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Sliding Mode in a Position Control Servo System
位置サーボ系におけるスライディングモード制御

Fumio HARASHIMA*, Okyay KAYNAK** and Michitaka OKUDA***
原島文雄・オキヤイカイナック・奥田通孝

Introduction

In many control applications, it has always been a challange to the control engineers to design systems that will be insensitive to parmeter variations and disturbances. A method of control, called the sliding mode control, is suggested to overcome these difficulties, in which the representative point of the system is constrained to move along a predetermined hyperplane or hyperplanes in state space. In this way parameter insensitivity and disturbance rejection can be ensured. In order to achieve such a sliding regime, the control law is required to have a discontinuous nature, i. e. the system structure needs to be changed in time. Such a system is called a variable structure system (VSS).

The theory of VSS has been well explored, almost exclusively by the scientists of the USSR, over the past two decades[1)~4)]. However only a few practical results can be seen in literature[5)~7)] and in those not all the possible control law structures are explored. There exists a gap between the theory and practice, as it always does, requiring more detailed practical investigations to demonstrate the effects of the various terms in the control law and to set rules of thumb for design.

In this paper, various different control laws, including one with a switched current feedback term, are employed to establish a sliding regime in a position control servo system subjected to a heavy disturbance. The experimental results are presented and discussed. It is shown that the control system can be made robust by a suitable choice of the control law yielding very small steady state and dynamic errors.

* Dept. of Power Electronics and Control Engineering Inst. of Industrial Science, Univ. of Tokyo.
** Electrical Engineering Dept. Boğaziçi Univ., Istanbul, Turkey.
*** Japan Radio Co. Design Dept.

Description of the System

The block diagram of the system is show in Fig. 1. A d. c. servo motor, modelled as a first order system neglecting the electrical time constant, is driven by a PWM power MOSFET chopper amplifier operating at a frequency of 10 kHz. A 10 bit digital shaft encoder together with a D/A convertor is used to sense the output position while a tachometer provides the speed signal. The block shown as the controller consists of various op-amps, comparators and relays to achieve the control law desired.

The arrangement shown in Fig. 2 was used to generate a disturbance, the maximum value of which could be varied by varying the mass M. Although the disturbance is now a sinusoidal function of the output position and it is one of the state variables of the system, the treatement of it as such would result in non-linear equations and complicate the analysis. It was decided to treat it as a constant, taking the maximum value, and all the tests were carried out around ∓90° output position which is the worst condition.

The phase variable state representation of the system can then be written as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -b\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ -a\Phi\end{bmatrix}[u]+\begin{bmatrix}0\\ 1\end{bmatrix}[f] \quad (1)$$

where

$$x_1=\text{pos. error},\quad x_2=\text{error vel.},$$
$$a=K_TK_E/R_AJ=1.75,$$
$$b=(K_TK_E+DR_A)/JR_A=95, \quad (2)$$
$$\Phi=60,\quad f=\mp\max|K_GF/J|=\mp 1.25.$$

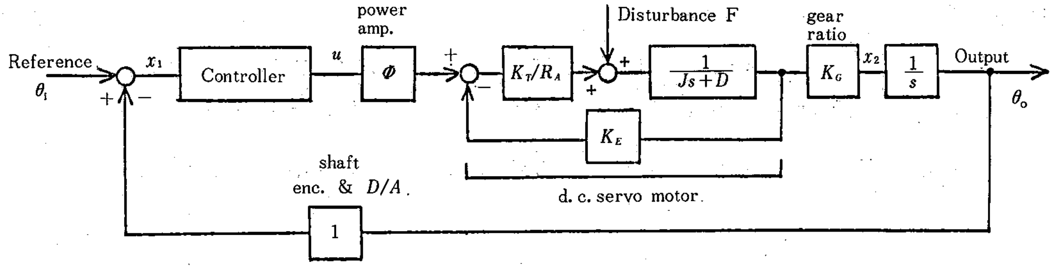


Fig. 1　Block diagram of the system.

