研 究 速 報
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Study on Reinforced Concrete Members Using Fiber Reinforced Concrete (5)
—— Method of Calculating Yield and Maximum Loads ——

鋼繊維補強鉄筋コンクリート構造部材に関する研究(5)

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Introduction

Recently there has been an interest in studying the use of steel fiber reinforced concrete (SFRC) in structural components such as beams, slabs and columns, etc. This results the engineer to face new problems like predicting the strengths and behaviour of structural members cast out of these concretes. Thus the need for theoretical calculations of strength, such as yield and maximum loads of these structural components, arises if one is to use them in actual structures.

Many experimental evidences show that flexural strength and the ductility of structural members increase due to the presence of steel fibers. As an example, the maximum moments of singly reinforced concrete beams with steel bar reinforcement of 2% gave an increment around 10% to 15%, when the steel fiber content was 1.5% by volume. Therefore it is necessary to seek for a method to calculate these strengths in a way different to the conventional methods adopted in ordinary beam analysis for reinforced concrete.

Complexity of the calculations is due to the non availability of simple stress-strain relations in tensile zone. However, accurate solutions to these problems are getting feasible with the proper understandings of the materials and also the use of speedy digital computers. Therefore the objective of this report is to explain some simple and accurate method of calculating the yield and maximum loads of SFRC beams and slabs which fail in pure flexure.

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1. Strength analysis

1.1 Beams

The analysis follows the compatibility and equilibrium conditions with the simplified stress-strain relationships. Concrete tensile stresses are considered for SFRC beams and neglected for plain concrete. The stress-strain model for SFRC in the tensile region is as described by K. Kobayashi et al (shown in Fig. 1). The stress-strain relation for the compressive zone of concrete is according to the formula proposed by S. Popovics. Fig. 2 illustrates the complete stress distribution across the depth of a SFRC beam at

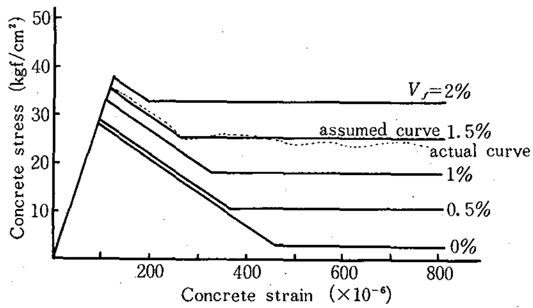


Fig. 1 Stress-strain curves for SFRC in tensile region.

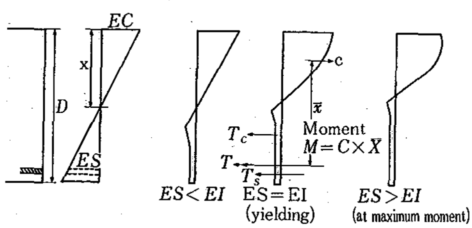
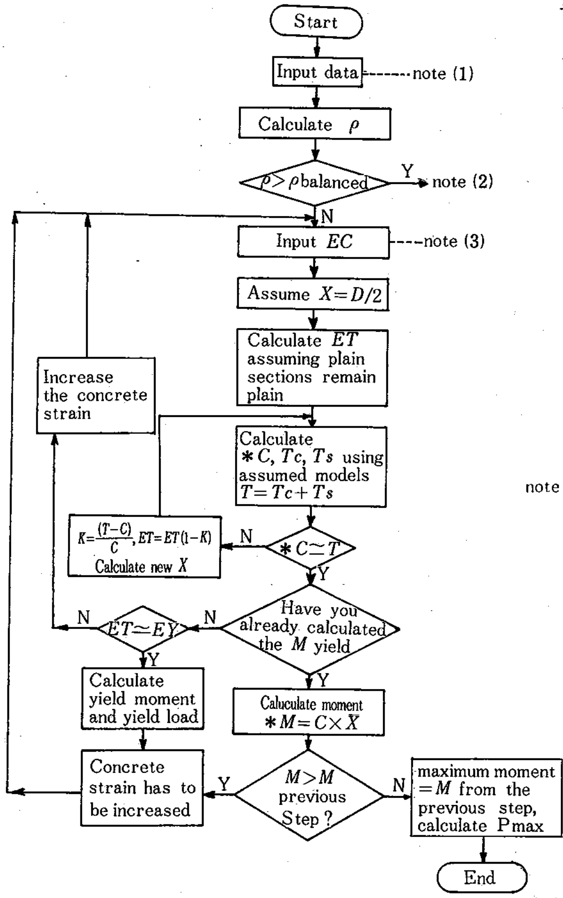


Fig. 2 Stress variation across the depth of a SFRC beam specimen at various stages of loadings.



