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Adsorption Uptake in a Shallow Bed Adsorber
微分型吸着層内の吸着量増加

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For recovering a dilute valuable which exists in diverse medium, such as uranium in sea water, adsorption sometimes plays an important role. When large amount of fluid are passed through a solid adsorbent bed, bed depth will be minimized so that the large pressure drop in the bed may be avoided. It is necessary, however, to get a good contact efficiency by selecting proper design parameters and operating conditions.

Since there have been few works designated to describe performances of shallow bed adsorber when the adsorption isotherm is non-linear and the rate of adsorption is controlled both by particle-to-fluid mass transfer and intraparticle diffusion, numerical solutions for Freundlich isotherm systems as well as analytical solutions for special cases are obtained for adsorption uptake in a shallow bed operation.

Basic Equations

For a shallow bed adsorber, concentration in the bed is assumed longitudinally uniform and then material balances in the bed, at particle surface and in the adsorbent particles are

$$F(C_0-C)=V\frac{\partial C}{\partial t}+k_f\cdot A(C-C_i|_{r=R}) \qquad (1)$$

$$k_f\cdot(C-C_i|_{r=R})=D_e\cdot\frac{\partial C_i}{\partial r}\Big|_{r=R} \qquad (2)$$

$$D_e\left(\frac{\partial^2 C_i}{\partial r^2}+\frac{2}{r}\cdot\frac{\partial C_i}{\partial r}\right)=\varepsilon_p\frac{\partial C_i}{\partial t}+\rho_p\frac{\partial q}{\partial t} \qquad (3)$$

with the initial conditions

$$t=0:\ q=C=C_i=0 \qquad (4)$$

Freundlich type adsorption isotherm is assumed where q_0 is the amount adsorbed in equilibrium with inlet concentration, C_0.

$$q/q_0=(C_i/C_0)^{1/n} \qquad (5)$$

In Eqs. (2) and (3), pore diffusion is assumed to be dominant as an intraparticle diffusion mechanism.

CASE I. General case-Adsorption Uptakes for Both Particle-to-fluid and Intraparticle Diffusion Controlling

Dimensionless formes for Eqs. (1) through (5) are as follows, assuming $V\frac{\partial C}{\partial t}=0$ and $\varepsilon_p\frac{\partial C_i}{\partial t}=0$.

$$1-X_L=\beta(X_L-X|_{\rho=1}) \qquad (6)$$

$$B_i'(1-X|_{\rho=1})=\frac{\partial X}{\partial \rho}\Big|_{\rho=1} \qquad (7)$$

$$\frac{\partial^2 X}{\partial \rho^2}+\frac{2}{\rho}\frac{\partial X}{\partial \rho}=\frac{\partial Y}{\partial \tau} \qquad (8)$$

$$Y=X^{1/n} \qquad (9)$$

$$Y=X=X_L=0 \quad \text{at} \quad \tau=0 \qquad (10)$$

where

$$B_i'=\frac{(R^2/D_e)}{\left\{\left(\frac{3W_s}{\rho_p F}\right)+\left(\frac{R}{k_f}\right)\right\}} \qquad (11)$$

Analytical solution for the Eqs. (6) through (11) with $n=1$ is given[1] as

$$\frac{q}{q_0}=1-\sum_{n=1}^{\infty}\frac{6B_i'^2\exp(-\beta_n^2\tau)}{\beta_n^2\{\beta_n^2+B_i'(B_i'-1)\}} \qquad (12)$$

where the $\beta_n s$ are the roots of

$$\beta_n\cot\beta_n+B_i'-1=0 \qquad (13)$$

Numerical solutions can be computed arbitrary n ($1<n<\infty$).

