研　究　速　報
UDC 530. 152. 1. 083. 2

New Method for the Calibration of the Inertia of Resonant-Column Devices

共振法土質試験機における貫性の新しい検定法

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Introduction

Resonant-column devices are popularly used to evaluate the shear moduli and damping characteristics of soils at small strain amplitudes. The most popular device is one where the base platen (pedestal) is fixed to a rigid base and the top mass is excited rotationally by a sinusoidal excitation force (Fig. 1). The Hardin-type resonant-column device (Hardin and Music, 1965) has a torsional spring which connects the top mass with an unmoving mass, while the Drnevich-type resonant-column device (Drnevich and Richart, 1970) does not have this spring.

In general, it is necessary to accurately know the value of the inertia of the top mass (J_A) of the apparatus in order to evaluate shear modulus values from resonant frequency valuse. However, the shape of this top mass is not uniform and it is very difficult to determine the value of J_A precisely from physical measurements and from values of the density of the mass. Therefore, this paper describes a new and simple method for calibrating J_A for a resonant column system such as illustrated in Figure 1.

Theory

To evaluate shear modulus values of soil specimens, a resonant-column device with a cylindrical specimen can be modeled by the system shown in Figure 1. The physical constants of the apparatus are the rotational inertia of the top mass (J_A), the torsional spring (K_s), the sinusoidal excitation force (M)

$$M=M_0e^{i\omega t}$$

where M_0 is the force amplitude, ω is the circular frequency, and t is the time.

The physical constants of the specimen are the rotational inertia (J), the density (ρ) the shear modulus (G) and the length (L). Note that the damping capacity of both the device and the specimen are not needed to describe the system shown in Figure 1. $\pi/2$ is the frequency at which the phase difference between the sinusoidal excitation force ($M=M_0e^{i\omega t}$) and the rotational acceleration at the top mass is not affected by the damping capacity of the device and/or the damping capacity of the soil specimen. Therefore, when the resonant-frequency (f) of the system is identified by the phase difference of $\pi/2$, the elastic solution for the system shown in Figure 1 can be used to evaluate the shear modulus of a specimen. This elastic solution is (Hardin and Music, 1965):

$$G=\rho(2\pi f_n L/F)^2 \qquad (1)$$

f_n is the frequency where the plase difference between the sinusoidal excitation force and the rotational acceleration at the top mass reaches $\pi/2$ and F is a dimensionless frequency factor which is obtained from the following equation:

$$F\tan F=\frac{J}{J_A-K_s/(2\pi f_n)^2} \qquad (2)$$

The value of K_s can be related to the resonant frequency of the system without a sample (f_{0T}) as follows (see Fig. 2).

$$f_{0T}=\sqrt{K_s/J_A}\,/2\pi \qquad (3)$$

Equation (2) can be rewritten by using equation (3) as

$$F\tan F=\frac{J}{J_A[1-(f_{0T}/f_n)^2]} \qquad (4)$$

Calibration Methods for Determining the Rotational Inertial of the Top Mass (J_A)

Drnevich et al. (1978) have proposed a method for calibrating the rotational inertia of the top mass (J_A) where a metal calibration sample was used. In their method, a value of J_A was evaluated by measuring the resonant frequency of the testing system including the calibration sample (Fig. 3). The top portion of the calibration sample

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was made the same size as the cap used for testing.

A top mass consists of a sample cap, whose inertia is J_1, and a moving mass attached to the device, whose inertia is ΔJ. Therefore, J_A is the summation of J_1 and ΔJ. The value of J_A can be evaluated from the resonant frequency of the system with the calibration sample(f_{n1})from equations (1) and (4) as:

$$J_A = \frac{J}{[1-(f_{0T}/f_{n1})^2]\cdot F_1 \tan F_1} \qquad (5)$$

where J is the rotational inertia of rod portion of calibration sample, f_{0T} is the resonant frequency of the system shown in Fig. 2, f_{n_1} is the resonant frequency of the system with a calibration sample, and F_1 is a constant and is obtained from the following equation:

$$F_1 = 2\pi f_{n1} L/\sqrt{G/\rho} \qquad (6)$$

in which L is length of rod portion of calibration sample, G and ρ is the shear modulus and the density of the calibration rod material, respectively.

