研　究　速　報
UDC　621.315.615:537.84.04

On the Electrohydrodynamic Instability of Dielectric Liquids under Space Charge Limited Conduction in the Unipolar Injection Case

単極注入空間電荷制限電導状態における絶縁性液体の電気流体力学的不安定性

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It has been well known that a sufficiently large electric current in a dielectric liquid causes macroscopic motion of the medium[1-3]. Such a macroscopic motion occurs when the applied voltage on a thin liquid layer exceeds a certain critical value, and it often exhibits a stationary regular hexagonal pattern[4-6]. The size of the hexagons or, more precisely, the lattice constant of the hexagonal convective pattern has been usually found to be proportional to and somewhat larger than the layer thickness[5,6]. Theoretical consideration on the phenomenon also showed that the lattice constant, or the inverse of the wave number of fluid convection, is proportional to the layer thickness[2,7]. We discuss here on the transition from the non-convective state to a convective one in the case of the space charge limited current in view of the facts mentioned above.

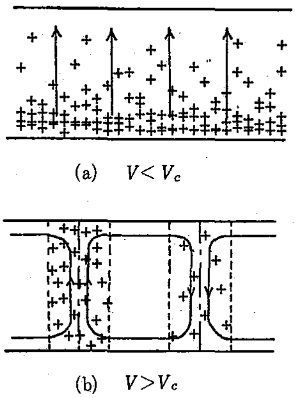

Fig. 1 Schematic illustration of the charge transport in a thin liquid layer: (a) static state of $V<V_c$ and (b) convective state of $V>V_c$.

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In the case where applied voltage, V, on a thin liquid layer is smaller than a certain critical voltage, V_c, there is no macroscopic fluid movement, and only the ions injected from one electrode are transported under an electric field (Fig. 1(a)). In this case, the space charge limited current density, i_1, is expressed by Langmuir-Child's law[8] as

$$i_1=\frac{9}{8}\frac{\varepsilon\mu}{c^3}V^2, \tag{1}$$

where μ is the mobility of the injected ions, ε is the dielectric constant of the fluid, and c is the thickness of the layer.

In the case of $V>V_c$, the fluid begins to move and eventually makes a stationary hexagonal pattern. Assuming the lattice constant, a, to be $a=2\times3^{-1/4}c=1.520\,c$[7], the energy W_i, which is dissipated by friction in the unit cell per unit time is expressed as

$$W_i=\int_{\substack{\text{unit}\\ \text{cell}}}\Phi(x,y,z)\,dV=\pi^2\left(1+\frac{1}{\sqrt{3}}\right)\eta v^2 c, \tag{2}$$

where $\Phi(x,y,z)$ is the viscous dissipation function[9], η is the viscosity of the liquid, and v is the maximum velocity of the flow.

In order to calculate the energy, W_e, which comes into the unit cell in an unit time from an external power supply, we use the following simple model for the convection. Along a stream line, there is no change in the charge density, ρ,

$$\frac{d\rho}{dt}=\boldsymbol{v}\ \mathrm{grad}\ \rho=\mathrm{div}(\rho\boldsymbol{v})-\rho\ \mathrm{div}\ \boldsymbol{v}=0,$$

except the vicinity of the two electeodes, by virtue of the continuity of the flow of the fluid (div $v=0$) and that of charge (div $(\rho v)=0$) at a stationary state. Now we assume that the unit cell consists of an ascending and descending flow regions and a static one (Fig. 1(b)). Let us consider the case where positive charge is injected from the anode at $z=0$. The charge density, ρ, in the ascending region is uniform and equal to $2\varepsilon V/c^2$. On the other hand, as we consider the unipolar injection case, the charge density, ρ', in the descending region is $\rho'=(1-\kappa)\rho$, where κ is the efficiency of discharge of cations at the cathode at $z=c$. Therefore, the electric field is expressed as

$$E(z)=\frac{\rho}{\varepsilon}z=\frac{2V}{c^2}z$$

for the ascending region and

$$E'(z)=\frac{\kappa V}{c}+(1-\kappa)\frac{2V}{c^2}z,$$

for the descending region. The force by the external field on the fluid in the stream tube of a unit cell along a stream line can be obtained as

$$F=\frac{1}{2}S\int_0^c \rho E(z)\,dz-\frac{1}{2}S\int_0^c \rho' E'(z)\,dz$$
$$=S\kappa\frac{\varepsilon V^2}{c^2},$$

where S is the sum of the cross section of the first and second regions, i.e. the conducting area in a unit cell and the two areas are assumed to be same to each other. Now we can calculate the work done by the external field on the fluid in a unit cell per unit time as

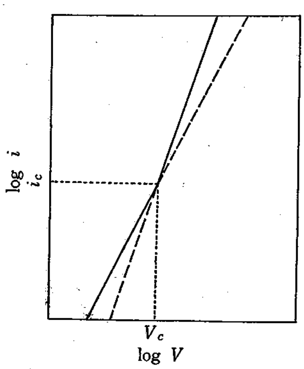

Fig. 2 Plot of current density versus applied field.
A: non-convective state; B: convective state.
The transition between the two states occurs at $V=V_c$.

$$W_e=vF=\kappa v\varepsilon V^2, \qquad (3)$$

where S is assumed to be a half of the total cross section of a unit cell i.e. $S=c^2$. Since W_e and W_i should be equal to each other in the steady state, we obtain

$$v=\frac{\kappa}{\pi^2\left(1+\frac{1}{\sqrt{3}}\right)}\frac{\varepsilon V^2}{\eta c}=0.064\frac{\kappa\varepsilon V^2}{\eta c}. \qquad (4)$$

Therefore, the average electric current density of the convective state is

$$i_2=\frac{1}{4}\kappa\rho v=0.032\kappa^2\frac{\varepsilon^2 V^3}{\eta c^3}. \qquad (5)$$

The current density in the non-convective state and that in the convective state are plotted against applied voltage in Fig. 2. The voltage, V_c, and the current density, i_c, at the crossing of the two curves are respectively

$$V_c=\frac{27.8}{\kappa^2}\frac{\eta\mu}{\varepsilon} \qquad (6)$$

and

$$i_c=\frac{1236}{\kappa^4}\frac{\eta^2\mu^3}{\varepsilon c^3}. \qquad (7)$$

Because a system takes the state that has largest conductivity under a constant applied field and temperature [10], the transition from the static state to the convective state takes place at $V=V_c$. Consequently, we can take

$$R=\frac{\varepsilon V}{\eta\mu}, \qquad (8)$$

as a dimentionless parameter for the transition. The parameter derived here coincides with one derived by Félici [1] and Schneider and Watson [2]. The critical value of the parameter is

$$R_c=\frac{27.8}{\kappa^2}. \qquad (9)$$

Eq.(9) means that the value of the dimensionless parameter at the critical point is inversely proportional to the square of the efficiency of discharge. Schneider and Watson estimated the parameter as $R_c=99$ which corresponds to $\kappa=0.53$. Although this numerical value for κ is considerably large as the efficiency of discharge under convection, it might be due to the fact that they took into consideration only the case of the infinitesimal convection.

The usual models which have been applied by several authors [2-4] can not properly treat a stationary convection but only infinitely small fluctuation. Although the present model seems too simple, it is easy to understand its physical meaning without manipurating complex calculations.

(Manuscript received, October 16, 1981)

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[10] This fact has been recognized on the basis of Thomson's theorem in the electrostatics. See for example: H. Takahashi, *"Electromagnetism"* p. 130, Shokabo, Tokyo, 1963