In the case of rectangular isotherm ($n=\infty$), basic equations are modified as follows

$$F(C_0-C)=V\frac{\partial C}{\partial t}+k_f\cdot A(C-C_i|_{r=R}) \qquad (1)$$

$$k_f(C-C_i|_{r=R})=D_e\frac{\partial C_i}{\partial r}\Big|_{r=R} \qquad (2)$$

$$D_e\left(\frac{\partial^2 C_i}{\partial r^2}+\frac{2}{r}\frac{\partial C_i}{\partial r}\right)=0 \quad \text{for} \quad r_i\le r\le R \qquad (14)$$

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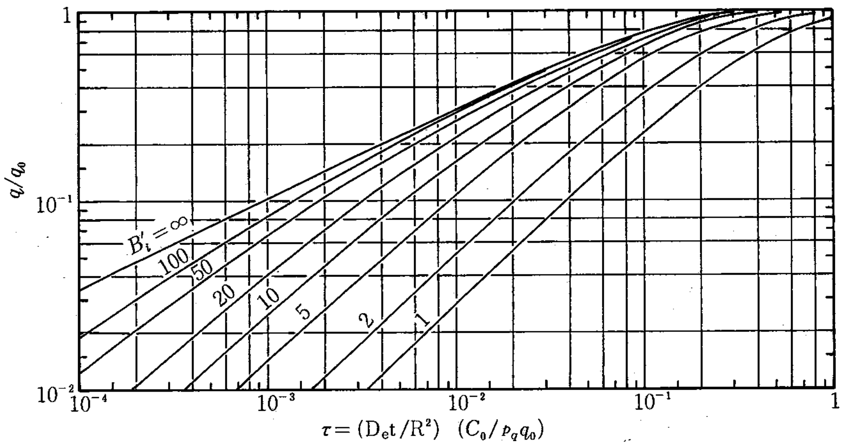

Figure 1 Uptake curve, q/q_0 versus $\tau = (D_e t/R^2)(C_0/\rho_p q_0)$
both particle-to-fluid mass transfer and pore diffusion controlling, for $n=1$ (Eq. (21))

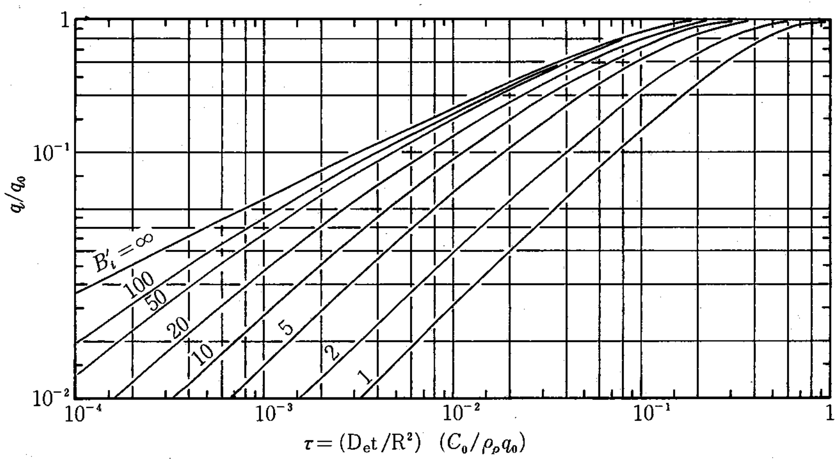

Figure 2 Uptake curve, q/q_0 versus $\tau = (D_e t/R^2)(C_0/\rho_p q_0)$
both particle-to-fluid mass transfer and pore diffusion controlling, for $n=2$

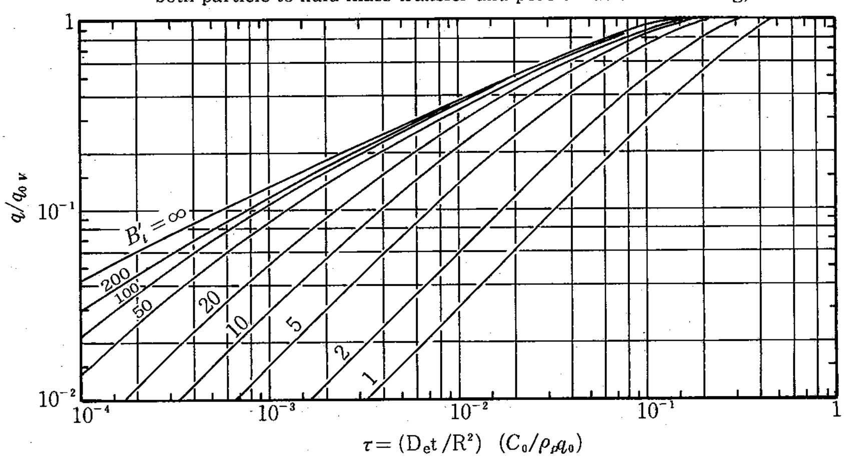

Figure 3 Uptake curve, q/q_0 versus $\tau = (D_e t/R^2)(C_0/\rho_p q_0)$
both particle-to-fluid mass transfer and pore diffusion controlling, for $n=\infty$

