order was most probably made for responding 'Urḏī's new calculation together with Jābir's criticism. Thus, it is clear that Ṭūsī recognized 'Urḏī's thesis on

the planetary order when he composed the *Tadhkira*. Because the *Mu'iniyya* is known to have been composed in 1235, we can conclude that Ṭūsī was acquainted with 'Urḏī at some point between 1235 and 1250s. As for the case of the *Zubda*, since it has no clear indication of the date when he composed it, we have not been able to posit this work up until now in the timeline of his astronomical research, but the fact that the *Zubda* presented the standard explanation of the planetary order indicates that he wrote the *Zubda* before his acquaintance with 'Urḏī. Therefore, we can now conclude that its composition date further preceded 1261, i.e., the year when he wrote (the first version of) the *Tadhkira*.

Here, we must note that, as I have mentioned beforehand, hay'a works written by Quṭb Din Shīrāzī, who was an eminent student of Ṭūsī, accepted 'Urḏī's new planetary order as the correct one. Next, I will overview Shīrāzī's opinion about the planetary order and elucidate how he included 'Urḏī's argument in his hay'a works.

5. 'Urḏī's planetary order in Shīrāzī's hay'a works

Beginning his career as a medical student, Shīrāzī met Ṭūsī in 1263 and studied with him philosophy and mathematical sciences in Marāgha,²² so that he became one of Ṭūsī's most prominent students. After his study with Ṭūsī, he went to Anatolia, where he composed his first hay'a work entitled *Nihāyat al-idrāk fī dirāyat al-aflāk* (henceforth, the *Nihāya*) in 1281; then, while revising the *Nihāya*, he wrote a Persian hay'a work *Ikhtiyārāt-i Muẓaffarī* (henceforth, the *Ikhtiyārāt*) in 1282; and finally, he completed *al-Tuḥfa al-shāhiyya* (henceforth, the *Tuḥfa*) in 1285.

In the *Nihāya*, Shīrāzī told in the introduction that he wrote it in the request of his patron named Muḥammad ibn 'Umar al-Badakhshānī, who asked Shīrāzī "that I [= Shīrāzī] follow the linguistic style of the *Tadhkira*, which nothing before has surpassed and nothing after has overtaken, and that I incorporate it in the course of the exposition if it is clear, and expound upon it if something in it is obscure",²³ so the *Nihāya* follows

the structure and wording of the *Tadhkira*. To show the similarity between the *Nihāya* and the *Tadhkira*, I will present the table of contents of the *Nihāya* Book IV “On finding the measurements of the distances and the bodies”, a volume containing discussions about the planetary order, together with the table of contents of the *Tadhkira* Book IV, as follows:²⁴

The *Nihāya*

Book IV: On finding the measurements of the distances and the bodies.

Chapter 1: Introduction [= the *Tadhkira* 1[1]]

Chapter 2: On the measure of the Earth, [=the *Tadhkira* 1[2-5]] and on the knowledge of the height of the sphere of air

Chapter 3: On finding the distances of the Moon from the centre of the World [= the *Tadhkira* 2]

Chapter 4: On the sizes of the diameters of the Moon [= the *Tadhkira* 3]

Chapter 5: On the measure of the diameters of the Sun and the ratio of the sizes of the two luminaries and the Earth [= the *Tadhkira* 4]

Chapter 6: On the rest of the distances of the Sun and the distances and body [sizes] of the two lower planets [= the *Tadhkira* 5]

Chapter 7: On the distances of the upper planets and their body [sizes] [= the *Tadhkira* 6]

Chapter 8 On the distance of the fixed stars and their body [sizes] [= the *Tadhkira* 7]

Chapter 9: On what is taken against the Ancients and the Moderns together on the distances and the bodies

Chapter 10: On the correct method of determining the distances and the bodies

The *Tadhkira*

Book IV: On finding the measurements of the distances and the bodies.

