論文の内容の要旨

Theoretical study on interacting fermionic topological phases

（相互作用するフェルミオントポロジカル相に関する理論的研究）

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Many important properties of quantum many-body systems with gapped local Hamiltonians are encoded in the entanglement property of its ground state, independent on the local details of the Hamiltonian. In particular, we can determine the gapped phase of such systems from the ground state, which is defined as an equivalence class of Hamiltonians under local deformations preserving the energy gap.

The classification of gapped phases is an important problem in condensed matter physics. Though the classification problem is difficult and unsolved in general, one can simplify the problem by considering a simplified class of systems with a unique gapped ground state on arbitrary closed spatial manifolds. Such phases are called invertible topological phases.

In the presence of a global symmetry *G*, one can consider a gapped phase with the specified global symmetry, based on the deformations respecting the symmetry. The global symmetry typically enriches the phase diagram, since the global symmetry in general constrains the possible deformations on the ground state. As a result, a single phase can be refined into distinct phases by taking the global symmetry into account. Invertible topological phases with the global symmetry are also called symmetry protected topological (SPT) phases.

In the case of free fermions, a complete classification of SPT phases has been obtained using K-theory. In the case of intrinsically interacting systems, the classification in general differs from the free phases, and we have to use completely different method to perform the classification. For example, a large class of interacting bosonic SPT phases with the global symmetry can be classified by utilizing group cohomology. For the case of fermionic systems, the SPT phases have a richer classification than the bosonic phases; for example, Gu and Wen found that a subclass of fermionic SPT phases is classified by a pair of cohomological data, generalizing the classification of bosonic phases. These fermionic SPT phases are called Gu-Wen phases or super-cohomology phases.

Later, a comprehensive classification scheme of SPT phases utilizing cobordism group is proposed. The cobordism group is thought to classify invertible field theories which effectively describe SPT phases. The cobordism theory predicts novel interacting phases beyond group cohomology and Gu-Wen phases.

In particular, while the Gu-Wen phases cover a large class of fermionic SPT phases, there are phases that are outside of the Gu-Wen subclass, and such “beyond Gu-Wen phases” turns out to contain various important systems. For example, a (1+1)-

dimensional topological superconductor with time reversal symmetry such that $T^2=1$ is classified by Z8, and the only Z4 subclass corresponds to the Gu-Wen phases. The generator of Z8 is well-known to be realized by a chain of a Majorana fermion called the Kitaev wire. Moreover, a (3+1)-dimensional topological superconductor with time reversal symmetry such that $T^2 = (-1)^F$ is classified by Z16, and the only Z4 subclass corresponds to the Gu-Wen phases.

One important feature of the SPT phase is the presence of nontrivial edge state that appears on the boundary of the SPT phase, which includes a well-known gapless edge state of a (2+1)-dimensional topological insulator. The nontrivial spectrum of the edge state originates from an 't Hooft anomaly of the boundary theory of the SPT phase.

An 't Hooft anomaly is an obstruction to coupling the global symmetry of the theory with the background gauge fields. As such, a boundary theory is not invariant under a gauge transformation of background gauge fields, but the variation is canceled by coupling with the bulk SPT phase. After all, the bulk-boundary system gives a fully gauge invariant theory. This cancellation of the 't Hooft anomaly by coupling with the SPT phase in the bulk is called anomaly inflow. Importantly, an 't Hooft anomaly has a dramatic consequence on the low-energy spectrum of the boundary theory. In particular, an ’t Hooft anomaly implies that, in a gapped phase, the symmetry must either be spontaneously broken, or the theory is described by a symmetry-preserving topologically ordered states matching the anomaly.

The aim of this thesis is to develop field theoretical understanding of fermionic topological phases and their boundaries, covering both Gu-Wen and beyond Gu-Wen phases.

We provide a path integral definition of Gu-Wen SPT phases and (1+1)-dimensional superconductor with time reversal symmetry. Since these phases are fermionic, the theory intrinsically depends on the choice of spin or pin structure of a spacetime. We show how one can construct a topologically invariant lattice path integral coupled with spin/pin structure. This is done by generalizing the bosonization and fermionization in (1+1) dimensions to higher spacetime dimensions in the presence of time reversal symmetry, which allows us to construct a path integral of fermionic phases starting with a given path integral for a bosonic theory.

This formulation has various physical applications. For instance, we provide a local lattice definition of symmetry-preserving topologically ordered states on boundary of Gu-Wen SPT phases protected by finite group symmetry which can contain time reversal symmetry. This means that an 't Hooft anomaly that corresponds to the boundary of Gu-Wen phases can be carried by subtle topological degrees of freedom, not by gapless particles and in particular the system having the anomaly can have an energy gap.

In addition, based on the lattice path integral of (1+1)-dimensional topological superconductor with time reversal symmetry, we study how one can diagnose the SPT classification given by Z8, for a given wave function of the SPT phase. We show that a non-local operation on the ground state called a “partial time reversal” can be utilized to diagnose the Z8 classification of the given wave function, based on our field theoretical formulation of the topological superconductor.

Moreover, we also discuss the construction of beyond Gu-Wen phases in higher spacetime dimensions. For example, we consider a (3+1)-dimensional topological superconductor with time reversal symmetry $T^2 = (-1)^F$ classified by Z16. We show that a wave function for the (3+1)-dimensional topological superconductor labeled by 2 in Z16 is constructed by decorating the Kitaev wire on the junction of T symmetry defects in codimension 2.