As it can be seen from equations (5) and (6), the accuracy of the value of J_A is based on the accuracy of the values of J and G determined for the calibration rod material. The value of J is determined from measurements of the small diameter of the calibration rod. Therefore, determination of a value for J is very sensitive to a minor error in measuring the diameter of the calibration rod. Furthermore, in some cases it is not easy to determine a shear modulus value for the calibration rod material.

The new proposed method for evaluating the value of J_A is not based on knowing values of J and G of the calibration rod. In this method, two calibration samples are used (Figs. 3 and 4). These two calibration samples are made of the same material and the rod section of the two samples must have the same dimensions. However, the top portions of two calibration samples should have different sizes as shown in Figures 3 and 4. These two different calibration samples will be called Calibration Sample I and Calibration Sample II, respectively.

The value of J_A can be obtained from measurements of the two resonant frequencies for the two calibration samples as follows:

For the Calibration Sample I, equation (1) becomes:

$$G=\rho(2\pi f_{n_1} L/F_1)^2 \qquad (7)$$

f_{n_1} is the resonant frequency for the system with Calibration Sample I. F_1 is obtained from the following equation which can be derived from equation (4):

$$F_1 \tan F_1 = \frac{J}{(J_1+\Delta J)[1-(f_{0T}/f_{n_1})^2]} \qquad (8)$$

J_1 is the rotational inertia of top portion of calibration sample and ΔJ is the rotational inertia of moving mass attached to the device (See Fig. 3).

For Calibration Sample II shown in Figure 4, equation (1) becomes

$$G=\rho(2\pi f_{n_2} L/F_2)^2 \qquad (9)$$

in ehich f_{n_2} is the resonant frequency for the system with Calibration Sample II. F_2 is obtained from the following

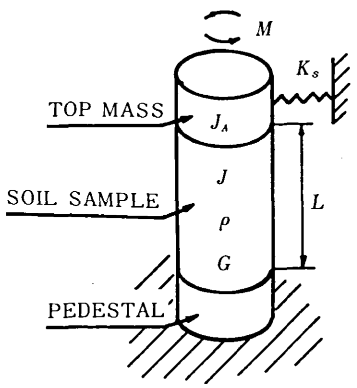


Fig. 1 Schematic figure of the system

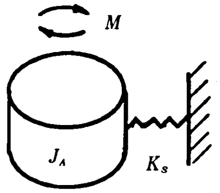


Fig. 2 System without sample

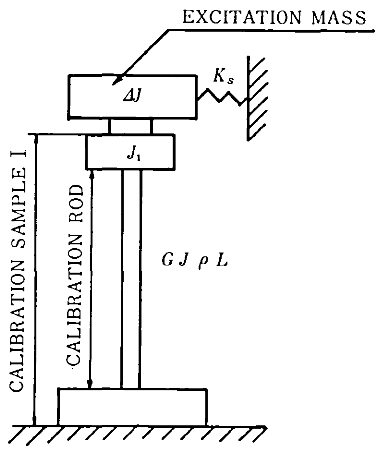


Fig. 3 System with calibration sample I

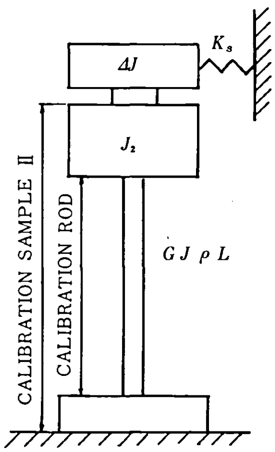


Fig. 4 System with calibration sample II

equation which can be obtained from equation (2):

$$F_2 \tan F_2 = \frac{J}{(J_2+\Delta J)-K_s/(2\pi f_{n2})^2} \quad (10)$$

where J_2 is the rotational inertia of top portion of the Calibration Sample II.