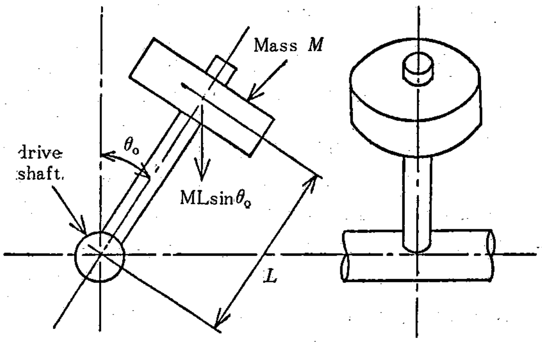


Fig. 2　Arrangement for generation of disturbance.

Design and Experimental Investigations

Four different control law structures were used for the experimental investigations and they are explained step by step below. The reachability, the existence and the stability of the sliding mode are also discussed.

The simplest control law that could be used would be of the form

$$u=\Psi_1 x_1 \tag{3}$$

where the coefficient Ψ_1 is discontinuous, given by

$$\Psi_1=\begin{cases}\alpha_1 & \text{when } sx_1>0\\ \beta_1 & \text{when } sx_1<0\end{cases} \tag{4}$$

s defining the switching line

$$s=x_2+cx_1,\quad c>0, \tag{5}$$

$c>0$ is a condition required for the stability of the motion along the switching line.

For the second order system under study, the sliding line is always reached. Therefore it is not considered any further.

The existence of the sliding mode requires[1),2)]

$$\lim_{s\to+0}\dot{s}<0 \text{ and } \lim_{s\to-0}\dot{s}>0 \text{ or } \lim_{s\to0}s\dot{s}<0 \tag{6}$$

which dictates that, taking $f=0$ for the time being,

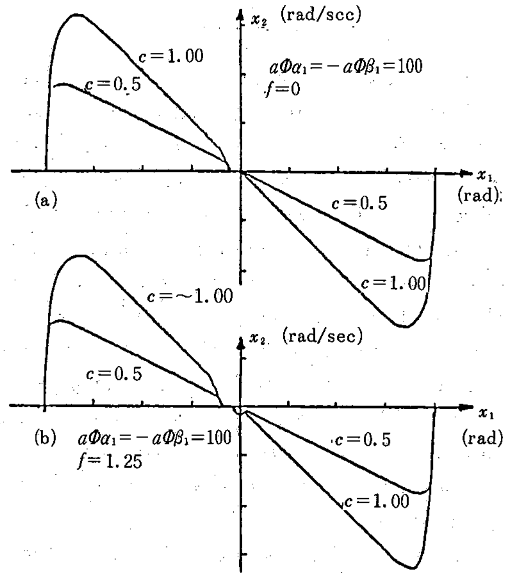


Fig. 3　Phase plane trajectory. Control law $u=\Psi_1 x_1$ [$\theta_i(-0)=1.00$, $\theta_i(+0)=2.00$ or $\theta_i(-0)=2.00$ $\theta_i(+0)=1.00$. Scale 1 div.$=0.25$ rad or rad/sec]

$$\begin{aligned}a\Phi\Psi_1&>bc-c^2 \quad \text{if } sx_1>0\\ a\Phi\Psi_1&<bc-c^2 \quad \text{if } sx_1<0\end{aligned} \tag{7}$$

Fig. 3a and 3b show the phase plane trajectories obtained for $a\Phi\alpha_1=-a\Phi\beta_2=100$, the disturbance f being set to zero and full respectively. Because of (7), the maximum value of c that can be used is 1.07.

Fig. 3 indicates a steady state error even in the case when the external disturbance is set to zero. This is due to the non-linearities in the system such as delays, hysterisis, dead zones, etc., which present a virtual disturbance to the system.

The disturbance rejection can be ensured by including a feedforward term in the control law with a constant gain if the disturbance is measurable. However in most practical cases this is not possible and the following control law is suggested[1)~3)].

