研　究　速　報

UDC 539.215
539.37.01

Stress-Strain Behavior by a Simple Elasto-Plastic Theory for Anisotropic Granular Materials 1 (Theory)

簡単な弾塑性理論による異方性のある粒状体の応力-歪特性 I（理論）

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1. Introduction

This paper presents a simple elasto-plastic theory for granular materials as a preliminary one incorporating the inherent anisotropy and the stress-system induced anisotropy. Several elasto-plastic theories to describe stress-strain characteristics of soils have been developed. Among them, Roscoe and his colleagues[11,13] originally developed the Cam Clay Model for stress-strain behaviors of cohesive soils and the Granta Gravel Model for those of cohesionless soils. In these models, soils are treated as isotropic work-hardening elasto-plastic materials, incorporating associated flow rule.

Poorooshasb et al.[8,9,10] have proposed another elasto-plastic theory for cohesionless soil based on a comprehensive series of tests in triaxial stress condition. They found that while plastic potential functions exist, they do not coincide with yield function. This means that normality or associated flow rule can not be applied to cohesionless soils. Besides this, they established an empirical hardening function from shear tests, not from consolidation test (Fig. 1). Tatsuoka[14] showed that the ratios of the components of the plastic strain increments for the q/p' = constant consolidation case are different from those for the

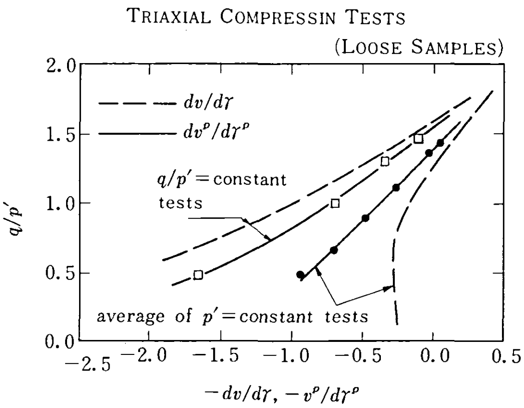

Fig. 2　The ratios of the components of the strain increments by q/p'=constant tests (p' =0.1~3.0kg/cm^2) and p'=constant tests (the measured values in the figure are the averaged ones of p'=1.0, 2.0 and 3.0kg/cm^2) on Fuji River Sand (Tatsuoka 1972)