Chapter 1: On the measure of the Earth

Chapter 2: On finding the distances of the Moon from the centre of the world

Chapter 3: On the sizes of the diameters of the Moon, the Sun and the shadow, and the distances of the Sun and the shadow from the Earth

Chapter 4: On the volumes of the two luminaries

Chapter 5: On the rest of the distances of the Sun and the distances and body [sizes] of the two lower planets

Chapter 6: On the distances of the upper planets and their body [sizes]

Chapter 7: On the distance of the fixed stars and their body [sizes] and a concluding discussion regarding this section

This table shows that Shīrāzī adopted the *Tadhkira*'s structure in the *Nihāya*, even copying almost all the chapter headings.

Moreover, we notice that Shīrāzī extensively copied the text of the *Tadhkira* in the body of his text. For example, the following are the beginning of the *Nihāya* Book IV Chapter 4 and the beginning of the *Tadhkira* Book IV Chapter 3:

The *Nihāya* Book IV, Chapter 4 (MS K, f. 139a)

Ptolemy observed two lunar eclipses during which the moon was at the epicycle apex. During one of them, one-quarter of its diameter was eclipsed and, during the other, half of it. By calculation, its latitude during the first eclipse was $48 \frac{1}{2}$ minutes and, during the second, $40 \frac{2}{3}$. Then, he took (fa-akhadha) the difference between them, namely $7+1/2+1/3$ minutes, which is obviously a quarter of the diameter, because it [= the difference] is the difference between a quarter of it [= the diameter] and a half of it. He thus knew ...

The *Tadhkira*, Book IV, Chapter 3 [1] (Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy*, vol. 1, pp. 320-321)

Ptolemy observed two lunar eclipses during which the moon was at the epicycle apex. During one of them, one-quarter of its diameter was eclipsed and, during the other, half of it. By calculation, its latitude during the first eclipse was $48 \frac{1}{2}$ minutes and, during the second, $40 \frac{2}{3}$. He took (wa-akhadha) the difference between them, namely $7+1/2+1/3$ minutes, which is obviously a quarter of the diameter. He thus knew ...

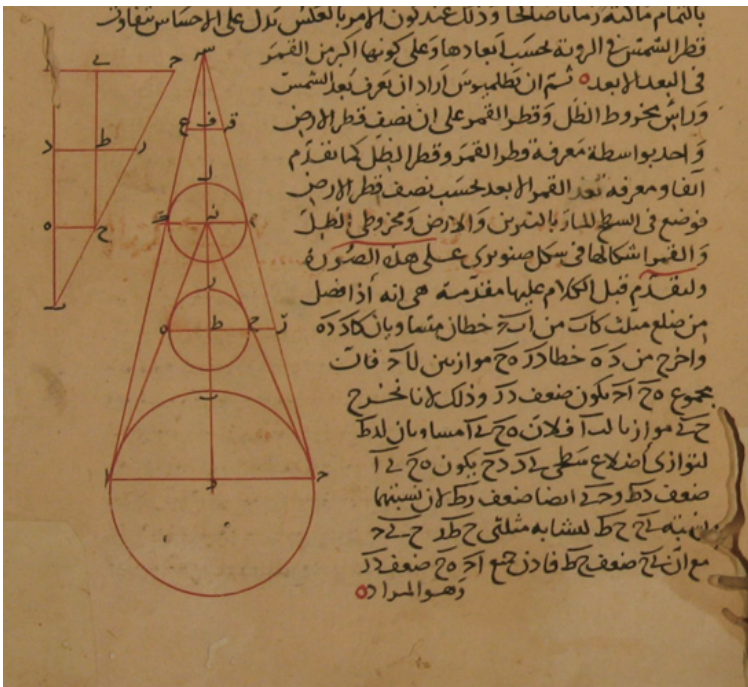
The above comparison illustrates that Shīrāzī reproduced the *Tadhkira's* text carefully, so that his thorough copying keeps the reading “Then, he took” (fa-akhadha) found in the Marāgha version (α) and MS M, which is different from the reading “He took” (wa-akhadha) in the Baghdad version (β).²⁵

From the *Nihāya's* text quoted above, however, we notice that, while Shīrāzī copied the *Tadhkira's* text faithfully, he inserted in the last part an explanation “because it [= the difference] is the difference between a quarter of it [= the diameter] and a half of it”. This shows that when he considered his reproduced text of the *Tadhkira* needed more explanation, he expanded it by adding his own words.

