研究速報
UDC 699. 841. 059. 4

Seismic Reliability Analysis of Lifeline Systems (2)

ライフラインの耐震信頼性解析 －第2報－

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Summary

In the previous reports[5)] we have discussed a methodology for the seismic risk assessment of lifeline system, by using the terminal reliability and the flow reliability as a measure of seismic risk. This report deals with a measure of seismic importance for lifeline systems, the main idea of which is based on a sensitivity analysis of terminal reliability. Some general characteristics of the "Importance" are also clarified.

1. Introduction

Prior to the discussion of "measures of importance" in reliability theory, we consider the system structure to be described in the following manner. Here we will distinguish between only two states: a functioning state and a failed state. This dichotomy applies to the system as well as each component. To indicate the state of the ith component, we assign a binary indicator variable x_i to the ith component:

$$x_i=\begin{cases}1 & \text{if the } i\text{th component is functioning,}\\ 0 & \text{if it is failed,}\end{cases} \qquad (1)$$

for $i=1, 2, \ldots, n$, where n is the total number of components in the system.

Similarly, the indicator variable ϕ denotes the state of the system:

$$\phi=\begin{cases}1 & \text{if the system is functioning,}\\ 0 & \text{if it is failed.}\end{cases} \qquad (2)$$

As the state of the system is determined only by each state of component, we can write

$$\phi=\phi(\boldsymbol{x})$$

where $\boldsymbol{x}=(x_1, x_2, \cdots, x_n)$.

The binary function ϕ is called the *structure fuction* of the system.

When the structure function $\phi(\boldsymbol{x})$ satisfies the following condition:

if $\boldsymbol{x}^1\leqq\boldsymbol{x}^2$, $(x_i^1\leqq x_i^2$ for $\forall i)$,

then $\phi(\boldsymbol{x}^1)\leqq\phi(\boldsymbol{x}^2)$,

we call the system a *coherent system*. In the other words, this condition means that to improve the performance of any component never causes the system to deteriorate.

If the following equations holds,

$$\phi(1_i, \bar{\boldsymbol{x}})^{***}-\phi(0_i,\bar{\boldsymbol{x}})=1 \quad \text{and}$$

$$\phi(1_j,\bar{\boldsymbol{x}})-\phi(0_j, \bar{\boldsymbol{x}})=0,$$

then we may conclude that the ith component is more important than the jth one. Because the ith component is very critical for the survival of the system. Whether the jth component survives or fails, it is irrelevant to the system's survival. So we define the *criticality* of the ith component such that

$$C(i)\triangleq E[\phi(1_i, \bar{\boldsymbol{x}})-\phi(0_i, \bar{\boldsymbol{x}})]$$

$$=h(1_i, \bar{\boldsymbol{p}})-h(0_i, \bar{\boldsymbol{p}})^{****} \qquad (3)$$

where the function h is the system reliability function; that is,

$h=E[\phi(\boldsymbol{x})]$.

We find that the criticality is the difference between two conditional reliability; one is the system reliability with the ith component completely success and the other is that with the ith component fail.

Now the structure function can be decomposed into the following, with the ith component as a pivot,

$$\phi(\boldsymbol{x})=x_i\phi(1_i, \bar{\boldsymbol{x}})+(1-x_i)\phi(0_i, \bar{\boldsymbol{x}}) \qquad (4)$$

The above decomposition is called the *pivotal decomposition.* If any components are statistically independent of each other, by taking the expectation of both sides of eq. (4), we have

$$h(\boldsymbol{p})=E[\phi(\boldsymbol{x})]=p_i h(1_i, \bar{\boldsymbol{p}})+q_i h(0_i, \bar{\boldsymbol{p}}) \qquad (5)$$

And we may derive the eq. (6) from eq. (5).

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$\bar{x}$*** : n-1 dim. binary vector which corresponds to n-1 component states.
$\bar{p}$**** : n-1 dim. probability vector which corresponds to the state.

$$\frac{\partial h}{\partial p_i}=h(1_i, \bar{p})-h(0_i, \bar{p}) \qquad (6)$$

When the system is assumed to be statistically independent, the criticality may be redefined such that

$$C(i) \triangleq \frac{\partial h}{\partial p_i} \qquad (7)$$

Additionally, from eq. (6), it is also clearly found that

$0 \leq C(i) \leq 1$ for $\forall i$.

