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Pattern of Hydrodynamic Dissipative Structure in a Thin Liquid Layer

液体薄層における流体力学的散逸構造のパターン

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1. Introduction

It is well known that a sufficiently large heat flow [1] or electric current [2] across a thin liquid layer causes macroscopic motion of the medium. This motion of fluid often exhibits a stationary regular pattern such as hexagonal, rectangular, square and plane wave type one. A remarkable example is the electrochemiluminescence pattern due to the convection, where the luminescing points make up a hexagonal lattice [3]. Such a regular pattern, which resembles that of the crystal lattice, is generated only by dissipating energy due to, *e. g.* heat flow or electrical flow, thus it is referred to the dissipative structure in contrast to the crystal structure. The size of the convective unit cell or, more precisely, the lattice constant of the convective pattern has usually been found to be proportional to and somewhat larger than the thickness of the layer [3.4]. This fact seems to be rather common regardless of the mechanism of energy dissipation. The pattern which appears most often is the hexagonal one, but other types of pattern can also observed especially in the case where the surface of the layer is not so large in comparison with the thickness [5], or when the heat or electrical flow across the layer becomes quite large [6]. We discuss here about a favorable type of lattice and its lattice constant for the convective pattern.

2. Molel and Results

Typical convective patterns appearing in a thin liquid layer are shown in fig. 1. We choose, in this figure, the sites of asceniing flow as the lattice points and illustrate the Wigner-Seitz unit cell with broken

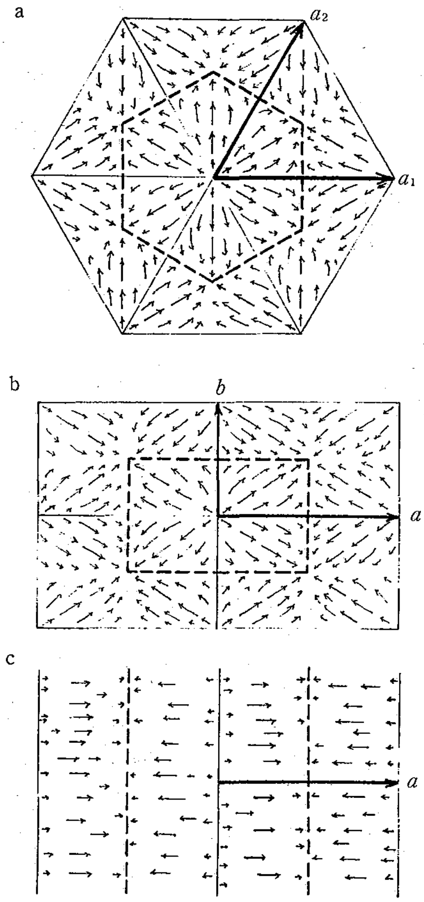

Fig. 1 Typical convective patterns appearing in a thin liquid layer. *a*, hexagonal convective pattern. *b*, rectangular convective pattern. *c*, plane wave type convective pattern. $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$, $\boldsymbol{a}$ and $\boldsymbol{b}$, and $\boldsymbol{a}$ are the fundamental unit vectors for the respective patterns. Fluid is confined within each Wigner-Seitz unit cell (illustrated with broken line), and its convection has the total symmetry in the cell.

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line. The sites of ascending flow make up a two-dimensional lattice, whereas the sites of descending flow make up another two-dimensional lattice, *i. e.* the dual lattice of the former one [7]. For a rectangular or a plane wave type lattice, each dual lattice is topologically identical with the original, *i. e.* they are self-dual. This means that the ascending and descending flow of these patterns are equivalent to each other. For the hexagonal lattice, on the other hand, its dual lattice is a heneycomb lattice (actually this is not a lattice) which differs from the original, and thus it is not nelf-dual. The number of the sites of the descending flow is twice that of the ascending flow.

These stationary fluid motions are expressed with the velocity of the fluid $v(\boldsymbol{r})$, and we assume that the z-component $v_z(\boldsymbol{r})$ can be separated into the horizontal (x, y) component and the vertical (x) component as

$$v_z(x, y, z)=v\cdot f(x, y)g(z), \qquad (1)$$

where the z-direction is taken perpendicular to the layer surface. Then the function $f(x, y)$ should satisfy the periodical condition of the lattice with lattice constants a and b, and the function $g(z)$ should satisfy the boundary condition at the surfaces ($z=\pm c/2$), where c is the layer thickness. As the components of a larger wave number in $f(x, y)$ and $g(z)$ are in general unfavorable for a stationary state because they might require a larger dissipation of energy, we take terms of the smallest wave number into consideration. Thus, by taking account of the total symmetry of the flow within the Wigner-Seitz unit cell, $f(x, y)$ is expressed for the hexagonal, rectangular and plane wave type lattices as

$$\begin{aligned} f_h(x, y, a)&=(1/6)\sum_{i=1}^{6}\exp(i\boldsymbol{a}_i^*\boldsymbol{r}_1),\\ f_r(x, y, a, b)&=A\cos \boldsymbol{a}^*x+(1-A)\cos b^*y, \qquad (2)\\ f_p(x, a)&=\cos a^*x,\end{aligned}$$