Remarkably, we find in the *Nihāya* Book IV that when Shīrāzī expanded Ṭūsī's explanation, he sometimes copied and pasted a text from Ṭūsī's *Tahrīr al-majisī* without notifying that he quoted it from the *Tahrīr al-majisī*.²⁶ For example, in the *Tadhkira* Book IV Chapter 3 [3],²⁷ where Ṭūsī explained the distances between the centres of the Sun, the Earth, and the Moon by utilizing two shadow cones cast by the Sun, he presented a figure (called “pine-tree figure”) without elucidating geometrical settings; in the *Nihāya* Book IV Chapter 4, on the other hand, after citing the *Tadhkira's* text, Shīrāzī quoted Ṭūsī's geometrical explanation (without the name of his source) about the pine-tree figure in the *Tahrīr al-majisī* where Ṭūsī offered an extra lemma. The following images that are taken from MS K (the *Nihāya*) and MS M (the *Tahrīr*) show the

correspondence between the *Nihāya*'s extra geometrical explanation about the pine-tree figure (Figure 1) and the *Tahrīr al-majisī*'s text (Figure 2), confirming Shīrāzī's quoting in the *Nihāya* a text from the *Tahrīr al-majisī*.

Figure 1: MS K, f. 139b, the *Nihāya* Book IV Chapter 4 (the pine-tree figure)



example proves that, through his editorial work of the *Nihāya*, he aimed to update the *Tadhkira*'s contents to make them more comprehensive and accessible to students by supplying quite a few extra explanations.

What is remarkable about Shīrāzī's supplements to the *Tadhkira* is that he also added information that seems not to contain a new aspect complementing the contents of the *Tadhkira*. For example, in almost all chapters of the *Nihāya* Book IV, he appended after quoting Ṭūsī's text Kūshyār ibn Labbān's (fl. the early eleventh century C.E.) explanation taken from his *zīj*.

Kūshyār ibn Labbān, an eminent scholar in the Iranian region, wrote several works on mathematics (e.g., *Principles of Hindu Reckoning*), astronomy (e.g., *Zīj al-Jāmi*'), and astrology (e.g., *Introduction to Astrology*).²⁹ His *zīj* contained a section on the distances and bodies of the planets, which also circulates as a separate work in several manuscripts.³⁰ In the *Nihāya*, Shīrāzī quoted Kūshyār's text several times after the text of the *Tadhkira*, sometimes with his criticism of its contents; note that he put the name "Kūshyār" at the first time when he cited it (MS K, f. 138b). His criticism confirms that he considered Ṭūsī's explanation to be better than Kūshyār's.

Even more remarkable is that Shīrāzī went beyond Ṭūsī in the *Nihāya* Book IV Chapter 9 "On what is taken against the Ancients and the Moderns together on the distances and the bodies" (MS K, 145a), where he criticized all existing opinions about the distances and bodies of the planets, so he concluded that Venus should be above the Sun. By comparing the text of the *Nihāya* and that of 'Urḍī's *Kitāb al-hay'a*, we realize that he composed this chapter by copying and pasting texts of 'Urḍī's *Kitāb al-hay'a*, with some modification but without the name of his source.

In fact, at the beginning of the *Nihāya* Book IV Chapter 9, Shīrāzī summarized 'Urḍī's presentation of the farthest and nearest distances of each planet according to Ptolemy's *Almagest* and *Planetary Hypotheses*, he reproduced 'Urḍī's criticism without mentioning the name of 'Urḍī, as follows:³¹

Nihāya Book IV Chapter 9 (MS K, 145b)

Also, he [= Ptolemy] and others made the farthest [distance] of the Moon be the nearest [distance] of Mercury, which is clearly wrong, because the two bodies of them [= the Moon and Mercury] have no sharing concerning the distance, since the Moon in its ascending does not reach the point which Mercury in its descending reaches...

وأيضاً جعل هو وغيره أبعد بعد القمر أقرب قرب عطارد وهو خطأ بين لأن جرميهما خاصة لا يشتركان في بعد واحد إذ القمر لا ينتهي في ارتفاعه إلى الموضع الذي ينتهي إليه عطارد في انحطاطه...