From equation (3) and from the expression $J_A = J_1+\Delta J$, we obtain

$$K_s = (2\pi f_{0T})^2 J_A = (2\pi f_{0T})^2 (J_1+\Delta J) \quad (11)$$

From equations (10) and (11), we obtain

$$F_2 \tan F_2 = \frac{J}{(J_2+\Delta J)-(J_1+\Delta J) f_{0T}^2/f_{n2}^2} \quad (12)$$

From equations (7) and (9) we obtain

$$F_1/F_2 = f_{n1}/f_{n2} \quad (13)$$

When the rotational inertia of the calibration rod is much less than the corresponding values of $J_1+\Delta J$ and $J_2+\Delta J$, values of F_1 and F_2 become much less than unity. We then get the following simplified equations:

$$F_1 \tan F_1 = F_1^2, \quad F_2 \tan F_2 = F_2^2 \quad (14)$$

From equations (8), (12), (13), and (14), we obtain:

$$(f_{n1}/f_{n2})^2 = (F_1/F_2)^2 = (F_1 \tan F_1)/(F_2 \tan F_2)$$
$$= \frac{(J_2+\Delta J)-(J_1+\Delta J)(f_{0T}/f_{n2})^2}{(J_1+\Delta J)[1-(f_{0T}/f_{n1})^2]} \quad (15)$$

After some calculation and by noting that $J_A = J_1+\Delta J$, we can obtain the expression

$$J_A = (J_2-J_1)/[(f_{n1}/f_{n2})^2-1] \quad (16)$$

Thus, we can obtain the value of the rotational inertia of the top mass (J_A) from equation (16) from only the values of the resonant frequencies f_{n1} and f_{n2} for two calibration samples and from calculated values of J_1 and J_2 determined from their dimensions and densities. It sould be noted that in this calibration method it is not necessary to know values of J, L and G for the thin calibration rod. Experience has shown that it is easier to determine accurate values of J_1 and J_2 for the top portion of the calibration samples than it is to determine values of J for the rod portion because values of J_1 and J_2 and determined from dimensions much larger than those of the calibration rod.

Note that equation (16) is not affected by the value of f_{0T} or the value of the torsional spring constant, K_s. Therefore, equation (6) is valid both for devices without a torsional spring and for those with a torsional spring having any value for K_s.

Hardin and Music (1965) derived equation (16) for the case where values of J for both calibration samples I and II were equal to zero. Therefore, the calibration method proposed in this paper combines methods proposed by Hardin and Music (1965) and methods proposed by Drnevich et al. (1978).

In using this calibration technique, the ratio J_1/J_2 should be chosen so that values of f_{n1} and f_{n2} are not close together. It is preferable to choose a ratio of J_1/J_2 so that the ratio of (f_{n1}/f_{n2}) is around 2.

Rotational Intertia of Top Mass (J_A) Measured by the Proposed Method

The methods described above for determining J_A were used to calibrate a Hardin-type resonant-column device (Hardin and Music, 1965). Two calibration samples were machined from high grade aluminum alloy (See Figs. 5 and 6). The resonant frequencies for two calibration samples were measured for a wide range of angular displacement values, θ, as shown in Figure 7. The shear strain amplitude γ shown in this figure was obtained from the following equation.

$$\gamma = D\theta/(3L) \quad (17)$$

in which θ is the angular single amplitude displacement, D is the diameter of the solid specimen which was taken to be equal to 6.1 cm and L is height of a solid specimen which was equal to 15.0 cm. These values of D and L were selected to give representative strain values obtained for soil in the Hardin-type resonant-column device.

It can be seen from Figure 7 that values of f_{n1} and f_{n2} were fairly constant. Slight decreases in values of f_{n1} and f_{n2} for strain values larger than around 5×10^{-5} were due to the non-linearity of K_s (See Fig. 8).

Values of J_1 and J_2 were determined from calibration sample dimensions and from their density, ρ, which was equal to 2.7 g/cm^3.

It was determined that

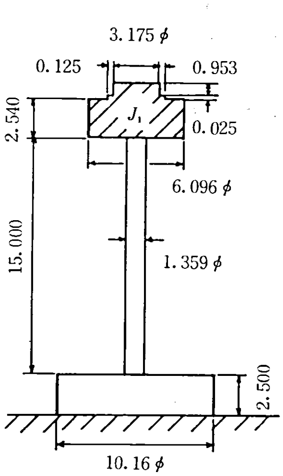
$$J_1 = 9.569 \times 10^2 \text{ g-cm}^2$$

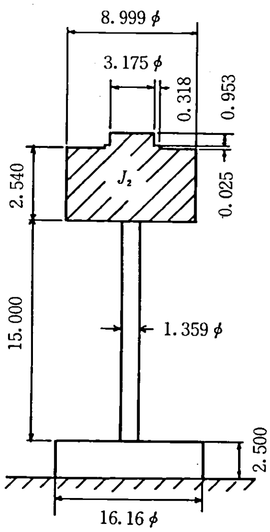
and $$J_2 = 8.720 \times 10^3 \text{ g-cm}^2$$

The value of J_A was obtained from equation (16) by using the values of J_1 and J_2 shown above and by using the values of f_{n1} and f_{n2} shown in Figure 7. The results are shown in Table 1. It can be seen that the value of J_A was fairly constant for a wide range of shear strain amplitude values, γ.