Notations
ρ……% of reinforement
EC……extreme concrete compressive strain
ET……steel tensile strain
EI……steel yielding strain
D……depth of the beam
X……neutral axis depth
M……moment of the section
*…refer Fig. 2

note (1) Input properties of materials and the geometry of specimen
(2) POPOVICS formula beyond the yielding of concrete has to be modified. Calculation is then proceeded. This case is not considered in this report.
(3) Starting value of concrete strain.

Fig. 3　Flow chart for calculation of yielding and maximum loads.

various stages of loading. The main steps involved in the analysis are shown in the form of a flow chart shown in Fig. 3.

1.2　Slabs

In calculating yield and maximum loads of slabs, possible cracking mechanisms are assumed. Loads for each mode of fracture can be calculated from the energy principles using yield line theory. The two types of mechanisms assumed are illustrated in Fig. 4.

When the concentrated load applied at the center is P and the moment at the fracture line is M per unit length, the load caused to form the relevant fracture modes are as given by equations (a) and (b) shown in Fig. 4. The solution to mode (II), expressed in equation (b) is obtained by considering a regular N sided slab forming a symmetrical deflected mechanism. The solution to circular simply supported slab can be obtained by finding the limit when N tends to infinity.

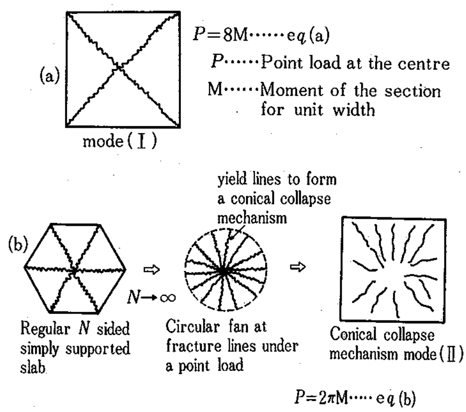


Fig. 4　Assumed fracture mechanisms for slabs.

2.　Discussion of Results

The details of beams and slabs of SFRC and plain concrete are given in Tables 1 and 2 respectively.

Table 1 Details of beams and observed values (W/C=50%, Vf=1.5%, Beam width=10cm)

Beam No	Effective Depth (cm)	p (%)	a/d	Yield Loads (Ton f)			Maximum Loads (Ton f)		
				Observed (1)	Calculated (2)	(1)/(2) (3)	Observed (4)	Calculated (5)	(4)/(5) (6)
B_1	8.5	1.84	5	2.25 (1.9)	2.33 (1.94)	0.96 (0.97)	2.65 (2.38)	2.56 (2.23)	1.04 (1.07)
B_2	8.5	1.84	4	2.68 (2.55)	2.91 (2.43)	0.92 (1.05)	3.27 (2.92)	3.2 (2.78)	1.02 (1.05)
B_3	8.5	1.84	3	3.78 (3.17)	3.88 (3.24)	0.97 (0.98)	4.37 (3.76)	4.26 (3.71)	1.03 (1.01)
B_4	18.2	2.21	5	5.7 (5.58)	5.44 (4.72)	1.04 (1.2)	6.28 (5.58)	6.10 (5.48)	1.03 (1.02)
B_5	18.2	2.21	3.5	8.0	7.76	1.03	9.29	8.72	1.06
B_6	8	3.54	5	3.62	3.43	1.05	3.9	3.6	1.08
B_7	8	3.54	4	4.48	4.28	1.04	4.71	4.46	1.05

Note:-(1) ( )Values are for Plain Concrete
(2) Average of Column (3)=1.01(1.05)
Column (6)=1.04(1.04)
(3) Standard deviation of
Column (3)=0.047(0.09)
Column (6)=0.02(0.02)
(4) Steel fiber used was indented cut wires with length 30 mm and diameter 0.5 mm
(5) p refers to perceutage of steel bar reinforcements
(6) a/d refers to shear arm length to depth ratio

Table 2 Details of slabs and observed values (W/C=50%, Vf=1.5%)

Slab No	S (mm)	p (%)	Yielding Loads (Ton f)					Maximum Loads (Ton f)				
			Observed Loads (1)	Calculated Loads				Observed Loads (6)	Calculated Loads			
				From eq(a) (2)	(1)/(2) (3)	From eq(b) (4)	(1)/(4) (5)		From eq(a) (7)	(6)/(7) (8)	From eq(b) (9)	(6)/(9) (10)
S1	100	2.01	—	—	—	—	—	8.82 (7.07)	9.9 (8.61)	0.89 (0.82)	7.77 (6.76)	1.13 (1.05)
S2	150	1.4	—	—	—	—	—	7.23 (5.62)	7.95 (6.11)	0.91 (0.92)	6.24 (4.8)	1.16 (1.17)
S3	200	1	—	—	—	—	—	7.226 (5.15)	6.54 (5.02)	1.00 (1.03)	5.14 (3.94)	1.41 (1.31)
S4	250	0.8	—	—	—	—	—	6.486 (4.28)	5.8 (4.24)	1.12 (1.1)	4.55 (3.33)	1.42 (1.28)
S5	300	0.6	—	—	—	—	—	5.674 (3.63)	5.02 (3.07)	1.13 (1.18)	3.94 (2.41)	1.44 (1.51)
S6*	150	1.4	4.5 (3.3)	5.82 (3.44)	0.77 (0.96)	4.57 (2.7)	0.98 (1.22)	7.978 (5.63)	7.95 (6.11)	1.00 (0.92)	6.24 (4.8)	1.28 (1.17)
S7*	250	0.8	4.2 (2.6)	5.05 (2.49)	0.83 (1.04)	3.97 (2.5)	1.06 (1.04)	5.93 (4.25)	5.8 (4.24)	1.02 (1.00)	4.553 (3.33)	1.30 (1.28)
				Average	0.8 (1.00)		1.02 (1.13)			1.025 (0.99)		1.3 (1.25)
				Stardard deviation	0.03 (0.04)		0.04 (0.09)			0.09 (0.11)		0.11 (0.13)