‘Urđi, *Kitāb al-hay’a* (Saliba, *Kitāb al-hay’ah*, p. 293)

وأيضاً فإنه قد جعل هو وغيره أيضاً أبعد بعد القمر هو أقرب قرب كوكب عطارد. وهذا خطأ بين لأن جرمي هذين الكوكبين خاصة لا يشتركان في بعد واحد لأن جرم القمر لا ينتهي في ارتفاعه إلى الموضع الذي ينتهي إليه عطارد في انحطاطه...

As this example makes clear, he wrote this chapter mainly by copying and pasting ‘Urđi’s words. Moreover, the comparison between the *Nihāya* Book IV Chapter 10 (MS K, f. 146b) and Urđi’s *Kitāb al-hay’a* reminds us that, following his editorial way in Chapter 9, he made Chapter 10 “On the correct method of determining the distances and the bodies” by reproducing ‘Urđi’s texts, where he presented ‘Urđi’s calculation of the distances and bodies of the planets as the correct method, again without mentioning ‘Urđi’s name.

The overview of the *Nihāya* Book IV illustrates how Shīrāzī edited the *Nihāya* and how he accepted ‘Urđi’s new planetary order: while Shīrāzī composed the *Nihāya* mainly by reproducing the text of the *Tadhkira* with explanatory materials when needed, he added Kūshyār’s explanation as a popular alternative to Tūsī’s, but in concluding part, i.e., Chapters 9 and 10, he criticized Kūshyār’s calculation (the popular method) as well as Tūsī’s (the most updated method in his days) by utilizing ‘Urđi’s text and

presented 'Urḏī's calculation as the correct one. Since he studied astronomy with Ṭūsī, Shīrāzī's experience of learning the *Tadhkira* with Ṭūsī might reflect his way of expanding the contents of the *Tadhkira* with the *Tahrīr al-majisī* and Kūshyār's *zīj*, as well as his focus on 'Urḏī's *Kitāb al-hay'a*. Remarkable is that although the Marāgha associates including Ṭūsī struggled with 'Urḏī's thesis and defended the standard planetary order, Shīrāzī finally accepted 'Urḏī's order, opposing to his teacher Ṭūsī.³²

6. Conclusion

In this article, I first overviewed Abharī's astronomical works, and the discussion about the planetary order in the works composed by Abharī and Kātibī led us to recognize in this topic 'Urḏī's impact to the Marāgha associate. Since anyone of Abharī's contemporaries including Kātibī, Ṭūsī and Shīrāzī did not explicitly mention Urḏī's name in their works, we have not been able to detect the impact of Urḏī's achievement among them, but the analysis of the influence of his new thesis about Venus' position on them clarified 'Urḏī's enormous impact on contemporary scholars around the Marāgha.

Moreover, we realized that this impact can be used as a criterion determining the dates of the compositions of astronomical works. With recourse to this finding, now we can posit the *Talkhīs* preceding the *Ikhtaṣara* in Abharī's astronomical career, and the *Zubda* preceding the *Tadhkira* in Ṭūsī's timeline.

Remarkable is that Ṭūsī's answer in the *Tadhkira* could not wipe out 'Urḏī's thesis on the planetary order. In fact, after learning the *Tadhkira* with Ṭūsī, Shīrāzī, an eminent student of him, accepted 'Urḏī's new order in his three hay'a works. Here, we must note that his experience of reading the *Tadhkira* with Ṭūsī might push him to accept 'Urḏī's planetary order.

In Marāgha, Ṭūsī organized a circle for the study of astronomical texts with his students including Shīrāzī. So, how Shīrāzī expanded and updated the contents of the

Tadhkira in the *Nihāya* might be connected to how Ṭūsī read with his students the *Tadhkira* in the circle: as Shīrāzī's editorial work of the *Nihāya* illustrates, students in the circle might read the *Tadhkira* under the supervision of Ṭūsī with the aid of the *Tahrīr al-majisfī*, and they might study and discuss other astronomical texts such as Kūshyār's *zīj* and Urḍī's *Kitāb al-hay'a* with the knowledge of the *Tadhkira*; based on his reading experience with Ṭūsī, Shīrāzī expanded and updated the contents of the *Tadhkira* in the *Nihāya* with recourse to various astronomical works including the *Tahrīr al-majisfī*, *Kitāb al-hay'a*, and Kūshyār's *zīj*. Thus, Shīrāzī's skill of thoroughly reading astronomical works trained in Ṭūsī's circle undoubtedly led him to go beyond Ṭūsī and accept 'Urḍī's thesis.