2. Measures of Seismic Importance

In the last report, we numerically evaluated the seismic risk of the lifeline system with the terminal reliability as one representative index of system performance. Here we make most of results in the last report and produce some measure that enables us to rank each facility in the lifeline system in the order of its importance.

Though the criticality mentioned in the previous section represents the importance of each component, we modify it into a new measure of importance.

Def. 1

The importance S_i is defined such that

$$S_i=\frac{p_i}{h(\boldsymbol{p})}\frac{\partial h}{\partial p_i}=\left(\frac{\partial \ln h(\boldsymbol{p})}{\partial \ln p_i}\right) \qquad (8)$$

where h and p_i denote the system reliability and the reliability of the ith component, respectively.

It is found from eq. (8) that S_i is produced by weighting a component reliability on the criticality and normalizing it with the system reliability, and the S_i is only the logarithmic form of the criticality. In the physical meanig, we understand that S_i means a contribution of the improvement of the ith component reliability to that of system reliability.

Another proposed measure of importance is the dual concept of S_i in the mathematical sense, which is shown in the following definition.

Def. 2

The dual measure of importance, denoted by S_i', is defined such that

$$S_i'=\frac{q_i}{\bar{h}(\boldsymbol{q})}\frac{\partial \bar{h}(\boldsymbol{q})}{\partial q_i}\left(=\frac{\partial \ln \bar{h}(\boldsymbol{q})}{\partial \ln q_i}\right) \qquad (9)$$

where $\bar{h}$ and q_i denote the system unreliability $(1-h)$ and the ith component unreliability, respectively.

module* : a kind of disjoint subset. See Refs. (1) and (2).

Just contrary to Def. 1, S_i' means a contribution of the deterioration of the ith component reliability to that of the system reliability.

Now to the same model lifeline showed in the previous report[5] we apply these two measures of importance for earthquake resistance character of lifeline systems.

Table 1 and Fig. 1 show each component importance S_i for the pair $S_1 \rightarrow D_1$ to be connective, where the node reliable case and the node vulnerable case are compared with each other. Table 2 and Fig. 2 give the results of dual importance S_i' for the same pair.

Table 1

		Node reliable case	Node vulnerable case
Terminal Reliability		0.631	0.379
Importance Ranking	1	Link 13 (1.0)	Link 13 Node 16 (1.0)
	2	Link 1 (0.572)	Link 1 Node 2 (0.581)
	3	Link 5 (0.271)	Link 5 (0.340)
	4	Link 4, 8 (0.143)	Link 4, 8 Node 5 (0.194)
	5	Link 2, 9 (0.091)	Link 2, 9 Node 3, 7 (0.073)
	6	Link 6 (0.057)	Link 6 (0.051)
	7	Link 3, 7, 10 (0.007)	Link 3, 7, 10 Node 4, 8 (0.005)
	8	Link 11, 12, 14, 15, 16, 17, (0.0)	Link 11, 12, 14, 15, 16, 17 (0.0) Node 9

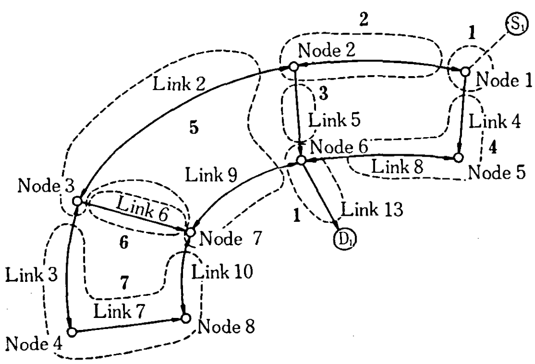
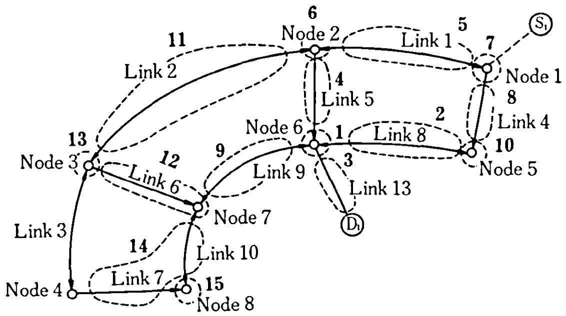


