研　究　速　報

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An Extension of Dempster & Shafer's Theory to Fuzzy Set for Constructing Expert Systems

エクスパートシステム構築のための Dempster & Shafer 理論のファジィ集合への拡張

Mitsuru ISHIZUKA*
石　塚　　満

1. Introduction

Efficient knowledge utilization of human experts is the central issue of expert system, in which artificial intelligence techniques are applied to solve complex problems in the real world. Studies relating to the construction of the expert system is called knowledge engineering. The expert system basically consists of knowledge base and inference machine. Useful knowledge for solving the problem is stored in the knowledge base in a stilized format. The inference machine deduces an answer for a given problem situation by using the knowledge. Figure 1 shows a simplified diagram of the expert system.

The basic mechanism of the inference machine is symbol manipulation which tries to search a way to reach a goal state from a given state. However, this basic mechanism alone is not sufficient, because uncertain and/or fuzzy information is involved in many real-world problems. This paper describes a rational inference mechanism for the expert systems which utilize uncertain and/or fuzzy knowledge and evidences[1,2]. This mechanism is based on the extension of Dempster & Shafer's theory and has been successfully employed in SPERIL[3,4,5] which is a rule-based damage assessment system for existing structures.

2. Problem Reduction with Combination Relation

In a complex problem, it is an efficient way to express relevant knowledge as a collection of many small pieces of knowledge. Problem reduction method can be used as a guideline to decompose a problem into simpler subproblems, which are further decomposed into even simpler subprobrems. Hence the whole problem can be described hierachically, and it has its own final goal to be achieved. Likewise, earch subproblem has its own subgoal to be achieved

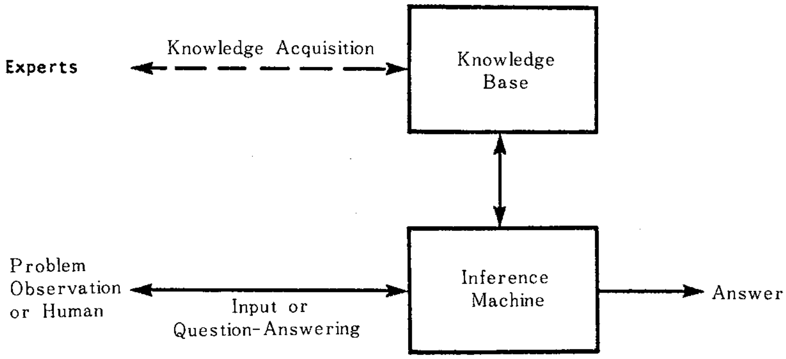

Fig. 1　Expert system.

* Dept. of Electrical Engineering and Electronics, Institute of Industrial Science, University of Tokyo.

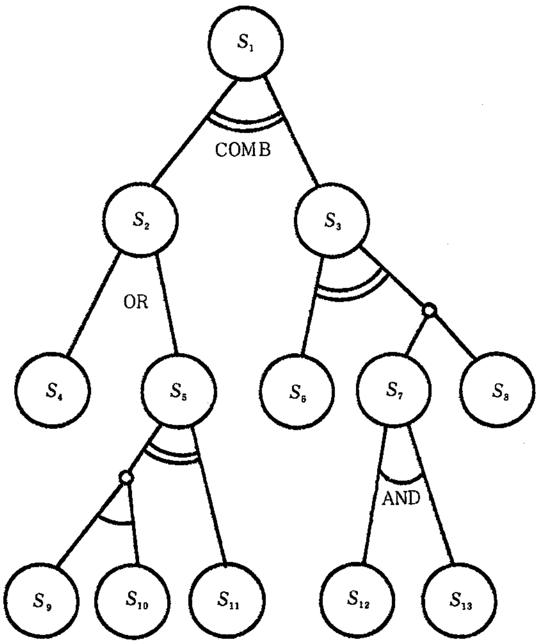

Fig. 2 An example of AND/OR/COMB graph for a problem with uncertainty.

