研　究　速　報

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Another Extension of Dempster & Shafer's Theory to Fuzzy Set for Constructing Expert Systems*

エクスパートシステム構築のための Dempster & Shafer 理論のファジィ集合への拡張

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1. Introduction

Instead of Bayesian theory, Dempster & Shafer's theory[1,2] plays an important role in dealing with subjective uncertainty. Recently, the importance of this theory is recognized for the design of inference machine in expert systems which utilize uncertain knowledge and evidences.[3-8] This paper describes one of rational inference mechanisms for the expert systems which utilize fuzzy knowledge as well as uncertain knowledge. This mechanism is based on a natural extension of Dempster & Shaher's theory to include fuzzy certainty.

2. AND/OR/COMB Graph and Inference Based on Dempster & Shafer's Theory

Ishizuka et al. proposed an AND/OR/COMB graph to describe hierarchical structures of problems with uncertainty.[3,4] Figure 1 shows an example of the AND/OR/COMB graph. The combination relation denoted by COMB refers to such a problem decomposition that a goal or subgoal is supported from plural uncertain knowledge and/or evidences which are independent with each other. There exists a consensus in AI (artificial intelligence) community that min and max operations on a certainty measure can be adopted for the inference to integrate the evidences with AND and OR relations, respectively.

As for the COMB relation, the following inference methods have been proposed to date. An intuitive combining function was employed in MYCIN to integrate the evidences with the COMB relation.[10] Duda et al. have proposed subjective Bayesian method for the same purpose.[11] Afterward, the importance of Dempster & Shafer's theory is recognized.[3-8] This theory enables us to deal with subjective uncertainty in a theoretical manner. Ishizuka et al. extended the Dempster & Shafer's theory to include fuzzy subsets[3,4,5] and have successfully employed it in the inference machine of SPERIL, which is a rule-based damage assessment system for existing structures particularly subjected to earthquake excitation.[9]

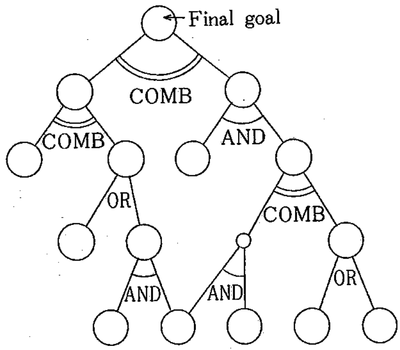

Fig. 1 AND/OR/COMB graph for a problem with uncertainty.

3. Knowledge with Fuzzy Certainty

The knowledge expressed with the fuzzy subsets and numerical certainty measure has been concerned so far.[3,4,5] There exists another situation where even human experts cannot express the certainty of their knowledge in a precise manner. One possible treatment for such certainty is to use fuzzy probability (or linguistic probability[12]). Therefore, in order to include such knowledge in expert systems, we will

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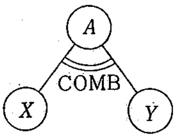

Rule 1 IF X_1,
THEN A_1 with C_1.
Rule 2 IF Y_1,
THEN A_2 with C_2.

Fig. 2　Rule representation for COMB relation.

consider another extension of the Dempster & Shafer's theory.

Let the knowledge for the COMB relation be expressed in the rule format as in Fig. 2, where X_1 and Y_1 are subsets of evidential states X and Y, respectively, A_1 and A_2 are subsets of subgoal state A, and C_1 and C_2 are certainty measures described in 3, 4, 5, 9).

In this paper, we will consider the case that the certainty measure is expressed in a fuzzy form, whearas the subsets in the premise and conclusion of the rule are non-fuzzy. More specifically, consider that C_1 and C_2 in the rules of Fig. 2 can be fuzzy basic probability to be assigned to the subsets A_1 and A_2, respectively, in the conclusion if the respective premise is completely satisfied. The basic probability is one of the central concepts of the Dempster & Shafer's theory. In this situation, extension principle[12] in fuzzy set theory plays key role to deal with the fuzzy probabilities, or more generally, fuzzy values.