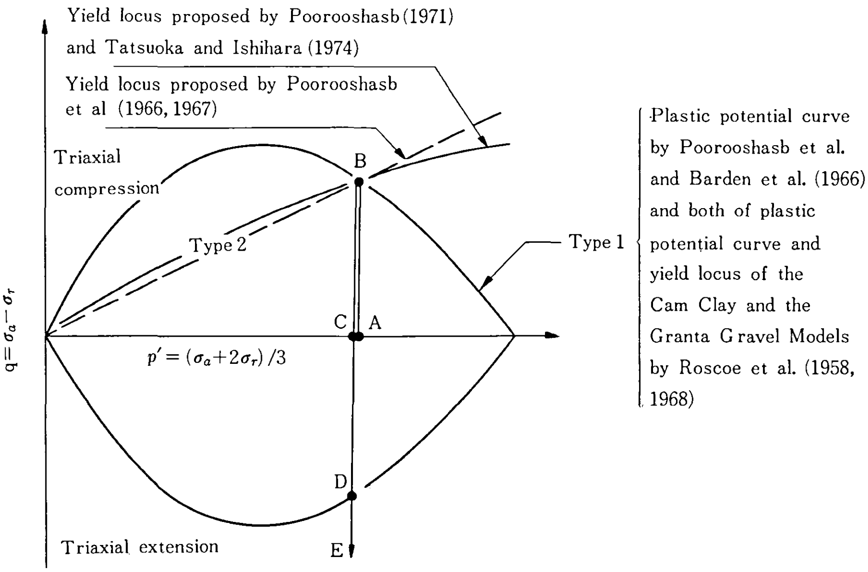

Fig. 1　Yield loci and plastic potential curves

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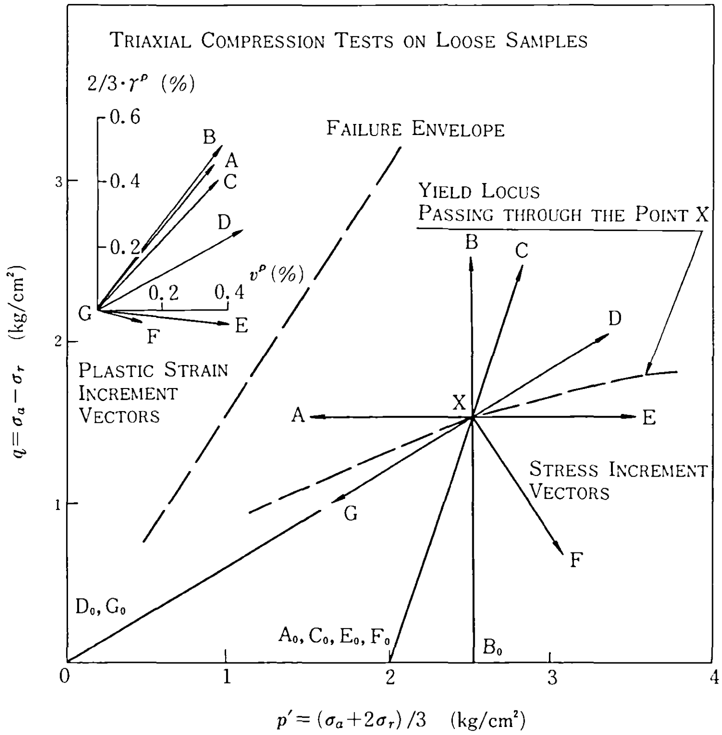

Fig. 3 The strain increment vectors by various stress increment probes for Fuji River Sand (Note that the strain increment vectors are not normal to the yield locus concerned.) (Tatsuoka and Ishihara (1974 a))

p' = constant shear case, especially for the low stress ratio as shown in Fig. 2. For the stress increments in which the increase in mean principal stress is relatively small, the ratios of the components of the plastic strain increments can be considered relatively independent of the ratios of the components of the stress increments as shown in Fig. 3. It may be induced from the facts described above that the unique plastic potential exists for plastic strains caused only by shearing, but if the plastic strain caused by the increase in mean principal stress is involved, the ratios of the components of the plastic strain increments depend on the ratios of the components of the stress increments.

In other words, we can divide the plastic strain increment $d\varepsilon^p$ as

$$d\varepsilon^p = d\varepsilon_s^p + d\varepsilon_c^p \qquad (1)$$

in which $d\varepsilon_s^p$ means the plastic strain increment caused by the shear process and $d\varepsilon_c^p$ means the plastic strain incre – ment caused by the consolidation process. In this paper, only $d\varepsilon_s^p$ will be treated.

Rowe and his colleagues[2),12)] have shown that the ratios of the components of the strain increments or the plastic strain increments caused by the shear process can be uniquely related with the stress ratio. In the triaxial compression case, this relationship is

$$\sigma_1/\sigma_3 = -K(2d\varepsilon_3/d\varepsilon_1) \qquad (2)$$

in which K is a material constant not affected by σ_3 and void ratio. Oda[7)] have shown that K is also independent of inherent anisotropy. From Equation (2), the plastic potential function for the triaxial case can be derived as

$$d\varepsilon_1 = (\partial\Psi/\partial\sigma_1)d\lambda,\ \ d\varepsilon_3 = (\partial\Psi/\partial\sigma_3)d\lambda,$$
$$\Psi = (\sigma_1^{2K}/\sigma_3) \qquad (3)$$

in which $d\lambda$ is a scalar.

Lade and Duncan[4)] performed cubical triaxial tests with general three-dimensional stress conditions on cohesionless sand and proposed an elasto-plastic model which can be used in general three-dimensional stress conditions. This model, in principles, follows that by Poorooshasb et al. This model uses the non-associated flow rule and the yield function f, the plastic potential function g and the empirical work-hardening law dW_p are expressed by stress invariants as

$$f = I_1^3/I_3,\ \ g = I_1^3 - \kappa_2 I_3$$
$$dW_p = a\cdot df/(1 - r_f(f - f_t)/(\kappa_1 - f_t))^2 \qquad (4)$$

(a, r_f, κ_1, f_t are specimen constants)

where $I_1=\sigma_1+\sigma_2+\sigma_3$ and $I_3=\sigma_1\sigma_2\sigma_3$ The hardening function dW_p is expressed by the isotropic stress function which can not express the inherent anisotropy as described later. The yield function f is expressed by the stress ratio of σ_a/σ_r or q/p' in the triaxial compression case as those by Poorooshasb et al. But it is also apparant that this yield function expresses the isotropic yielding because the yield surface expands isotropically from the hydrostatic axis of $\sigma_1=\sigma_2=\sigma_3$.

