論文の内容の要旨

Renormalization group flow and fixed-point in tensor network representations

(テンソルネットワークによる繰り込み群フローと固定点の研究)

上田 篤

We propose the integration of an energy-based finite-size scaling methodology with tensor network renormalization (TNR) techniques. TNR, serving as a numerical implementation of real-space renormalization group (RG) methods, provides a pathway to access the low-lying energy spectrum of various systems. By melding TNR with conformal perturbation theory, we can effectively calculate running coupling constants. This combined methodology is particularly valuable in practical calculations, as it adeptly navigates around the numerical errors commonly encountered in TNR applications.

A primary objective of numerical simulations in the study of lattice models is often the precise determination of their phase diagrams. The demarcation of phase transition points within these diagrams often necessitates simulations of systems with very large sizes. For example, accurately identifying the phase boundary of the Ising model through spontaneous magnetization typically requires simulating thousands of lattice sites. While TNR is capable of handling large system sizes, it is also known to suffer from amplified numerical errors as the size of the system increases.

In contrast, our proposed methodology requires only a few steps of RG, thereby inducing fewer numerical errors and reducing computational costs. The energy-based finite-size scaling approach does not rely on large system sizes, unlike conventional methods that use observables such as magnetization and heat capacity to determine phase transitions. This approach is not only more efficient but also more resilient to the challenges posed by the scale of the simulations, offering a significant advantage in the study of critical phenomena in lattice models.

Additionally, we will delve into the origins of numerical errors in TNR simulations from a field-theoretical perspective. This analysis will shed light on how these errors scale

with the approximation parameter, denoted as D. Understanding this scaling is critical for accurately estimating and managing errors in simulation results.

Through the application of this methodology, we aim to provide a more accurate and computationally efficient means of exploring phase transitions and the critical properties of lattice models, enhancing our understanding of these complex systems.

In the subsequent discussion, we delve into the tensor structure of fixed points in the context of lattice models. A significant challenge in this area arises from the effects of finite bond dimensions, which make the true fixed-point tensor practically unattainable through direct numerical methods. To circumvent this issue, we adopt an analytical approach, employing conformal mappings to study the fixed-point tensors.
This analytical exploration leads to a revealing insight: the tensor elements of the fixed-point tensor correspond to the four-point functions of primary operators within the framework of conformal field theory (CFT). This correspondence is not just a theoretical conjecture; it is corroborated by empirical observations showing that tensors renormalized for finite sizes tend to align with our theoretical predictions.
The significance of this finding cannot be overstated. It suggests that the tensor representations of fixed points in lattice models embody the universality of non-trivial infrared (IR) physics at the lattice level. Our approach thus bridges the gap between the abstract theoretical constructs of CFT and the practical, computable structures in lattice models. By demonstrating this universal behavior, we provide robust support for the concept of universality in critical phenomena, particularly as it manifests in the intricate world of lattice models.
Through this investigation, we aim to offer a deeper understanding of the fundamental principles underlying critical phenomena, specifically highlighting how the universal aspects of CFT are reflected in the practical, numerical realm of lattice model simulations.