Table 2

Importance Ranking by S_i' (node vulnerable case: $S_1 \rightarrow D_1$)

Terminal Reliability		0.379	
Importance Ranking	1	Node 6	(0.193)
	2	Link 8	(0.158)
	3	Link 13	(0.141)
	4	Link 5	(0.089)
	5	Link 5	(0.087)
	6	Node 2	(0.068)
	7	Node 1	(0.053)
	8	Link 4	(0.034)
	9	Link 9	(0.024)
	10	Node 5	(0.022)
	11	Link 2	(0.012)
	12	Link 6 Node 7	(0.008)
	13	Node 3	(0.004)
	14	Link 10, 7	(0.002)
	15	Node 8	(0.001)
	16	others	(0.0)



3. General Characteristics of Importances

Suppose a system of n components which contains a module* of m components. Let P_M be the reliability of the module, then we write

$$P_M = \begin{cases} \prod_{j=1}^{m} p_j & \text{; if the module is a series subsystem,} \\ 1 - \prod_{j=1}^{m} q_j & \text{; if it is a parallel subsystem.} \end{cases} \quad (10)$$

whether the module is in parallel or in series, the variable p_i (i = 1, 2, . . . , m) in h must always appear in the lumped form. If the kth component is in the module, for the above reason

$$\frac{\partial h}{\partial p_k} = \frac{\partial h}{\partial P_M} \frac{\partial P_M}{\partial p_k}. \quad (11)$$

From Def. 1, we have

$$S_k = \begin{cases} \dfrac{\partial h}{\partial P_M} \dfrac{P_M}{h} & \text{; in the series module} \\ \dfrac{\partial h}{\partial P_M} \dfrac{1-P_M}{h} \left\{ \dfrac{1}{q_k} - 1 \right\} & \text{; in the parallel module.} \end{cases} \quad (12)$$

so we find the following characteristics.

> Proposition I:
> (1) The importance S_i is always constant in any series module.
> (2) In any parallel module, the ith component is more important than the jth, if $p_i > p_j$.

((Proof))

Eq. (11) means that $S_1 = S_2 \ . \ . \ . = S_M$ in the series module. And in the parallel module,

$$S_i - S_j = \frac{p_i - p_j}{q_i q_i} \frac{1-P_M}{h} \frac{\partial h}{\partial P_M}$$

As the derivative $\frac{\partial h}{\partial P_M}$ should be surely non-negative, the following inequality holds:

$$S_i \geq S_j \quad \text{if} \quad p_i > p_j \ (i, j = 1, 2, \cdots, m)$$

(Q.E.D.)

As mentioned in the previous section, S_i' is a dual concept of S_i. This duality yields the following proposition.

> Proposition II:
> (1) The dual importance S_i' is always constant in any parallel module.
> (2) In any series module, the less reliable component is the more important; that is, $S_i' \leq S_j'$ if $p_i > p_j$.

Next we try to see the relationship between two kinds of reliability importance S_i and S_i'. The next proposition will be also derived from previous discussions.

> Proposition III:
> (1) If $p_i < h$, then $S_i \leq S_i'$.
> (2) If $p_i > h$, then $S_i \geq S_i'$.

((Proof))

$$S_i - S_i' = \left\{ \frac{p_i}{h} - \frac{1-p_i}{1-h} \right\} \frac{\partial h}{\partial p_i}$$

$$= \frac{p_i - h}{h(1-h)} \frac{\partial h}{\partial p_i}$$

where we use $\frac{\partial h}{\partial p_i} = \frac{\partial \overline{h}}{\partial q_i}$ The sign$(S_i - S_i')$ is the same as the sign $(p_i - h)$, because $\frac{\partial h}{\partial p_i} \geq 0$.

(Q.E.D.)

4. Conclusion

In the report, we have been discussing the measures of importance and their general characteristics. Some numerical examples of importance ranking in the lifeline system are also exhibited. By using such an importance ranking, we would be able to improve the aseismicity of lifeline system effectively.

(Manuscript received, October 23, 1978)

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