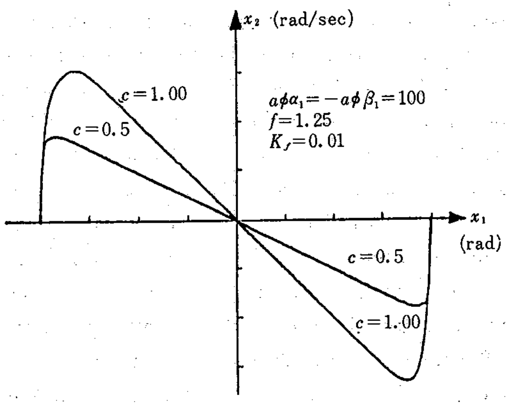


Fig. 4 Phase plane trajectory. [$\theta_i(-0)=1.00$, $\theta_i(+0)=2.00$ or $\theta_i(-0)=2.00$, $\theta_i(+0)=1.00$. Scale 1 div. $=0.25$ rad or rad/sec]

$$u=\Psi_1 x_1+k_f \operatorname{sgn} s, \quad \operatorname{sgn} s=+1 \text{ if } s>0 \quad (8)$$
$$\operatorname{sgn} s=-1 \text{ if } s<0$$

where Ψ_1 has the same structure as before and k_f is a positive number satisfying the inequality

$$a\Phi k_f>\max f. \quad (9)$$

The control law of (8) includes a relay term and like any relay system a too high value of k_f can cause intolerable self sustained oscillations around the origin.

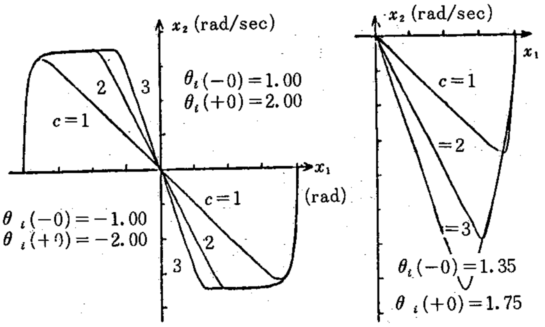
Fig. 4 shows the phase plane trajectory for $k_f=0.010$. This value is less than the theoretically required value of $k_f=0.012$ ($a\Phi=105$) but it however resulted in an error less than the resolution of the 10 bit encoder, i.e. 0.4°. A small amount of self sustained oscillations were experienced on the d.c. motor side but the amplitude was not so big as to overcome the backlash in the gear train. Even the LSB of the encoder was not effected.

With a control law of the form (8), the time constant of the system is given by $1/c$ neglecting the time up to the moment of hitting the sliding line. c cannot be increased indefinitely due to the constraints (7). These constraints can be relaxed by the following control law[1].

$$u=\Psi_1 x_1+\Psi_2 x_2$$
$$\Psi_1=\alpha_1 \text{ if } sx_1>0, \quad \Psi_2=\alpha_2 \text{ if } sx_2>0, \quad (10)$$
$$\Psi_1=\beta_1 \text{ if } sx_1<0, \quad \Psi_2=\beta_2 \text{ if } sx_2<0.$$

The conditions of (6) result in the requirements

$$s\dot{s}=s(\dot{x}_2+c\dot{x}_1)=s(-bx_2-a\Phi\Psi_1 x_1-a\Phi\Psi_2 x_2+cx_2)$$



a) 1 div. $=0.25$ rad or rad/sec　　b) 1 div. $=0.1$ rad or rad/sec.

Fig. 5 Phase plane trajectory. $u=\Psi_1 x_1+\Psi_2 x_2+k_f$ sgn s, $a\Phi\alpha_1=-a\Phi\beta_1=50$, $a\Phi\alpha_2=10$, $a\Phi\beta_2=-100$, $k_f=0.007$]

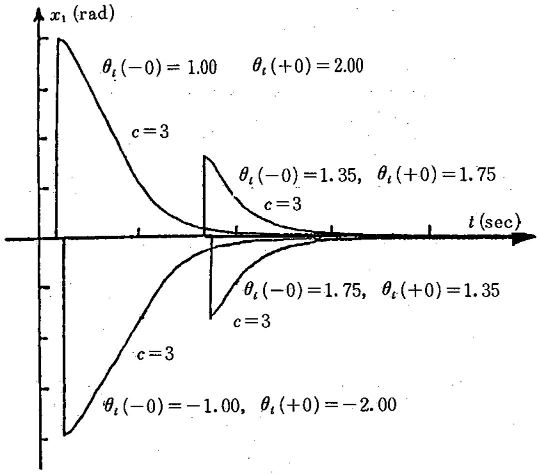


Fig. 6 Error waveform. $u=\Psi_1 x_1+\Psi_2 x_2+k_f$ sgn s. $a\Phi\alpha_1=-a\Phi\beta_1=50$, $a\Phi\alpha_2=10$, $a\Phi\beta_2=-100$, $k_f=0.007$, $f\mp1.25$. Scale 1 div. $=0.25$ rad or 1 sec.

