

論文の内容の要旨

論文題目 dS/CFT correspondence for heavy scalar field

(dS/CFT 対応における重いスカラー場)

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1 Background and motivation

The existence of the inflationary phase is strongly supported by observations of the cosmic microwave background (CMB). This phase, which closely resembles a de Sitter (dS) phase, is believed to account for the origin of the large-scale structure of the universe. Furthermore, the present accelerated expansion of the universe also resembles the early period of inflation. Given the relevance of de Sitter space to at least two epochs of the universe, understanding quantum gravity in dS space is considered crucial.

String theory, which is seen as the most promising UV-complete theory of quantum gravity, aligns with Einstein gravity at low energies and has been advanced significantly with the AdS/CFT duality conjecture. This duality conjecture states that certain conformal field theories (CFTs) can describe UV-complete gravitational theories in asymptotically anti de Sitter (AdS) spacetimes [1], illustrating the holographic principle where CFT spacetime has fewer dimensions than its gravitational counterpart. However, the observable universe does not present itself as asymptotically AdS, which leads to the exploration of a possible holographic duality for de Sitter space.

It was found that some key features of AdS/CFT, such as the matching of symmetry groups between the dual theories, also find parallels in de Sitter space. This led to the development of the dS/CFT framework, suggesting that a dS wavefunction is equivalent

to a CFT generating function [2, 3]. Additionally, it was shown that the bulk correlation functions in dS space have properties analogous to those of the CFT correlation functions.

2 Difficulties and issues

However, dS/CFT is significantly less studied as compared to its AdS counterpart, due to various conceptual and technical challenges it presents. For instance, in dS/CFT, the holographically emergent dimension is temporal rather than spatial. Additionally, unlike AdS/CFT, there are only a few concrete examples of this correspondence.

Furthermore, most of the research on dS/CFT has focused on light fields because they correspond to fields with mass squared above the Breitenlohner-Freedman (BF) bound in AdS space after analytic continuation, which leads to a straightforward extension from AdS/CFT to dS/CFT. On the other hand, heavy fields have much more complicated features. They correspond to AdS fields with mass squared below the BF bound and they do not share the same asymptotic behavior as fields in AdS around the boundary. For instance, a famous dS/CFT dictionary for light scalar fields is provided as [2, 3]

$$\langle \phi_{\mathbf{k}}(\tau_*) \phi_{-\mathbf{k}}(\tau_*) \rangle' = -\frac{1}{2 \operatorname{Re} \langle O_{\mathbf{k}} O_{-\mathbf{k}} \rangle'}, \quad (2.1)$$

where τ_* is a late time where the wavefunction is evaluated. For example, two-point functions of light scalars with the mass $0 \leq m < \frac{d}{2}H$ in the superhorizon regime $-k\tau_* \ll 1$ read

$$\langle \phi_{\mathbf{k}}(\tau_*) \phi_{-\mathbf{k}}(\tau_*) \rangle' \propto (-\tau_*)^d (-k\tau_*)^{-2\nu} \quad \text{with} \quad \nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}, \quad (2.2)$$

where H is the Hubble constant. This can naturally be identified with the inverse of two-point functions of the dual CFT operator with the scaling dimension $\Delta = \frac{d}{2} + \nu$. In contrast to light fields, heavy fields oscillate outside the cosmological horizon and so do their two-point functions:

$$\langle \phi_{\mathbf{k}}(\tau_*) \phi_{-\mathbf{k}}(\tau_*) \rangle' \propto (-\tau_*)^d \left| 1 + e^{-\pi\mu} e^{i\alpha(\mu)} (-k\tau_*)^{2i\mu} \right|^2 \quad \text{with} \quad \mu = \sqrt{\frac{m^2}{H^2} - \frac{d^2}{4}}, \quad (2.3)$$

where $\alpha(\mu)$ is a mass-dependent phase factor. At least naively, it is hard to identify them with inverse of (a real part of) conformal two-point functions.

However, such heavy fields are inevitable once we consider UV completion of the bulk theory. The study of heavy fields is crucial for gaining insight into the applicability and limitations of dS/CFT, and hence for advancing our understanding of quantum gravity.

3 Purpose, method and results

In this thesis, we perform a comprehensive study on heavy scalar fields in de Sitter space and aim to explore their quantum properties in relation to dS/CFT correspondence.

First we study wavefunctions of heavy scalars on de Sitter spacetime and their implications for dS/CFT correspondence. We analyze them by imposing various boundary conditions: Dirichlet boundary conditions and a type of mixed boundary conditions, which eliminate one asymptotic mode on the boundary and keep the other. We will demonstrate that dS wavefunctions with mixed boundary conditions are naturally identified with generating functions of the would-be dual CFTs. Then we show that wavefunctions with the Dirichlet boundary conditions are dual to their double-trace deformations. Furthermore, we provide a dictionary that applies for scalar fields with general masses between dS two-point functions and CFT two-point functions of the form,

$$\langle \phi_{\mathbf{k}}(\tau_*) \phi_{-\mathbf{k}}(\tau_*) \rangle' = -(-\tau_*)^{2d} \frac{\langle O_{\mathbf{k}} O_{-\mathbf{k}} \rangle'_{M+} \langle O_{\mathbf{k}} O_{-\mathbf{k}} \rangle'_{M-}}{\langle O_{\mathbf{k}} O_{-\mathbf{k}} \rangle'_{M+} + \langle O_{\mathbf{k}} O_{-\mathbf{k}} \rangle'_{M-}}, \quad (3.1)$$

where $\langle O_{\mathbf{k}} O_{-\mathbf{k}} \rangle'_{M\pm}$ are two-point functions in QFTs dual to wavefunctions with mixed boundary conditions, which can be identified with CFT two-point functions. The subscripts \pm are associated to the time ordered and anti-time ordered integration contours in the in-in formalism.

We also explore a different type of dS/CFT for global de Sitter, which associates states on global spacelike slices instead of the past/future boundary, using a quasinormal mode basis. It is shown that for light scalar fields, the Euclidean vacuum is a thermofield double state in the dual CFT description, and that the global de Sitter geometry arises from quantum entanglement between two copies of the CFT [4]. This is an analogy of eternal AdS black holes in AdS/CFT and resembles the concept of ER=EPR, which is a conjecture stating that two entangled particles (Einstein-Podolsky-Rosen pair) are connected by a wormhole (Einstein-Rosen bridge). We extend the discussion in Ref. [4] to heavy scalar fields and demonstrate that the thermofield double state description for Euclidean vacuum holds for heavy scalar fields as well. Namely, the Euclidean vacuum can be expressed as

$$|0_E\rangle \propto e^{-e^{-\pi\mu} N^{AB} \hat{\Phi}_A^{S\dagger} \hat{\Phi}_B^{N\dagger}} |0_N\rangle \otimes |0_S\rangle, \quad (3.2)$$

where $|0_N\rangle$ and $|0_S\rangle$ are vacua for the ‘northern modes’ and ‘southern modes’, and $\hat{\Phi}_A^{N\dagger}$ and $\hat{\Phi}_A^{S\dagger}$ are the corresponding creation operators. Tracing over one side of the modes produces a mixed thermal state describing a single static causal patch. However, the boundary field corresponds to a mixed form of CFT operators with different conformal dimensions, which makes identification with a CFT difficult and requires further study.

References

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- [2] A. Strominger, *The dS / CFT correspondence*, JHEP **10** (2001) 034 [[hep-th/0106113](#)].
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- [4] J. Cotler and A. Strominger, *Cosmic $ER=EPR$ in dS/CFT* , [2302.00632](#).