Recently, Oda[6),7)] and Authur and Menzies[1)] have shown that sands have different deformabilities and strengths for different directions even for the same combination of principal stresses or for the same stress invariants. This is called inherent anisotropy. This means that the deformation of sands having inherent anisotropy can not be expressed only by stress invariants, but it is necessary to incorporate some parameters expressing inherent anisotropic into the hardening function.

Tatsuoka and Ishihara[16)] have shown that sand yields not isotropically but anisotropically as follows. Assume that a sample is sheared reversingly in the triaxial compression and extension stress conditions as schematically illustrated in Fig. 1. Assume that this sample have reached the stress condition A only by the isotropic consolidation. Then this sample is sheared and unloaded as A → B → C. If the yielding of this sample is isotropic and expressed by the yield function 1 in Fig. 1, this sampel starts yielding at the stress point D when sheared as C → D → E. The actual sand specimen, however, starts yielding at the stress point C. It was also shown by them that when the strain caused during the stress path A → B is not excessive, the deformability of the specimen is almost identical to that of the virgin specimen which is sheared as A → D → E without the stress history of A → B → C. On the other hand, when the strain caused during the stress path A → B is excessive, the deformability of the specimen is larger than that of the virgin specimen. This performance is called induced anisotropy.

2. Fundamental Postulates

The following fundamental postulates will be utilized to develop an elasto-plastic theory for granular materials.

(i) Strain increment $d\varepsilon$ can be divided into two parts as

$$d\varepsilon=d\varepsilon^e+d\varepsilon^p$$

where $d\varepsilon^e$ means the elastic part and $d\varepsilon^p$ means the plastic part. It is to be noted that this is not necessarily true when the stress-strain curves during unloading and reloading make hysteresis loops. For the simplicity, $d\varepsilon^e$ will be neglected and $d\varepsilon^p$ will be denoted as $d\varepsilon$ hereafter.

(ii) Following Matsuoka[5)], it will be assumed that each of three principal plastic strain increments $d\varepsilon_i$, $d\varepsilon_j$ and $d\varepsilon_k$ in the three-dimensional stress condition of $\sigma_i>\sigma_j>\sigma_k$ can be divided into two parts as

$$d\varepsilon_i=d\varepsilon_{i_{ij}}+d\varepsilon_{i_{ik}},\ d\varepsilon_j=d\varepsilon_{j_{ij}}+d\varepsilon_{j_{jk}},\ d\varepsilon_k=d\varepsilon_{k_{ik}}+d\varepsilon_{k_{jk}} \quad (5)$$

Note that σ_i, σ_j and σ_k are not necessarily σ_1, σ_2 and σ_3, respectively, but these mean three principal stresses, each of which has the fixed direction such as the vertical or horizontal direction, for example. In this study, the case where the directions of principal stress rotate continuously is not treated. $d\varepsilon_{i_{ij}}$ and $d\varepsilon_{j_{ij}}$ mean the major and minor strain increment components in the idealized two-dimensional slipping caused by the stress system of $\sigma_i>\sigma_j$, respectively. In total, six different idealized two-dimensional slippings can be possible as

$\sigma_i>\sigma_j$, $\sigma_j>\sigma_i$, $\sigma_j>\sigma_k$, $\sigma_k>\sigma_j$, $\sigma_k>\sigma_i$ and $\sigma_i>\sigma_k$

for a given element under the three- dimensional stress condition (σ_i, σ_j and σ_k).