$$=-a\Phi\Psi_1 sx_1+(c-b-a\Phi\Psi_2)sx_2<0$$
$$\text{i.e. } \alpha_1>0,\ \beta_1<0,\ a\Phi\alpha_2>c-b,\ a\Phi\beta_2<c-b. \quad (11)$$

Fig. 5a and 5b shows the results obtained for the case $f=0$. The conditions of (11) require that $a\Phi\alpha_2>-95$ and $a\Phi\beta_2<-95$. A small positive value of α_2 was used to result in positive feedback, reducing the time required to hit the sliding line. For high values of the initial error the power amplifier is saturated resulting in a saturated value of $x_2=8.3$ rpm or around 3700 rpm motor speed since $K_G=1/450$.

Fig. 6 shows some typical responses for $c=3$ and $f=1.25$. It should be noted that the application of the change in reference does not necessarily occur at the

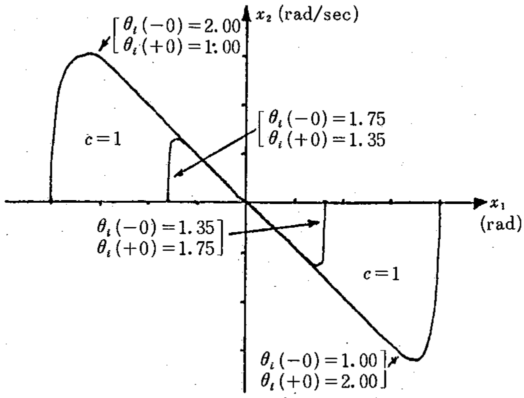


Fig. 7 Phase plane trajectory. $u=\Psi_1 x_1+\Psi_3 i$, $a\Phi\alpha_1=-a\Phi\beta_1=100$, $a\Phi\alpha_3=-a\Phi\beta_3=8$, Scale 1 div.=0.25 rad or rad/sec.

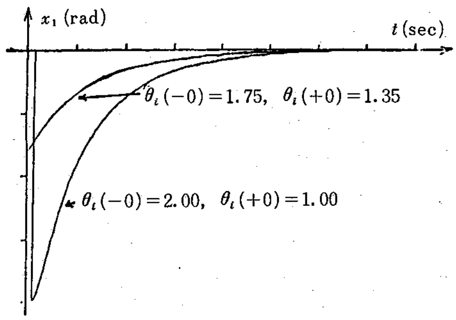


Fig. 8 Error waveform. $u=\Psi_1 x_1+\Psi_3 i$, $a\Phi\alpha_1=-a\Phi\beta_1=100$, $a\Phi\alpha_3=-a\Phi\beta_3=8$, Scale 1 div.=0.25 rad or 1 sec.

origin.

The disturbance rejection can also be effected by switching a feeedbeck around the actuator[1),2)]. In the present system the electrical part of the circuit can be thought as the actuator since it is the part that produces the torque. It was therefore decided to study the effects of a control law of the structure

$$u=\Psi_1 x_1+\Psi_3 i$$
$$\Psi_3=\alpha_3 \text{ if } si>0, \quad \Psi_3=\beta_3 \text{ if } si<0 \qquad (12)$$

Fig. 7 and Fig. 8 show some typical responses obtained, indicating the effectiveness of the current feedback. Although some oscillations were still observed on the motor side around the origin, the amplitude was somewhat less than the case for the control of (8).

Conclusions

This paper demonstrates that a robust system can be arrived at by establishing a sliding regime. All the responses reported showed very small error and the effects of the oscillations on the motor side around the origin was negligible. Higher switching frequency would decrease it even further. In cases where the system is not subjected to heavy disturbances, the control law of the form (8) with a small value of k_f will be sufficient. If a faster system response is required, the error velocity term needs to be included in the control. If the disturbance is high, the form of (12) with a current feedback, which has not attracted any attention yet, results in a very good response. More theoretical work should be carried out in this respect particularly with respect to electrical drives.

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