A value of J_A determined by the method proposed by Drnevich et al. (1978) was also determined by using the test data obtained from tests on Calibration Sample I. The rotational inertia of the calibration rod (J) in this perticular case was:

$$J = \frac{\pi}{32}\rho D^4 L = \pi/32 \times 2.7 \times 1.359^4 \times 15.0$$





(ALL DIMENSIONS IN CENTIMETERS)

Fig. 5　Calibration sample I　　Fig. 6　Calibration sample II

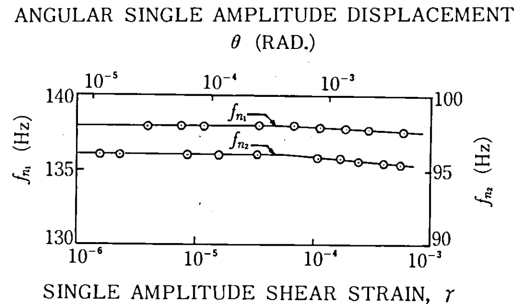


Fig. 7　Measured resonance frequency of the systems with calibration samples I and II

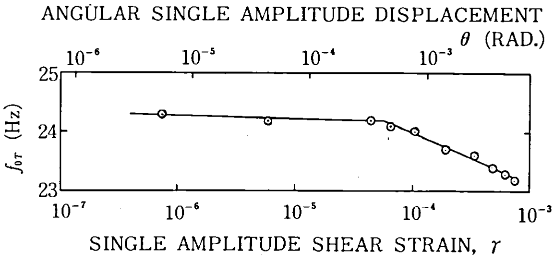


Fig. 8　Measured resonance frequency of the system without sample

Table 1　J_A by the method suggested in this paper

γ	f_{n_1} (Hz)	f_{n_2} (Hz)	J_A (g-cm^2)
10^{-6}	136.0	97.9	8.35×10^3
10^{-5}	136.0	97.9	8.35×10^3
10^{-4}	135.8	97.8	8.38×10^3
5×10^{-4}	135.4	97.6	8.40×10^3

Table 2　J_A by the method suggested by Drnevich et al[2] (1978)

γ	f_{0T} (Hz)	f_{n_1} (Hz)	F_1	J_A (g-cm^2)
10^{-6}	24.3	136.0	0.04095	8.35×10^3
10^{-5}	24.2	136.0	0.04055	8.35×10^3
10^{-4}	24.0	135.8	0.04077	8.37×10^3
5×10^{-4}	23.4	135.4	0.04077	8.41×10^3

$$= 13.562 \quad \text{g-cm}^2 \tag{18}$$

The shear modulus of aluminum was assumed to be 2.70 x 10^5 kgf/cm^2 or 2.646 x 10^7 kN/m^2. Thus, the value of the dimensionless frequency factor F_1 was obtained by equation (6) as

$$F_1 = 2\pi \times 15.0/\sqrt{2.7\times10^8\times980/2.7}\times f_{n_1}$$
$$= 3.011\times10^{-4} f_{n_1} \tag{19}$$

in which the value of f_{n_1} was obtained from Figure 7. The values of J_A were obtained by equation (5) using values of F_1 from equation (19), f_{0T} from Figure 8, f_{n_1} from Figure 7, and J from equation (18). The results are shown in Table 2. By comparing the values of J_A in Tables 1 and 2, it can be seen that both methods gave approximately the same value of J_A

Conclusions

This paper describes a new method for evaluating the rotational inertia of the top mass of resonant-column device (J_A). By this method, a value of J_A can be determined simply without having to know the material properties of the calibration specimen.

Acknowledgements

This study was performed while the senior author was a visiting professor at the University of Illinois at Chicago Circle. The authors wish to thank the stuff for their help in this work.　(Manuscript received September 21, 1979)

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