Endnotes

* This article is based on my paper "Mu'ayyad al-Dīn al-'Urḍī's Impact on Quṭb al-Dīn al-Shīrāzī concerning Planetary Order" presented at the International Quṭb Al-Dīn Al-Shīrāzī Symposium at Faculty of Theology, Sivas Cumhuriyet University and Faculty of Humanities, Istanbul Medeniyet University (online), 14 October 2021. And it is also partially based on my presentation "Scholarly Exchanges in the Marāgha Observatory: Reassessment of 'Urḍī's Impact on Marāgha Associates" at the International Conference on Traditional Sciences in Asia 2017 "East-West Encounter in the Science of Heaven and Earth", at Kyoto University, 25-28 October 2017.

1 On his biography, see the introduction to George Saliba, *Kitāb al-hay'ah: the astronomical work of Mu'ayyad al-Dīn al-'Urḍī: a thirteenth century reform of Ptolemaic astronomy*, Bayrūt, Lubnān: Markaz Dirāsāt al-Waḥdah al-'Arabīyah, 1990.

2 See George Saliba, "The Original Source of Quṭb al-Dīn al-Shīrāzī's Planetary Model", *Journal for the History of Arabic Science* 3 (1979): 3-18.

3 See Bernard R. Goldstein and Noel Swerdlow, "Planetary Distances and Sizes in an Anonymous Arabic Treatise Preserved in Bodleian Ms. Marsh 621", *Centaurus* 15 (1971):135-170.

4 For the identification of the manuscripts, see Dirk Grupe “Thābit ibn Qurra’s Version of the *Almagest* and Its Reception in Arabic Astronomical Commentaries (based on the presentation held at the Warburg Institute, London, 5th November 2015)”, in *Ptolemy’s Science of the Stars in the Middle Ages*, eds. by David Juste, Benno van Dalen, Dag Nikolaus Hasse, and Charles Burnett, Turnhout, Belgium: Brepols Publishers, 2020, pp. 139-157, pp. 153-154. In this article, I use MS B for reading the *Talkhīs*.

5 See Grupe, “Thābit ibn Qurra’s Version of the *Almagest*”, p. 153, fn. 45.

6 See Taro Mimura, “Too Many Arabic Treatises on the Operation of the Astrolabe in the Medieval Islamic World—Athīr al-Dīn al-Abharī’s Treatise on Knowing the Astrolabe and His Editorial Method”, *Medieval Encounters* 23 (2017), 365-403.

7 See Aydın Sayılı, *The Observatory in Islam and its Place in the General History of the Observatory*, Ankara: Türk Tarih Kurumu basımevi, 1988, p.205.

8 On his biography, see M. Mohaghegh, “Al-Katibi”, EI2.

9 See Sayılı, *The Observatory in Islam*, p.205. B.A. Rosenfeld and Ekmeleddin Ihsanoğlu, *Mathematicians, Astronomers, and Other Scholars of Islamic Civilization and Their Works (7th–19th c.)*, Istanbul: IRCICA, 2003, p. 221 entries one astronomical work called *Revision of “Almagest”* to Kātībī; however, I confirmed that the manuscript mentioned by Rosenfeld and Ihsanoğlu as the unicum of the *Revision of “Almagest”* contains Ṭūsī’s *Tahrīr al-majisī*, not Kātībī’s work.

10 For the sake of convenience, I number quotations in this article as e.g. [Q1].

11 In the following Arabic text, when I emend a reading, I note the original reading at the bottom of the text, where I denote MS B as “ب”.

12 In the following Arabic text, when I emend a reading, I note the original reading at the bottom of the text, where I denote MS C as “ج”.

