論文の内容の要旨

Construction of Exact Quantum Many-Body Scar States

(厳密な量子多体傷跡状態の構成)

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Thermalization is such a ubiquitous phenomenon in macroscopic systems that we should observe it every day. However, understanding its origin at the microscopic level, or from quantum mechanics, is far from established yet in spite of a long history of research. Recent progress in quantum engineering enabled us to tackle this long-standing problem experimentally. For instance, several experimental groups directly observed thermalization in quantum many-body systems including ultracold atoms and superconducting qubits. On the other hand, surprisingly, experimentalists also revealed non-thermal behavior with, e.g., Bose gases, trapped ions, and Rydberg atoms. Can we explain theoretically the reason why many systems eventually thermalize, but some exceptional ones do not?

Theoretical approaches to explaining macroscopic behavior from the underlying microscopic dynamics date back to Boltzmann. While his studies were based on classical mechanics, von Neumann carried this problem into quantum mechanics. In 1929, he has already mentioned the concept that is now referred to as typicality. After his work, several variants of typicality have been proved. However, it turned out that typicality cannot explain that an atypical initial state becomes thermalized after a sufficiently long time. It does not tell us the differences between systems with and without thermalization, either.

Recently, several theoretical studies revealed a plausible scenario to explain thermalization of quantum systems, namely, the eigenstate thermalization hypothesis (ETH), which states that all energy eigenstates are locally indistinguishable from the microcanonical ensemble. It is known that the ETH is a sufficient condition for thermalization of isolated quantum systems. Although there is no rigorous proof, it is widely believed to hold for a large class of interacting systems, as evidenced by several numerical studies. Well-known counterexamples to the ETH are integrable systems and many-body localized (MBL) systems. In such systems, there are an extensive number of conserved quantities or local integrals of motions. The existence of such quantities induces ergodicity breaking, hence a violation of the ETH. As a result, these systems can fail to thermalize. Another class of ETH-violating systems is also reported, in which the Hilbert space fractures into exponentially many disconnected subsectors due to constrained dynamics. This phenomenon is called Hilbert space fragmentation.

A pioneering experiment in the Rydberg atom system [H. Bernien *et al*, Nature **551**, 579 (2017)] revealed another mechanism of non-thermal behavior. In this system, many typical states thermalize rapidly, whereas certain particular states do not for an anomalously long time. These peculiar phenomena are referred to as *quantum many-body scars* (QMBS), since they are reminiscent of one-body quantum scars. The mechanism of ETH violation in a QMBS system is distinct from those mentioned above. First, the system has no local conserved quantities except the total energy, which implies the system is non-integrable. Second, the system can be homogeneous and disorder-free, which rules out the possibility of MBL. Third, the system does not exhibit Hilbert space fragmentation. In short, QMBS is an essentially new counterexample to the ETH.

The experimental observation triggered a number of theoretical studies on QMBS. In particular, an effective model of this experiment, dubbed the PXP model, has been intensively studied. In this model, numerical studies revealed that particular initial states exhibit slow relaxation and periodic revivals of the fidelity, whereas other typical states thermalize more rapidly, which corroborates the experimental observation.

Another approach is to construct models with perfect QMBS, whose exact expression can be written down, and perfect revivals in many-body quantum dynamics can be shown analytically. In fact, there is an exact scar state also in the PXP model, which can be written as a matrix product state. Another example is exact scar states in the Affleck-Kennedy-Lieb-Tasaki (AKLT) model, a celebrated $S = 1$ spin chain model. While it was originally proposed to support Haldane's conjecture, it turned out to be also a good example of exact QMBS models.

Despite such intensive studies on QMBS, its general framework and origin remain unclear. Thus, we aim to expand the frontier of understanding of them by providing a new family of analytically tractable QMBS models. To be more specific, most of the reported exact

QMBS models were limited to particular classes. For example, in most models, the dimension of the local Hilbert space is at most two or three. In addition, most models assume translational invariance. Therefore, it is a natural and important question to ask whether one can construct QMBS models under more general conditions, such as models with the higher local dimension or inhomogeneous ones.

Motivated by this question, in this thesis, we propose a new class of exact QMBS models. The key to the construction is the so-called Onsager algebra. While it was originally used to obtain exact solutions of the two-dimensional classical Ising model [Onsager, Phys. Rev. **65**, 117 (1944)], we utilize it for constructing exact QMBS models. We start with spin chain models that respect the Onsager symmetry. Although they cannot be called QMBS models due to their integrability, by focusing on a certain Onsager-algebra element, we can add appropriate perturbations that destroy the integrability but keep particular athermal states to be still eigenstates.

Our models have three remarkable features:

1. The scar states in our model are not product states but have a finite area-law entanglement. That is, our scars are not trivially ETH-violating states such as a vacuum state in the Fermi-Hubbard model.
2. Our models are easy to generalize to those with an arbitrary integer or half-integer spin quantum number. Scar states can also be generalized to multi-parameter ones.
3. We do not impose translational invariance on our models. To the best of our knowledge, this is the first explicitly constructed example of the disordered QMBS models.

The organization of the thesis is as follows. In Chapter 1, we briefly introduce the fundamental concepts, i.e., typicality, equilibration, thermalization, and the ETH. Then, we review the existence/absence of thermalization in various quantum many-body systems from both theoretical and experimental points of view. Although the ETH is believed to be valid in a large class of quantum many-body systems, its violations have been intensively studied theoretically and also observed experimentally. We focus on the three kinds of counterexamples to the ETH, namely, integrable systems, MBL systems, and systems with Hilbert space fragmentation.

In Chapter 2, we review QMBS with several previous works from both theoretical and experimental sides. After providing a brief historical context, we review the pioneering experiment in the Rydberg atom system and introduce an effective model, the PXP model, as an idealization of the experimental setup. We see that this model has an exact QMBS state written as a matrix product state. Next, we focus on other exact results on QMBS, namely, the systematic construction of QMBS Hamiltonians by embedding athermal states into the middle of the energy spectrum and a particular algebraic structure of scar states appearing in some models, providing their concrete examples.

In Chapter 3, we propose an $S = 1/2$ spin chain model with exact QMBS states as the simplest example of our construction. We explicitly write down the exact QMBS states and explain how to construct the QMBS Hamiltonian. We then show the numerical result of the level-spacing statistics, which are frequently used to confirm non-integrability. The statistics of our model strongly suggest that our model is non-integrable. Nonetheless, eigenstate expectation values of an observable and a half-chain bipartite entanglement entropy (EE) clearly illustrate the ETH violation. Moreover, the dynamics of the fidelity and EE is calculated for scar states and other typical states. Both results indicate that coherent states cannot escape from the scarred subspace and never thermalize, whereas other typical states rapidly get thermal.

In Chapter 4, we generalize our model constructed in Chapter 3 via the Onsager algebra. First, we briefly review the Onsager algebra and introduce a one-dimensional model that possesses the Onsager symmetry. Then, we provide the generalized construction of our QMBS Hamiltonian to higher spin S or multi-parameter scar states. We also show several numerical results parallel to those in Chapter 3, which demonstrate the validity of our generalized construction. In addition, we find intriguing situations that do not appear in the simplest case discussed in Chapter 3. For higher S, we show that additional scar states may emerge from the one-magnon subspace. For the multi-parameter generalization with higher Onsager-algebra elements, we see more scar states of the order of $\mathcal{O}(L^m)$ (with $m \geq 2$ and the system size L) than the one-parameter case where there are just $\mathcal{O}(L)$ scar states.

Finally, a summary of the thesis is given in Chapter 5. Some supplemental materials are provided in Appendices.