Note: (1) S refers to spacing of 10 mm steel bar
(2) p refers to percentage of steel bar reinforcenments
(3) * have bars crossing the center
(4) Steel fiber used was indented cut wires with length 30 mm and diameter 0.5 mm
(5) ( )Values are for Plain Concrete

Table 1 shows the yield and maximum loads of both type of beams, obtained from the experiments as well as the computed values by using the method explained above. It is interesting to note that the values calculated for all the beams agree farely well with the observed values, within 5% in most cases. The averages of ratios of test load to calculated load for SFRC and plain beams are 1.01, 1.04 and 1.05, 1.04 for yield and maximum loads respectively. It can also be observed that the calculated values of SFRC beams agree better with observed values compared to plain concrete beams. This may be due to the SFRC behaving better as a composite material than plain concrete. These results show that the analysis explained above is accurate enough to calculate yield and maximum loads of SFRC beams subjected to flexure.

The analysis is based on a simplified model regarding the shape of the stress-strain relation in the tensile zone after cracking. This has got to be verified for every fiber concrete as the shape will change with fiber volume, type of fiber, fiber length, water-cement ratio, age and also curing conditions.

When the extreme concrete tensile strains are farely high, since the strain capacities of fiber concretes are limited, fiber pullout will occur at very high strains and would not transfer any stress. Therefore care has to be taken if one is to use this kind of analysis for large tensile strains such as in deep beams.

The calculated yield and maximum loads for slab series are presented in Table 2. Observed and calculated values are compared. Unlike for beams, farely high differences can be observed. This may be due to the geometric complexity compared to beams and also difficulty in obtaining analytical solutions for loads. Results also show in calculating the yield loads, the mechanism shown in Fig. 4(b), the conical mode of failure is more valid. Hence equation (b) would give better results in calculating yield loads of SFRC slabs. Since only slabs No. S6 and No. S7 have the bars crossing the center, only in those cases the accurate yielding loads could be obtained. However, it can be noticed if the same equation is used to calculate the maximum loads, a lowerbound solution with an error of 20% to 30% is possible. On the other hand, equation (a) gave a better result in calculating

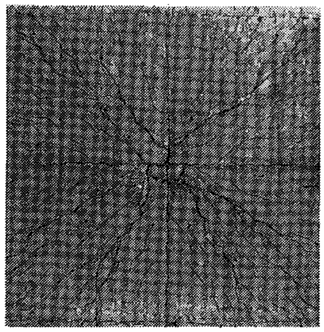


Fig. 5　Fracture pattern of SFRC slab at failure.

maximum loads. In this case, error was about 10%. Hence equation (a) is preffered in calculating the maximum loads of SFRC slabs.

Comparing the calculated and observed values of plain concrete slabs, equation (a) which was obtained by considering mode (I) gave better results in calculating both yield and maximum loads. Infact, in practice this equation is used to calculate the strengths of plain concrete slabs.

The use of two different equations for SFRC in yield and ultimate state may be due to the difference in cracking patterns the slabs exhibit in each case. As can be seen from Fig. 5, although SFRC exhibit a conical fracture pattern with large number of closely spaced cracks at yield conditions, ultimate cracking is governed by large cracks appearing diagonally.

3. Conclusions

The conclusions obtained from this study are as follows:

1) The yielding and maximum loads of SFRC beams can be accurately calculated by taking into account the stresses of concrete in the tensile zone.

2) In order to get accurate solutions for strengths of SFRC, stress-strain relationships for tension have to be verified, as they will change with type of fiber, fiber content, fiber length, curing conditions, etc..

3) The yielding loads of SFRC slabs can be calculated within 3% accuracy, by assuming yield lines to form a conical collapse pattern.

4) The maximum loads of SFRC slabs can be calculated to an accuracy of around 10% by assuming a cracking pattern assumed as for plain concrete. However, accuracy becomes higher for slabs having the reinforcements crossing the center.

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