13 Aydın Salih ed., *Hikmat al-'ayn*, 2002, p. 126

14 The Arabic text is in Ibn Sīnā, *al-Shifā'*, al-Riyadiyat, 4: ‘ilm al-hay’a, al-Qāhira: Dār al-Kātīb al-'Arabī lil-Ṭibā'a wa-al-Nashr, 1405H, p. 463

15 See Richard Lorch, “The Astronomy of Jabir ibn Aflah”, *Centaurus* 19 (1975): 85-107,

97-99.

- 16 See Lorch, “The Astronomy of Jabir ibn Aflah”, p. 99.
- 17 See Abbas Muhammad Hasan Sulayman ed, *Zubdat al-idrak fi hay’at al-aflak*, Alexandria: Dar al-Ma’rifat al-Jami’a, 1994.
- 18 On Kamāl Dīn as a teacher, see F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on Astronomy*, 2 vols., New York: Springer-Verlag, 1993, vol. 1, pp. 7-9.
- 19 See Sajjad Nikfahm-Khubravan and Fateme Savadi eds, *Naṣīr al-Dīn Muḥammad al-Ṭūsī al-Risāla al-Mu’niyya (al-Risāla al-Mughniyya) and its Supplement*, Volume I: Critical Edition of the Persian Texts, Tehran: Written Heritage Research Institute (Miras-e Maktoob), 2020.
- 20 Ragep, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on Astronomy*, vol. 1, p.110.
- 21 Ragep, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on Astronomy*, vol. 1, pp. 314-341.
- 22 On Shīrāzī’s biography and astronomical research, see Fateme Savadi, “The Historical and Cosmographical Context of *Hay’at al-arḍ* with a Focus on Quṭb al-Dīn Shīrāzī’s *Nihāyat al-Idrāk*”, Ph. D dissertation submitted to McGill University, 2018, pp. 21-44.
- 23 F. Jamil Ragep, “Shīrāzī’s *Nihāyat al-idrāk*: Introduction and Conclusion.” *Tārīkh-e Elm* 11 (2013): 41-57.
- 24 In this article I use Istanbul, Köprülü, MS Fazıl Ahmed Paşa 956 (= MS K) for reading the *Nihāya*.
- 25 On the versions of the *Tadhkira*, see Ragep, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on Astronomy*, vol. 1, pp. 70-75.
- 26 In this article I use Istanbul, Köprülü, MS Fazıl Ahmed Paşa 932 (= MS F) for reading the *Tahrīr al-majistī*.
- 27 Ragep, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on Astronomy*, vol. 1, pp. 320-321.
- 28 See George Saliba, “The Height of the Atmosphere according to Mu’ayyad al-Dīn al-’Urdī, Quṭb al-Dīn al-Shīrāzī and Ibn Mu’adh”, in *From Deferent to Equant: a Volume of Studies in the History of Science in the ancient and medieval Near East in honor of E. S. Kennedy*, eds. by David A. King and George Saliba, New York, N.Y.: New York Academy of Sciences,

1987, pp. 445-466.

29 On Kūshyār's biography, see Mohammad Bagheri, "Kūshyār Ibn Labbān," in *The Biographical Encyclopedia of Astronomers*, eds. Thomas Hockey et al., New York, NY: Springer, 2007, pp. 560-561; see also Mimura, "Too Many Arabic Treatises", pp. 366-371.

30 This text is edited with English translation by Mohammad Bagheri, Jan P. Hogendijk, and Michio Yano, "Kūshyār ibn Labbān Gilani's treatise on the distances and sizes of the celestial bodies", *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* 19 (2010-2011): 77-120.

31 To clarify his copying of 'Urḍī's text, I append the text of 'Urḍī's *Kitāb al-hay'a* corresponding to the quotation from the *Nihāya*.

32 Unfortunately, I lack space in this article for analysing Shīrāzī's other two hay'a works, in both of which he repeated almost the same contents as in the *Nihāya* concerning the distances and bodies of the planets with his heavy editorial work. I will discuss how he edited the *Tuḥfa* based on the text of the *Nihāya* on another occasion. Note that I am currently editing and translating the *Tuḥfa* Book IV (on the distances and bodies of the planets) with two colleagues of mine.