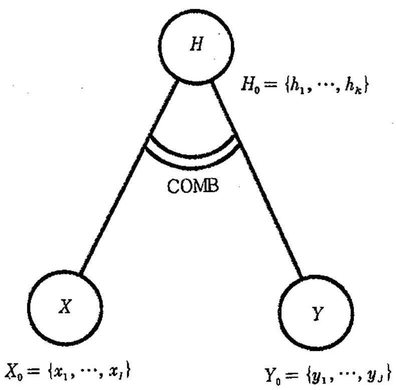

Fig. 3 Inference from the combination of two different evidences.

from available information. The production system provides a convenient formalization to express a piece of knowledge in a rule format for the inference process which infers a higher subgoal from observed evidences and lower subgoals.

There exist two kinds of uncertainties in many real-world problems. One is the uncertainty associated with the observed evidences; the other one is the uncertainty associated with the expressed rule. In such a situation, combination relation denoted by COMB in 1) becomes important in addition to AND/OR relations. The combination relation refers to such a decomposition that the goal is supported separately from plural uncertain evidences and rules. As a result, the problem can be described by AND/OR/COMB graph as shown in Fig. 2.

Inferences for AND and OR relations are rather simple; min and max operations on a certainty measure can be adopted, respectively. Therefore, a rational inference for COMB relation is required to be defined along with the certainty measure.

Consider the fundamental case of Fig. 3 where two independent evidential states X and Y are observed or already inferred from preceding inference. Suppose that we have the following rules;

Rule 1

IF : X is X_1
THEN : H is H_{x1} with C_{x1}
ELSE IF : X is X_2
THEN : H is H_{x2} with C_{x2}
⋮
ELSE : H is H_0,

Rule 2

IF : Y is Y_1
THEN : H is H_{y1} with C_{y1}
ELSE IF : Y is Y_2
THEN : H is H_{y2} with C_{y2}
⋮
ELSE : H is H_0,

where $X_1, X_2, Y_1, Y_2, H_{x1}, H_{x2}, H_{y1}$ and H_{y2} are assumed to be subsets of finite universe sets of X_0, Y_0 and H_0, respectively, and C_{x1}, C_{x2}, C_{y1} and C_{y2} are certainty measures of [0, 1]. Now the question is that how should we infer the certainty measure of hypothetical or subgoal state of H.

An intuitive combining function is employed in MYCIN[6] for this purpose. An inference procedure named subjective Bayesian method has been also proposed[7]. Combining functions for the Bayesian and a modified Bayesian probabilities have been reported by the author et al[8),9)]. On the other hand, the usefulness of Dempster & Shafer's theory is recently recognized[1)] and employed in SPERIL. This theory enables us to deal with subjective uncertainty in a rational manner.

Once the inference mechanism for the COMB

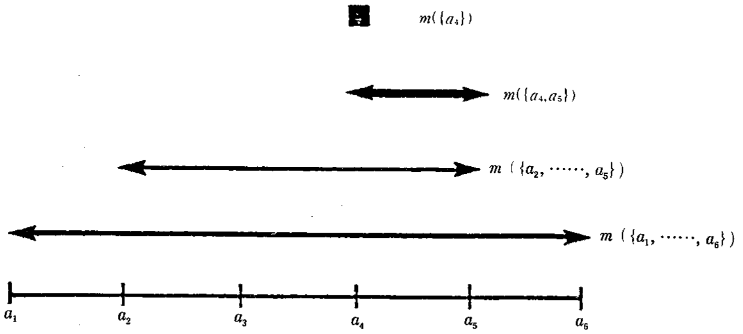

Fig. 4 An image of basic probability $m(A_i)$. $A_0=\{a_1, \cdots, a_6\}$

relation is defined as well as AND/OR relations, the certainty measure can propagate through the hierachical inference network. Eventually, we can obtain the degree of certainties of hypotheses in the final goal, which will provide a reasonable answer for decision-making purpose.

3. Dempster & Shafer's Theory and its Extension

The main criticism regarding the use of the Bayesian probability to express subjective uncertainty is that it cannot be used to deal with ignorance in an effective manner. In other words, the Bayesian theory cannot distinguish between the lack of belief and disbelief, because it requires the relation of $P(A)+P(\bar{A})=1$.