$$-\rho_p q_0 \frac{\partial r_i}{\partial t} = D_e \frac{\partial C_i}{\partial r}\Big|_{r=r_i} \tag{15}$$

$$C_i = 0 \quad \text{at} \quad r = r_i \tag{16}$$

$$q = q_0 \quad \text{for} \quad 0 \leq C_i \leq C_0 \tag{17}$$

$$r_i = R \quad \text{at} \quad t = 0 \tag{18}$$

Analytical solution can be obtained for the above set of equations as follows.

$$\tau = \frac{1}{B_i'}\left(\frac{1-\xi^3}{3}\right) - \frac{\xi^2}{6}(3-2\xi) + \frac{1}{6} \tag{19}$$

$$\xi = (1 - q/q_0)^{1/3} \tag{20}$$

The above equations can be derived from a general solution of batch adsorption uptake curves in rectangular isotherm systems given by Suzuki and Kawazoe.[2] Adsorption uptake curves for $n=1,2$ and ∞ are given in Figures 1 to 3.

CASE II. Special Case (I)-Particle-to-fluid Mass Transfer Controlling

The basic equations (1)-(3) are simplified, when particle-to-fluid mass transfer is rate determining, $B_i = k_f R/D_e \ll 1$. Accumulation in void space of the bed and in pores of adsorbent particles are neglected because of its small contribution $\left(V\frac{\partial C}{\partial t} = 0,\ \varepsilon_p \frac{\partial C_i}{\partial t} = 0\right)$.

$$F(C_0 - C) = k_f \cdot A(C - C^*) = W_s \frac{\partial q}{\partial t} \tag{21}$$

C^* is an equilibrium concentration with the amount adsorbed q.

Dimensionless forms of Eqs. (4), (5) and (21) become

$$3\frac{\partial Y}{\partial \tau_F} = 1 - X^* \tag{22}$$

$$1 - X_L = \beta \cdot (X_L - X^*) \tag{23}$$

$$Y = (X^*)^{1/n} \tag{24}$$

$$Y = X^* = X_L = 0 \quad \text{at} \quad \tau_F = 0 \tag{25}$$

where

$$\tau_F = \frac{t}{\left(\frac{3W_s}{\rho_p F} + \frac{R}{k_f}\right)} \cdot \left(\frac{C_0}{\rho_p q_0}\right) \tag{26}$$

$$\beta = k_f \cdot A/F = \left(\frac{3W_s}{\rho_p F}\right) \Big/ \left(\frac{R}{k_f}\right) \tag{27}$$

In the case of a linear isotherm ($n=1$) the analytical solution is possible for the set of Eps. (22)-(27).

$$q/q_0 = 1 - \exp(-3\tau_F) \tag{28}$$

Also analytical soution for $n=\infty$ is

$$q/q_0 = 3\tau_F \tag{29}$$

Adsorption uptakes for $n=1$ and $n=\infty$ in this case are show in Fig. 4. Numerical solutions can be obtained for any n $(1<n<\infty)$, uptake curves for which would be placed between the two curves given in Figure 4.

CASE III. Special Case (II)-Intraparticle Diffusion Controlling

When particle-to-fluid mass transfer is rapid and also fluid flow rate is large enough, the concentration at the particle surface is assumed to be C_0. Then Eqs. (8) and (9) with $X|_{\rho=1}=1$ are solved numerically for several n to give Figure 5.