(iv) Each of the strain increment components in the two-dimensional slippings is derived as

$$d\varepsilon_{i_{ij}}=(\partial\Psi_{ij}/\partial\sigma_i)d\lambda_{ij},\ d\varepsilon_{j_{ij}}=(\partial\Psi_{ij}/\partial\sigma_j)d\lambda_{ij}$$
$$d\varepsilon_{i_{ji}}=(\partial\Psi_{ji}/\partial\sigma_i)d\lambda_{ji},\ d\varepsilon_{j_{ji}}=(\partial\Psi_{ji}/\partial\sigma_j)d\lambda_{ji} \quad (6)$$

Ψ_{ij} and Ψ_{ji} mean the plastic potential functions for shear process and can be expressed as

$$\Psi_{ij}=\sigma_i^K/\sigma_j,\ \Psi_{ji}=\sigma_j^K/\sigma_i \quad (7)$$

These are based on Equation (3). The parameter K can be considered independent of void ratio, confining pressure and inherent anisotropy. It will be shown later that K is affected by the intermediate principal stress as pointed out by Barden et al.[2)] $d\lambda_{ij}$ and $d\lambda_{ji}$ in Equation (6) are scalars and denoted as

$$d\lambda_{ij}=h_{ij}\cdot df_{ij},\ d\lambda_{ji}=h_{ji}\cdot df_{ji} \quad (8)$$

h_{ij} and h_{ji} are the hardening functions and will be derived empirically later as

$$h_{ij}=\alpha_{ij}/(K\cdot\sigma_i^{K-1}/\sigma_j\cdot(1-\beta_{ij}(\sigma_i/\sigma_j-1))^2),$$
$$h_{ji}=\alpha_{ji}/(K\cdot\sigma_j^{K-1}/\sigma_i\cdot(1-\beta_{ji}(\sigma_j/\sigma_i-1))^2) \quad (9)$$

As shown later, the inherent anisotropy will be expressed by the different values of α_{ij} and β_{ij} for the different directions. f_{ij} and f_{ji} are the yield function and can be expressed as

$$f_{ij}=F_{ij}\cdot(\sigma_i/\sigma_j),\ f_{ji}=F_{ji}\cdot(\sigma_j/\sigma_i) \quad (10)$$

F_{ij} and F_{ji} are affected by the confining pressure and the intermediate principal stress or the parameter

$$b=(\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$$

for a given specimen. In general, F_{ij} increases with the increase in the mean principal stress and have smaller values for $1>b>0$ than for $b=1.0$ or $b=0.0$. How the b value affects F_{ij} is not clarified yet. It will be assume hereafter that

$$F_{ij}=1.0 \quad \text{for} \quad \sigma_i+\sigma_j+\sigma_k=\text{a constant} \qquad (11)$$

Equation (9) can be derived from the empirical hyperbolic stress-strain postulate for monotoneous loadings as

$$\sigma_i/\sigma_j=1+(\varepsilon_{i_{ij}}/(\alpha_{ij}+\beta_{ij}\varepsilon_{i_{ij}})) \qquad (12)$$

The parameter $1/\alpha_{ij}$ means the initial tangent in the $\sigma_i/\sigma_j\sim\varepsilon_{i_{ij}}$ relation and the parameter $1/\beta_{ij}$ means the ultimate value of σ_i/σ_j at $\varepsilon_{i_{ij}}=\infty$ Both α_{ij} and β_{ij} can be considered to be considerably affected by void ratio, inherent anisotropy and the b-parameter. The confining pressure affects α_{ij} but does not affect β_{ij} considerably. From Equation (12),

$$d\varepsilon_{i_{ij}}=\alpha_{ij}/(1-\beta_{ij}(\sigma_i/\sigma_j-1))^2\cdot d(\sigma_i/\sigma_j) \qquad (13)$$

And from Equations (6), (7), (8), (10) and (11)

$$d\varepsilon_{i_{ij}}=K\cdot\sigma_i^{K-1}/\sigma_j\cdot h_{ij}\cdot d(\sigma_i/\sigma_j) \qquad (14)$$

Equations (13) and (14) give Equation (9). The parameters

Table 1　Parameters in the theory

Parameter	Void Ratio	Mean Stress	Inherent Anisotropy	$b=\frac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3}$
K	X	X	X	A
F_{ij}	A	A	A	A
α_{ij}	A	A	A	A
β_{ij}	A	X	A	A

A: affected by this factor,
X : not or negligibly affected by this factor.

which appear in the theory is summarized in Table 1.

3. Acknowledgements

The author is grateful to Miss Michie Torimitsu of Institute of Industrial Science, University of Tokyo for her laborious work of typing the manuscript.

(Manuscript received April 12, 1978)

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正　誤　表（6月号）

頁	段	行	種 別	正	誤
229	左	↓ 14	本 文	実験結果と対比して	実験結果との対比して
232	左	↓ 10	本 文	不備を補えば	不備を捕えば