Let F_1 and F_2 be fuzzy values characterized by membership functions or possibility distributions $\mu_{F_1}(x_1)$ and $\mu_{F_2}(x_2)$ along real numbers x_1 and x_2, respectively,. According to the extension principle, the operations of addition(+), multiplication(*), minimum and maximum on F_1 and F_2 can be defined as follows;

$$\mu_{F_3}(x_3)=\max_{\substack{x_1,x_2\\x_3=x_1+x_2}}\min\{\mu_{F_1}(x_1),\mu_{F_2}(x_2)\},\quad \text{if } F_3=F_1+F_2, \tag{1}$$

$$\mu_{F_3}(x_3)=\max_{\substack{x_1,x_2\\x_3=x_1*x_2}}\min\{\mu_{F_1}(x_1),\mu_{F_2}(x_2)\},\quad \text{if } F_3=F_1*F_2, \tag{2}$$

$$\mu_{F_3}(x_3)=\max_{\substack{x_1,x_2\\x_3=\min\{x_1,x_2\}}}\min\{\mu_{F_1}(x_1),\mu_{F_2}(x_2)\},\quad \text{if } F_3=\min\{F_1,F_2\}, \tag{3}$$

$$\mu_{F_3}(x_3)=\max_{\substack{x_1,x_2\\x_3=\max\{x_1,x_2\}}}\min\{\mu_{F_1}(x_1),\mu_{F_2}(x_2)\},\quad \text{if } F_3=\max\{F_1,F_2\}, \tag{4}$$

where $\mu_{F_3}(x_3)$ is the possibility distribution of the resultant fuzzy value F_3 after the respective operations, and x_3 is a real number.

Then, an rational inference for the COMB relation of Fig. 2 can be realized by extending the procedure described in 3, 4, 5, 9) to include fuzzy probability. The inference procedure of 3, 4, 5, 9) is based on the calculation of the lower probabilities of the Dempster & Shafer's theory and Dempster's rule of combination for the basic probabilities. If the basic probabilities $m(X_j)$ (j : integer) are fuzzy, the fuzzy lower probability can be calculated as,

$$P_*(X_i)=\sum_{X_j\subseteq X_i} m(X_j), \tag{5}$$

where summation (addition) is performed according to Eq. (1). This Eq. (5) allows us to calculate the fuzzy lower probability of the premise clause, e. g., $P_*(X_1)$ in Rule 1 of Fig. 2 providing that the (fuzzy) basic probabilities of the evidential state X have been obtained. By multiplying two fuzzy values $P_*(X_1)$ and C_1 according to Eq. (2), we can determine fuzzy basic probability assignment $m_1(A_1)$ to subset A_1 of Rule 1. The residual amount is assigned to the universe set A_0 of the subgoal state as,

$$m_1(A_0)=1-m_1(A_1), \tag{6}$$

which possibility distribution can be given as,

$$\mu_{m_1(A_0)}(x)=1-\mu_{m_1(A_1)}(1-x). \tag{7}$$

Similarly from Rule 2 and the evidential state Y, another set of (fuzzy) basic probability assignment $m_2(A_2)$ and $m_2(A_0)$ regarding A can be deduced.

These probability assignments deduced from independent evidences X and Y regarding the same subgoal state A can be integrated by the following extended Dempster's rule of combination;

$$m(A_k)=\frac{\sum_{A_i\cap A_j=A_k} m_1(A_i)*m_2(A_j)}{1-\sum_{A_i\cap A_j=\phi} m_1(A_i)*m_2(A_j)}, \tag{8}$$

($A_k\neq\phi$(empty)),

($i=1, 0$ and $j=2, 0$ in this case),

where the summation and multiplication are performed according to Eq. (1) and (2). The division in Eq. (8) can be treated similar to Eq. (2) by changing * to /.

If the relation of two evidential states is AND (OR), the fuzzy basic probability assignment of the subgoal can be obtained by taking minimum (maximum) of two fuzzy probabilities according to Eq. (3)

(4).

Thus the rational inference method has been defined for uncertain and fuzzy problems represented by the AND/OR/COMB graph. As a result of the inference, we can have a reasonable answer based on the (fuzzy) certainty measure of final goal.

4. Conclusion

The rational inference mechanism based on the extended Dempster & Shafer's theory has been described for constructing expert systems which utilize uncertain and fuzzy information. When using the Dempster's rule of combination, the independency among evidences is critical problem in practice. If the independency is doubtful, inference procedure based on fuzzy logic becomes useful.

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