

Abstract 論文の内容の要旨

論文題目 Structural Characterizations of Rooted Subdivisions
 on Four Vertices in Graphs
(グラフにおける四頂点上の根付き細分に対する
 構造的特徴付け)

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One of the central topics in graph theory is to study substructures of graphs, such as paths and cycles. Starting with the celebrated theorem of Karl Menger, a number of variants of problems of finding disjoint paths in a graph have been studied from both theoretical and practical interests. The k -disjoint paths problem is, given a graph and k pairs of vertices of it, to find k disjoint paths that link the pairs in the graph. This problem is NP-hard if k is part of the input. On the other hand, as a byproduct of the Graph Minor project, Robertson and Seymour proved that for any fixed k there is a polynomial time algorithm to solve it. This algorithm involves a huge constant and it remains a challenge to devise a practical one. The case $k = 2$, however, admits a simple algorithm and its structural characterization is obtained independently by Thomassen, Seymour and Shiloach in 1980: In a 4-connected graph G , two pairs of vertices $(s_1, t_1), (s_2, t_2)$ are linked by two disjoint paths of G if and only if G cannot be drawn in a disc with s_1, s_2, t_1, t_2 on the boundary in order.

The rooted subdivision problem is a natural generalization of the k -disjoint paths problem. This problem asks internally disjoint paths in a graph that link given pairs of vertices. As an extension of the “two-paths theorem”, we mainly focus on rooted subdivisions with four branch vertices. Namely, for a fixed graph H with four vertices, we consider the following problem: Given a graph G and an injective map from $V(H)$ to $V(G)$, is there a subdivision of H in G with four branch vertices specified by the map? Hence the case $H = 2K_2$ (two copies of K_2) corresponds to the 2-disjoint paths problem.

In this dissertation, for any fixed H with four vertices, we give a complete structural characterization of 6-connected graphs G with no such subdivision of H . Roughly speaking, such graphs G can be decomposed into a planar graph and some local areas of non-planarity, giving us a glimpse of an extension of the two-paths theorem. As a corollary, we prove that every 7-connected graph contains a subdivision of K_4 with prescribed branch vertices. This generalizes a result of McCarty, Wang and Yu, who proved that every 7-connected graph is 4-ordered. We also prove that every triangle-free 6-connected graph contains a subdivision of K_4 with prescribed branch vertices. This solves a special case of a conjecture of Mader.

We also consider a relaxed version of the above problem for $H = K_4^-$, where K_4^- is the graph obtained from K_4 by removing one edge: Given a graph G and a subset Z of $V(G)$ of size 4, is there a subdivision of K_4^- in G with the four branch vertices in Z ? In this problem there is no requirement about which vertex of Z works as which vertex of K_4^- . We characterize 3-connected graphs G with no such subdivision of K_4^- . The proof is based on Mader's S -paths theorem.