

# 論文の内容の要旨

## Construction of Quantum Many-Body Scars in Spin Models with Multibody Interactions

(多体相互作用をもつスピン模型における量子多体傷跡状態の構成)

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The main theme of this thesis is the construction of models with Quantum Many-Body Scars (QMBS). QMBS are defined as nonthermal states embedded in the spectra of nonintegrable quantum systems, exhibiting nonthermal behavior that cannot be explained by either many-body localization or Hilbert space fragmentation.

The concept of thermalization, for both classical and quantum systems, revolves around the notion that isolated systems eventually reach thermal equilibrium. Historically, classical thermalization is explained by the ergodic hypothesis, which states that an isolated many-body system explores all accessible states in phase space over time, leading to an equilibrium state that is independent of its initial state. However, the essence of classical thermalization is more accurately represented by typicality, where almost all states in phase space are identical in terms of macroscopic physical quantities. It can explain the relaxation to thermal equilibrium in finite time.

In quantum systems, the relaxation to a diagonal ensemble follows directly from the Schrödinger equation. However, if we choose an initial state from the energy shell, we know empirically that the state will relax into a microcanonical ensemble. Boltzmann suggested the quantum ergodic theorem to explain the relaxation to the microcanonical ensemble. This is similar to the typicality in classical systems. When we prepare the set of macroscopic observables  $\tilde{\mathcal{M}} = \{\hat{\mathcal{M}}_1, \hat{\mathcal{M}}_2, \dots, \hat{\mathcal{M}}_n\}$ , which are mutually commuting, almost all of the Hilbert space is dominated by a single simultaneous eigenspace of  $\tilde{\mathcal{M}}$ . In modern framework, this definition is called macroscopic thermal equilibrium (MATE). There is another class of thermal equilibrium, microscopic thermal equilibrium (MITE), which is formulated such that a given state is indistinguishable from a microcanonical density matrix when traced out the degrees of freedom outside a sufficiently small region.

In recent developments, the eigenstate thermalization hypothesis (ETH) is known as the framework of thermalization in typical quantum many-body systems. The ETH is further classified into two categories, namely the strong and weak ETH, based on the strength of the claim. The strong ETH posits that in a large quantum system, all eigenstates exhibit properties of thermal equilibrium. In contrast, the weak ETH replaces the word *all* with *almost all*. This allows some eigenstates to not satisfy the ETH. Although there is a corresponding ETH for each of MATE and MITE, it is the MITE-ETH that is often discussed, and when we simply refer to ETH, we are referring to that one. Although we do not have a rigorous proof of this hypothesis, there is substantial evidence that the strong ETH holds true in numerous quantum many-body systems. On the other hand, several classes of quantum many-body systems do not satisfy the strong ETH. Examples of systems violating ETH include quantum integrable systems, many-body localized systems, and systems with Hilbert space fragmentation. These exceptions are characterized by an abundance of integrals of motion (IOMs) or similar structures that disrupt typical thermalization patterns. In practice, we employ level-spacing statistics to determine whether a system exhibits thermal behavior. If the system is not integrable, the normalized level-spacing distribution

obeys the Wigner-Dyson distribution. If not, it obeys the Poisson distribution. Additionally, entanglement entropy serves as an indicator to determine if a given state is thermal. The entanglement entropy of thermal states usually obeys a volume law. On the other hand, that of nonthermal states will scale as an area law or a subvolume law.

A recent experiment with Rydberg atoms has identified another class of quantum many-body systems that violate ETH, and nonthermal states that appear in the experiment are named quantum many-body scars (QMBS). Theoretical studies found that its effective model, the PXP model, has a structure that effectively confines the states in a small subspace. Today, systems with QMBS are characterized by a small number of nonthermal energy eigenstates in the spectra of nonintegrable quantum systems. Theoretical studies of QMBS suggested not only the analysis of the PXP model, which is the effective model of the Rydberg atom experiment, but systematic methods to construct models with QMBS, including the Shiraishi-Mori embedding, the Onsager algebra, and the restricted spectrum generating algebra.

In this thesis, we introduce several classes of models with multibody interactions that exhibit QMBS. They can be divided into two categories based on differences in construction methods. In Chapter 3, we describe the first approach that uses the traditional restricted spectrum generating algebra. It allows us to construct a large class of spin-1 models involving scalar spin chirality. We propose three classes of models with QMBS. The first and second classes of models are obtained by perturbing the spin-1 scalar spin chirality by tailored disorder. In the first model, we choose the random single-ion anisotropy as a perturbation. The second model considers a slightly more complex form of perturbation. This model can be extended to higher-dimensional lattices. As an example, we introduce a model on a two-dimensional triangular lattice. These models provide us with towers of scar states generated by restricted spectrum generating algebra. It allows us to demonstrate that a superposition of these scar states shows perfectly periodic revivals in the dynamics. Interestingly, in the second model, two different kinds of QMBS towers coexist, and we observe that their combined state exhibits complex but periodic dynamics. The third class is the combination of the Affleck-Lieb-Kennedy-Tasaki (AKLT) Hamiltonian and the scalar spin chirality. It is a deformation of the system constructed by the method mentioned in Chapter 4.

In Chapter 4, we explain the other method that uses integrable boundary states. Integrable boundary states are characterized as states that are annihilated by all parity-odd conserved charges of an integrable system. These are utilized to construct models with QMBS as follows: First, we consider an integrable boundary state  $|\Psi_0\rangle$ . It is annihilated by all parity-odd charges  $Q_{2k+1}$  ( $k = 1, 2, \dots$ ) of an integrable system. Then, if we find a nonintegrable Hamiltonian  $H_{\text{NI}}$  such that  $|\Psi_0\rangle$  is an eigenstate, the state  $|\Psi_0\rangle$  is also an eigenstate of the combination  $H_{\text{scar}} = H_{\text{NI}} + \sum_k t_k Q_{2k+1}$  with  $t_k \in \mathbb{R}$ . If this new Hamiltonian is non-integrable and the energy of  $|\Psi_0\rangle$  is in the middle of the spectrum, then  $|\Psi_0\rangle$  is likely to be a scar state. Then,  $H_{\text{scar}}$  is the Hamiltonian that we want. This method allows us to construct deformations of the Majumdar-Ghosh and AKLT models—prototypes of frustration-free systems. The former is related to the spin-1/2 scalar spin chirality, which is the third conserved charge of the Heisenberg model, and the latter is related to the third conserved charge of the SU(3) Sutherland model. In addition, we construct models starting from the spin-1/2 scalar spin chirality and its deformation. The scalar spin chirality and its deformation are the third conserved charges of the XXZ model and the XYZ model, respectively. In these models, the tilted Néel ( $q$ -Néel) states serve as integrable boundary states. Unlike the two previous models, i.e., the Majumdar-Ghosh and AKLT-based models, these models contain towers consisting of multiple QMBS. We observe that the Néel state, which is a combination of these QMBS, shows perfectly periodic revivals in its dynamics. In addition, we discuss the extension of these models to any bipartite lattice. As an example, we provide a model on a two-dimensional square lattice.