respectively, where $\boldsymbol{r}_1\equiv(x, y)$, $\boldsymbol{a}_i^*(i=1, 2, \cdots, 6)$ are the six reciprocal fundamental unit vectors with length $4\pi/\sqrt{3}a$, and a^* and b^* are the reciprocal lattice constants. By applying a rigid boundary condition $v_z=\partial v_z/\partial z=0$ at $z=\pm c/2$, $g(z)$ is expressed as

$$g(z, c)=(1/2)[1+\cos(c^*z)], \qquad (3)$$

where $c^*\equiv 2\pi/c$. From eqs. (2) and (3) we have differential equations for f and g as

$$\begin{aligned}(a^{*-2}\partial^2/\partial x^2+b^{*-2}\partial^2/\partial y^2+1)f_i&=0,\\ (\partial^2/\partial z^2-c^{*2})g&=-(1/2)c^{*2},\end{aligned} \qquad (4)$$

where i represents h, r or p for the haxagonal, rectangular or plane wave type pattern, respectively, and $a^*=b^*$ when $i=h$. From eq. (4), together with the continuity condition of incompressible fluid flow div $\boldsymbol{v}=0$, each velocity component of the fluid motion is obtained as

$$\begin{aligned} v_x&=va^{*-2}(\partial f_i/\partial x)(dg/dz),\\ v_y&=vb^{*-2}(\partial f_i/\partial y)(dg/dz), \qquad (5)\\ v_z&=vf_ig.\end{aligned}$$

A flow with a certain value takes in general the path that requires the minimum work [8]. If the total flow of heat or electrical charge by the convection expressed by eq. (5) remains unchanged during the lattice constants displacement, the lattice constant of a pattern should be so determined that the work which is needed to maintain the flow should be minimum. The amount of the heat generated by the convection of eq. (5) in a unit volume and unit time is given by the viscous dissipative function [9], $\phi=(1/2)\eta\sum(\partial v_i/\partial x_i+\partial v_j/\partial x_i)^2$. When a convective state is stationary, the mean dissipative function, $\langle\Phi\rangle\equiv\int_{unit\ cell}\Phi(x, y, z)dV/\int_{unit\ cell}dV$, corresponds to the work as mentioned above. It can be shown that this function has a minimum value of $\langle\Phi\rangle_m=(\pi^2/2)(1+3^{-1/2})\eta v^2/c^2$ at $a=2\times 3^{-1/4}c$ for the hexagonal pattern, $\langle\Phi\rangle_m=(\pi^2/2)3^{1/2}\eta v^2/c^2$ at $a=b=3^{1/4}c$ for the rectangular one and $\langle\Phi\rangle_m=\pi^2 3^{1/2}\eta v^2/c^2$ at $a=3^{1/4}c$ for the plane wave type one. These results are tabulated in table 1.

3. Discussion

It should be noted that although the lattice constant

Table 1 Lattice constants and mean viscous dissipative functions

	lattice constant (layer thickness unit)	$\langle\Phi\rangle_m$ ($\eta v^2/c^2$ unit)
hexagonal	1.520	7.784
square	1.316	8.547
plane wave	1.316	17.095

might take various values according to the convective patterns, the wave numbers of these patterns have the same magnitude of $k_c = 2\pi 3^{-1/4}/c$. That is, three stationary plane wave components with the wave number k_c, which are equivalent to each other, make up the hexagonal pattern, and similarly two equivalent stationary plane wave components with k_c make up the square pattern, and one statitnary component with k_c itself means the plane wave type pattern. In other words, the kind of pattern just depends on the number of the plane waves which are superimposed, and the size of the pattern is decided by the critical wave number k_c. According to the criterion mentioned above, table 1 indicates that the hexagonal pattern might be the most favorable for the convection. This seems likely if we consider the fact that the hexagonal pattern has the highest symmetry in the two-dimentional lattice. Of cource, this conclusion premises that the three stationary plane wave components should be equivalent to each other. If these components cannot be equivalent owing to a boundary condition by side wall, symmetry of the pattern will be degraded. So the regular hexagonal convective pattern can most frequently be observed in a thin layer cell with semi-infinite surface area. A similar degradation in the pattern symmetry has also been known to take place when the flow becomes quite large. The most common example is a transition from the hexagonal convective pattern to the plane wave type (or the roll type) one that takes place when the applied voltage on a thin liquid layer cell is fairly increased [6]. This might be a sort of "the Jahn-Teller effect" distinct from the boundary condition effect mentioned above. Thus, we may conclude that the hexagonal convective pattern with the wave number of $4.774/c$ will appear in a thin liquid layer with a sufficiently large surface area just after the transition from the non-convective state, and there may be another transition from the hexagonal pattern to the plane wave type one when the flow becomes larger.

Which of the ascending and descending flow sites makes up the hexagonal lattice or its dual lattice, the honeycomb lattice, is another problem. In the present case, a stream tube is a closed one, and the fluid is circulating in this tube. The driving force for this circulation, of course, depends on each mechanism, but this force might be applied mainly on either the ascending or descending parts of the tube, or on both of them. If the force is applied on both of them equally, we cannot say anything about this problem. If the force is applied on, e. g. the ascending flow parts, these will make up the hexagonal lattice and the descending flow sites will make up the honeycomb lattice, because the ascending flow rate at the hexagonal lattice points, the number of which is a half of the number of its dual lattice points, is twice the descending flow rate at the honeycomb lattice points.

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