Again solutions for $n=1$ and $n=\infty$ are already

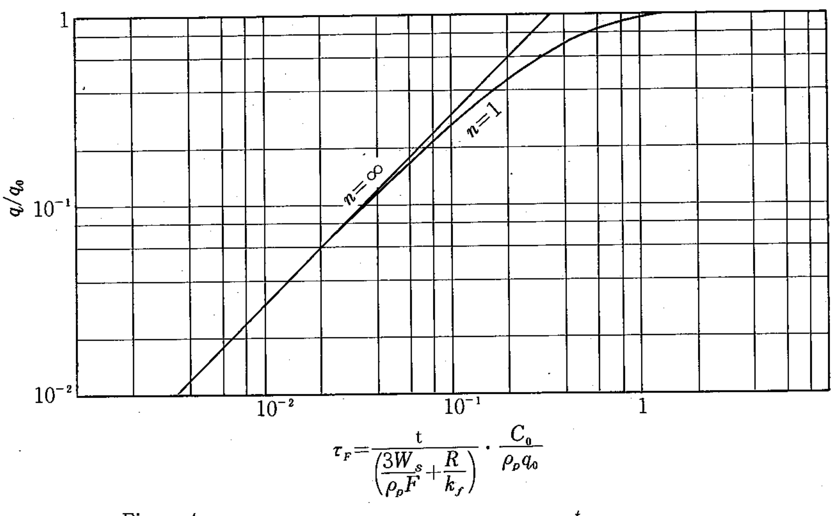

Figure 4 Uptake curve, q/q_0 versus $\tau_F = \frac{t}{(3W_s/\rho_p F + R/k_f)} \cdot (C_0/\rho_p q_0)$ particle-to-fluid mass transfer controlling

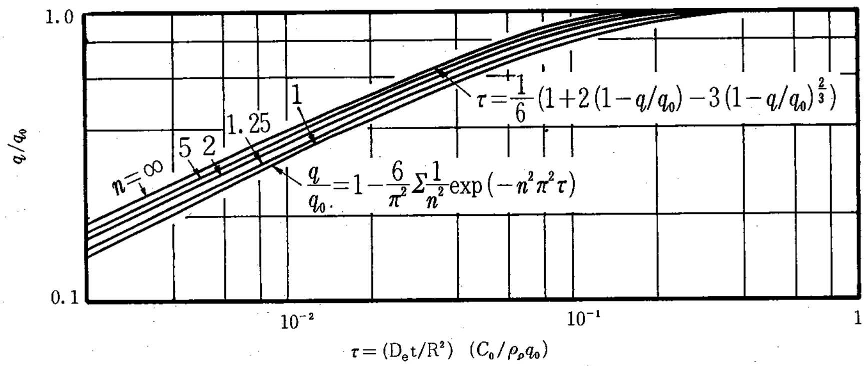

Figure 5　Uptake curve, q/q_0 versus $\tau=(D_e t/R^2)(C_0/\rho_p q_0)$
pore diffusion controlling, $C=C_0=$const or $B_i'=\infty$

given.[1),2)]

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Nomenclature

A : total external surface of adsorbent particles $=(W_s/\rho_p)(3/R)$ (cm³)
B_i : Biot number $(=k_f R/D_e)$ (−)
B_i' : defined by eq. (11) (−)
C : adsorbate concentration in fluid (g/cm³)
C_i : C in the pore of adsorbent (g/cm³)
C_0 : C at the inlet of adsorber (g/cm³)
C^* : C in equilibrium with q (g/cm³)
D_e : effective intraparticle diffusivity (cm³/sec)
F : flow rate (cm³/sec)
k_f : particle-to-fluid mass transfer coefficient (cm/sec)
n : Freundlich constant (−)
q : amount adsorbed (g/g)
q_0 : amount adsorbed in equilibrium with C_0 (g/g)
R : radius of adsorbent particle (cm)
r : radial position in a particle (cm)
r_i : radial position of adsorption front (cm)
t : time (sec)
V : volume of adsorber (cm³)
W_s : amount of adsorbent (g)
X : C_i/C_0 (−)
X_L : C/C_0 (−)
X^* : C^*/C_0 (−)
Y : q/q_0 (−)
β : defined by eq. (27) (−)
β_n : nth root of eq. (13) (−)
ε_p : porosity of particle (−)
ξ : r_i/R (−)
ρ : r/R (−)
ρ_p : particle density (g/cm³)
τ : $(D_e t/R^2)(C_0/\rho_p q_0)$ (−)
τ_F : defined by eq. (26) (−)

Literature Cited

1) Crank, J.: Mathematics of Diffusion, Oxford (1956).
2) Suzuki, M. and K. Kawazoe: J. Chem. Eng. Japan, vol. 7, No. 5 (1974).