In 1967, Dempster[10] proposed a useful concept named lower and upper probabilities. Shafer refined the Dempster's theory in his book[11].

According to Shafer, a basic probability $m(A_i)$ can be visualized as a semi-mobile probability mass which is confined to subset A_i but can move freely to every points of A_i. This can be depicted graphically as shown in Fig. 4. The lower probability is defined as,

$$P_*(A_i)=\sum_{A_j\subseteq A_i} m(A_i), \qquad (1)$$

that is, the sum of the basic probabilities confined within the subset A_i.

If m_1 and m_2 are the basic probabilities inferred from independent evidences, then Dempster's rule of combination tells that a new basic probability can be obtained by combining m_1 and m_2 as,

$$m(A_k)=\frac{\sum_{A_{1i}\cap A_{2j}=A_k} m_1(A_{1i})m_2(A_{2j})}{1-\sum_{A_{1i}\cap A_{2j}=\phi} m_1(A_{1i})m_2(A_{2j})}. \quad (A_k\neq\phi) \qquad (2)$$

The application of this theory to the inference in the expert system is rather straightforward. An inference procedure is as follows. For Rule 1, first calculate the lower probability of each premise $P_*(X_i)$ $(i=1,2,\cdots)$, then multiply this by the certainty measure C_{xi} and assign this amount to the basic probability of H_{xi} as,

$$m(H_{xi})=P_*(X_i)C_{xi}. \qquad (3)$$

Similarly, from Rule 2 and the evidential state Y, $m(H_{yi})$ can be deduced. These probability assignments regarding H from independent evidences can be integrated by using Eq. (2).

In addition to uncertainty, it is sometimes appropriate to express the rules with fuzzy subsets rather than crisp subsets. For example, the expression of slight, moderate or severe damage used in SPERIL is not well defined but meaningful for human experts. Thus we will extend the Dempster & Shafer's theory to include fuzzy subsets without losing its essence.

Define the degree that a fuzzy subset A_1 is included in another fuzzy subset A_2 of the same universe set A_0 as,

$$I(A_1\subseteq A_2)=\frac{\min_a\{1, 1-\mu_{A_1}(a)+\mu_{A_2}(a)\}}{\max_a\{\mu_{A_1}(a)\}}. \qquad (4)$$

where $\mu_{A_1}(a)$ and $\mu_{A_2}(a)$ are membership functions chracterizing A_1 and A_2, respectively.

Define the degree of intersection of two fuzzy subsets A_1 and A_2 as,

$$J(A_1, A_2)=\frac{\max_a\{\mu_{A_1\cap A_2}(a)\}}{\min\{\max_a\{\mu_{A_1}(a)\}, \max_a\{\mu_{A_2}(a)\}\}}, \qquad (5)$$

where

$$\mu_{A_1\cap A_2}=\min\{\mu_{A_1}(a), \mu_{A_2}(a)\}. \qquad (6)$$

The degree that the intersection of A_1 and A_2 is ϕ (empty) is defined as $1-J(A_1, A_2)$.

Using these definitions, Eqs. (1) and (2) can be generalized respectively to,

$$P_*(A_i)=\sum_{A_j} I(A_j \subseteq A_i) m(A_j), \quad (7)$$

$$m(A_k)=\frac{\sum_{A_{1i}\ A_{2j}=A_k} J(A_{1i}, A_{2j}) m_1(A_{1i}) m_2(A_{2j})}{\sum_{A_{1i}, A_{2j}} \{1-J(A_{1i}, A_{2j})\} m_1(A_{1i}) m_2(A_{2j})}. \quad (8)$$

Thus the inference procedure with uncertainty and fuzziness is given theoretically.

4. Concluding Remarks

A rational inference procedure based on the extension of Dempster & Shafer's theory has been developed to construct the expert system which utilizes uncertain and fuzzy information. As an alternative of the statistical inference methods which often require idealized conditions such as independency of evidences, inference procedures based on fuzzy logic become effective[1),12)]. Some possible applications of fuzzy set theory in civil engineering problems are described in 13), 14), 15).

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